Dipendenza dal flavour delle distribuzioni TMD non polarizzate

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Scopo

Estrazione della dipendenza dal flavour

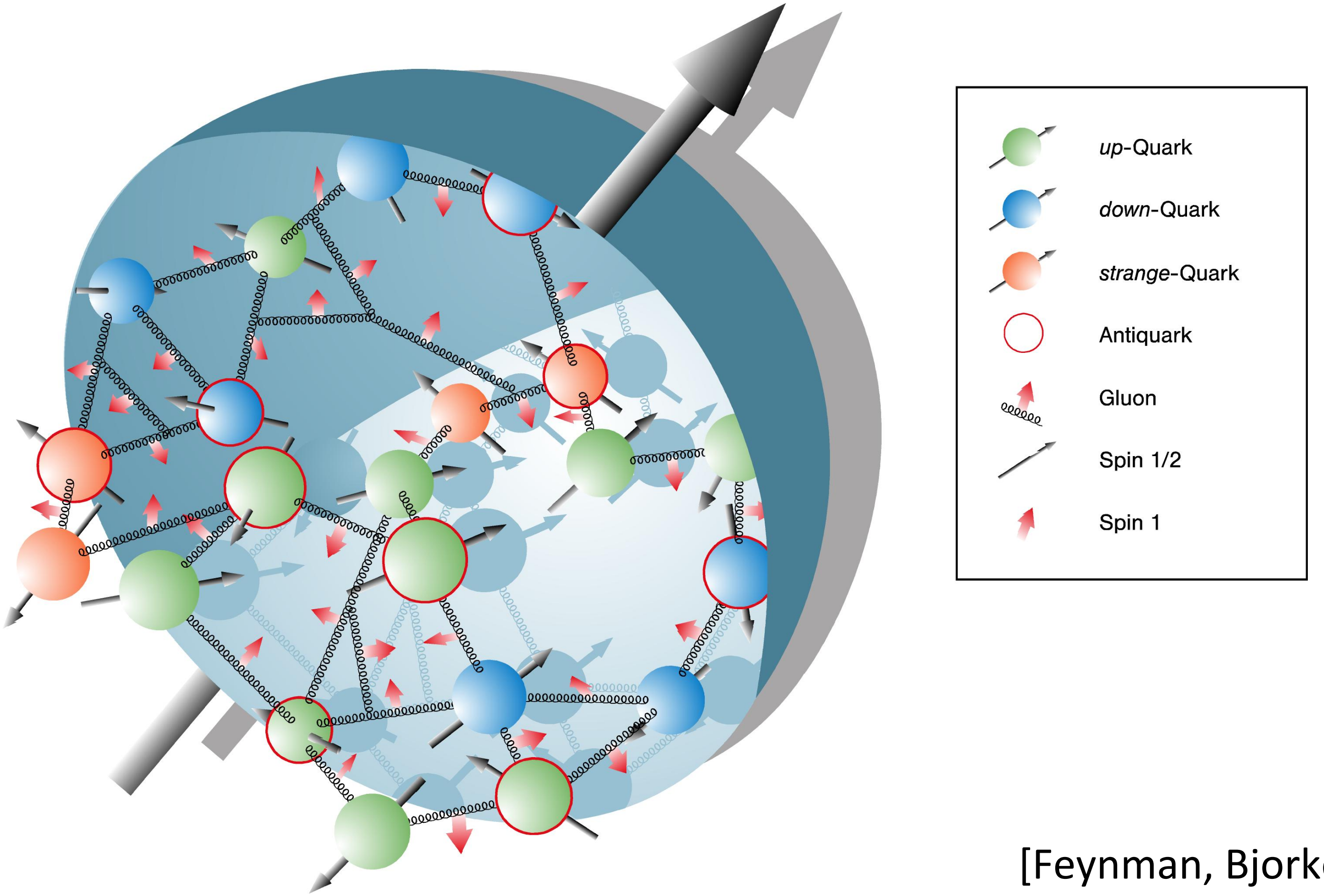
delle distribuzioni partoniche

dipendenti dal momento trasverso (TMDs)

da produzione di adroni semi-inclusiva

all’esperimento COMPASS

Quark Parton Model



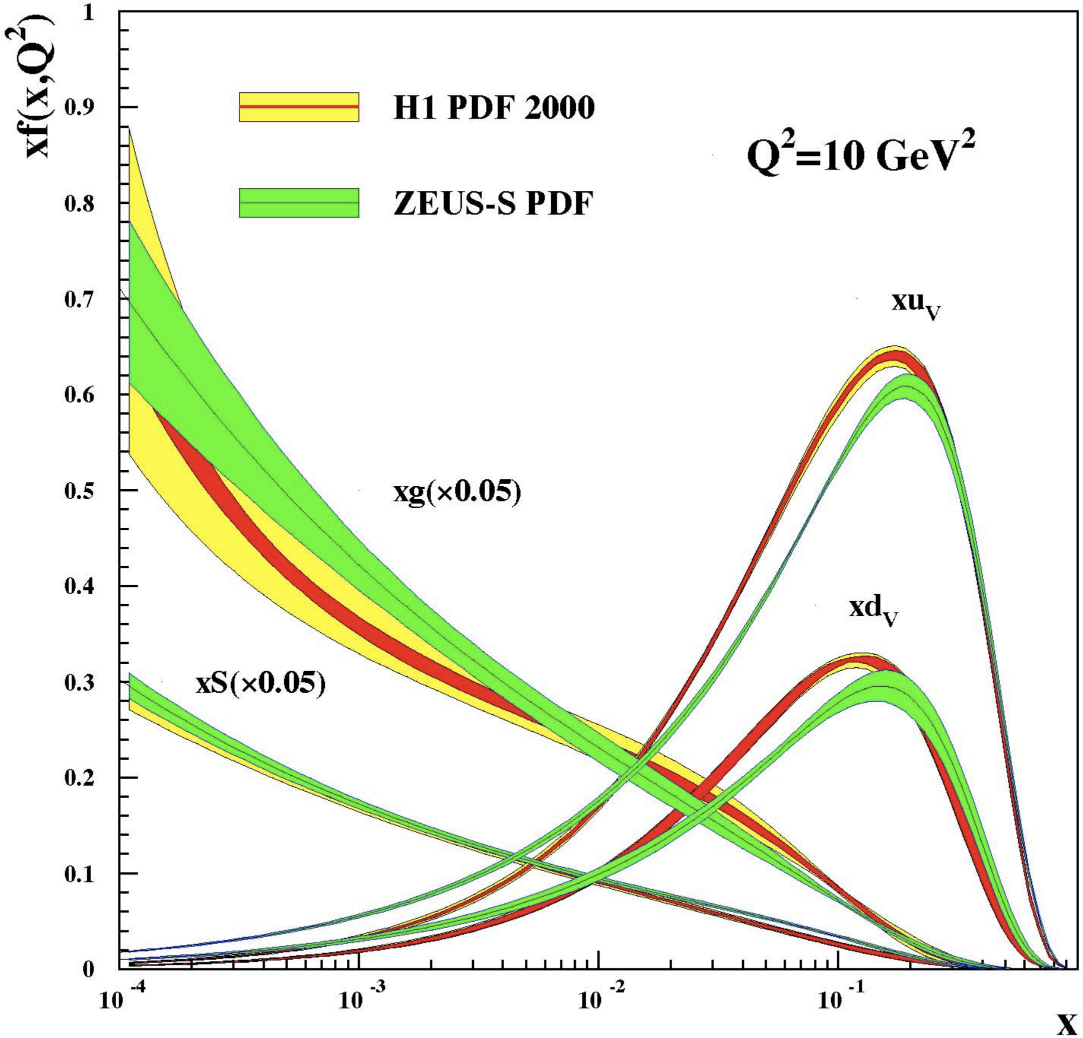
[Feynman, Bjorken '60-'70]

Che cosa sono le

transverse-momentum-dependent distributions

(TMDs) ?

Distribuzioni partoniche (PDFs) collineari

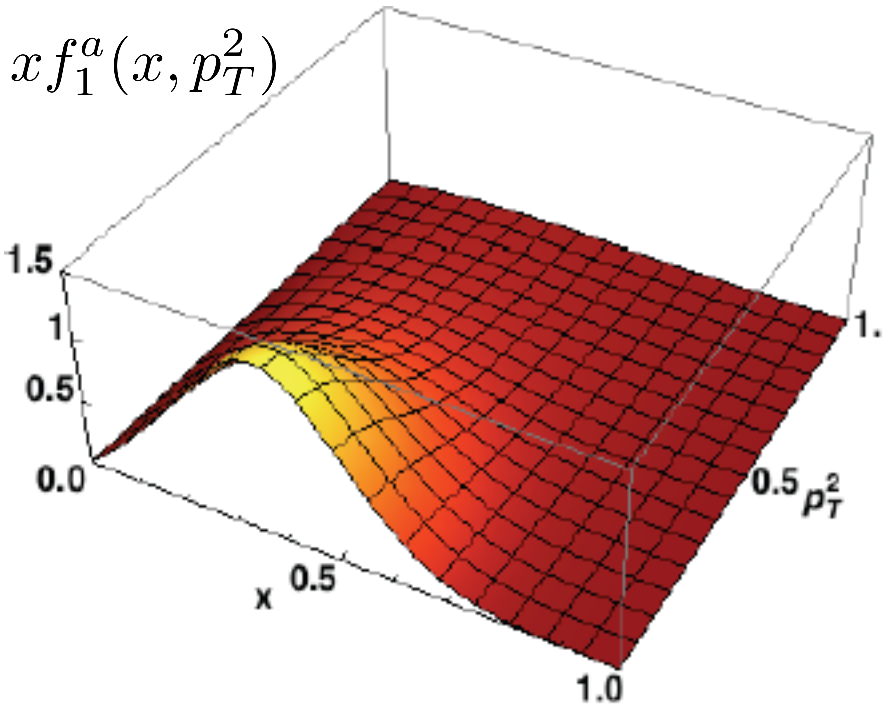


PDFs: struttura **monodimensionale** del protone nello spazio degli impulsi

$$f_1^a(x)$$

Densità di probabilità di trovare un partone con flavour a all'interno del protone

TMD PDFs



TMD PDFs: analisi della struttura **3-dimensionale** del protone nello spazio degli impulsi

$$f_1^a(x, p_T^2)$$

"Transverse-momentum distributions in a diquark spectator model"
PhysRevD.78.074010

A cosa servono le TMDs ?

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- Tomografia 3-dim degli adroni

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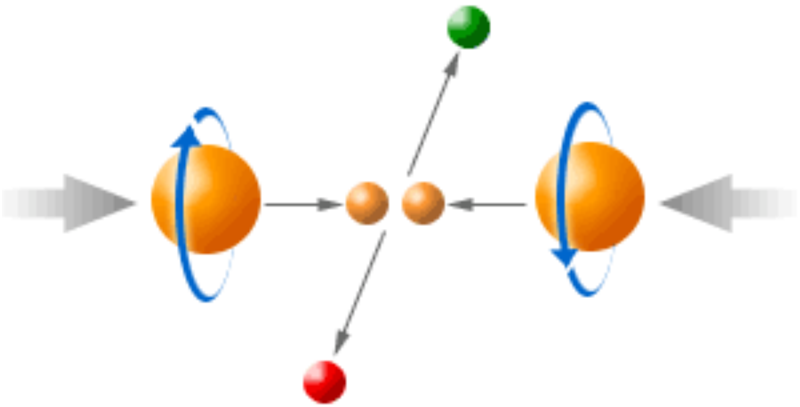
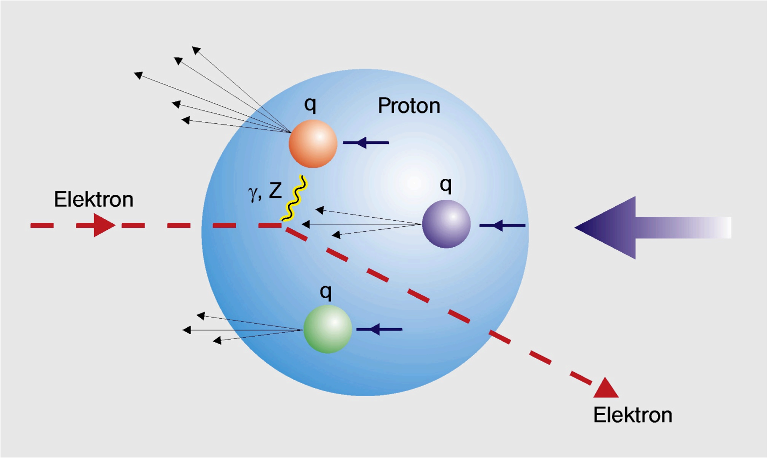
- Tomografia 3-dim degli adroni

- Strumento fondamentale per la fisica delle alte energie in generale





Come accedere alle TMDs ?

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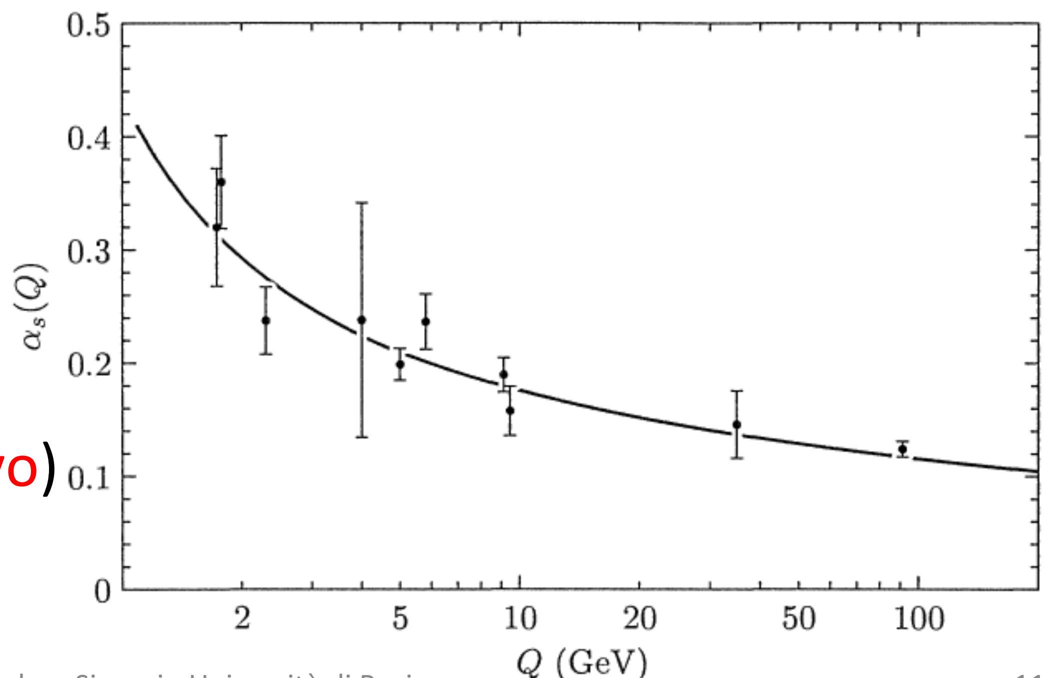
- Alla scala della massa del protone la QCD non è attualmente calcolabile sulla base di metodi perturbativi

$$\alpha_S(1\ \mathrm{GeV}) \sim 0.35$$

(problema non perturbativo)



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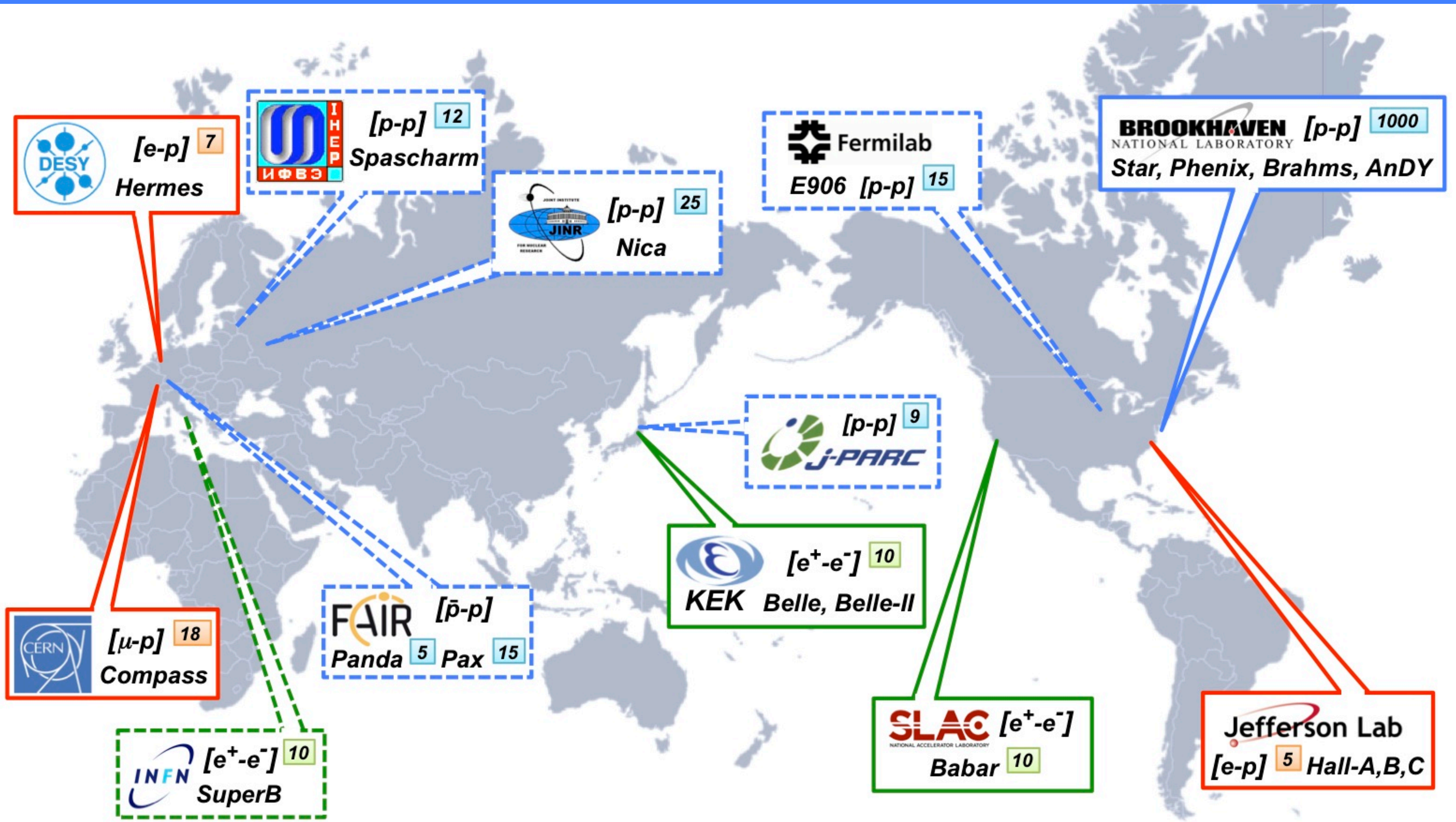
- Scegliamo di estrarre informazioni dai dati sperimentali con procedure di fit

Come accedere alle TMDs ?



Come accedere alle TMDs ?

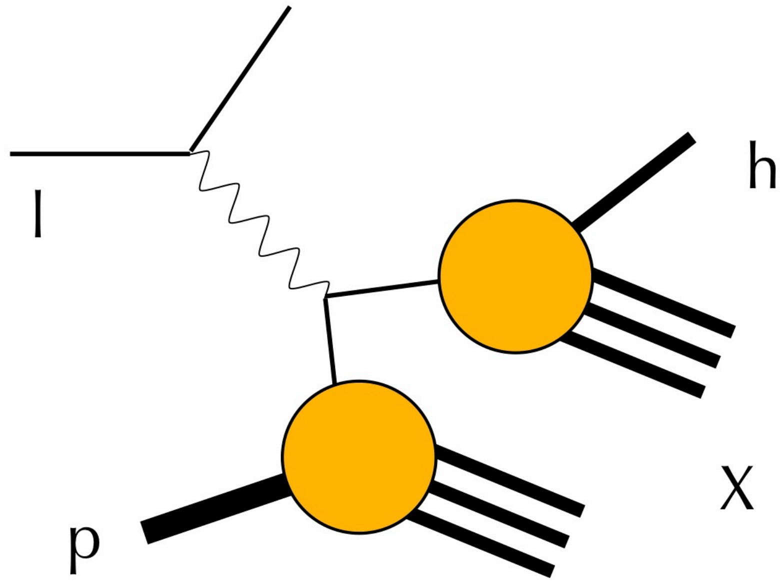
Analisi di scattering profondamente anelastico semi-inclusivo (SIDIS) di muoni non polarizzati

su nucleoni non polarizzati

all'esperimento COMPASS

Come accedere alle TMDs ?

Analisi di scattering profondamente anelastico semi-inclusivo (SIDIS) di muoni non polarizzati su nucleoni non polarizzati all'esperimento COMPASS

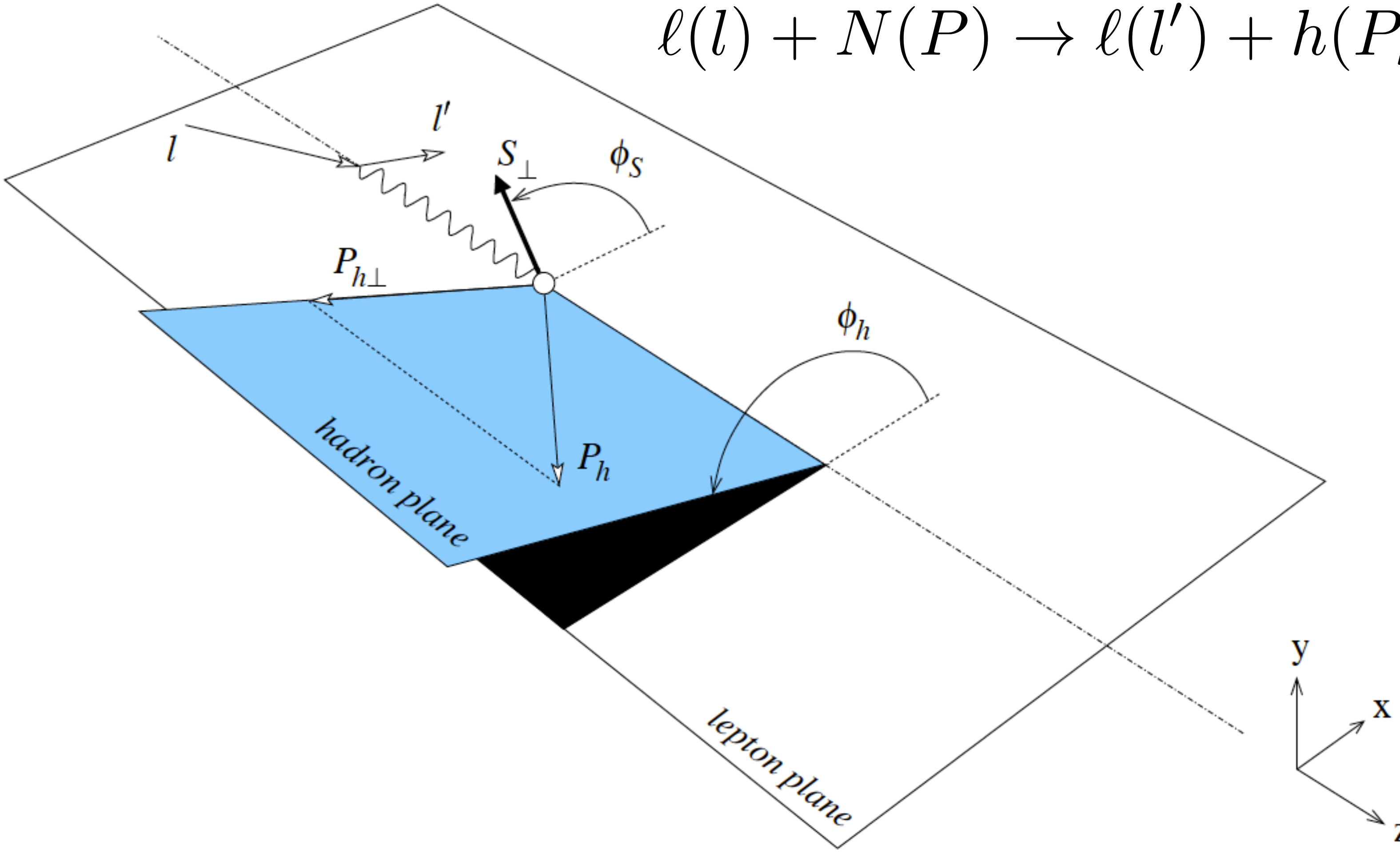


SIDIS $$\frac{d\sigma}{dP_{h\perp}^2} \sim \sum_a e_a^2 \; f_1^a(x, p_T^2) \otimes D_1^a(z, K_T^2)$$

Semi-inclusive
Deep Inelastic Scattering

SIDIS

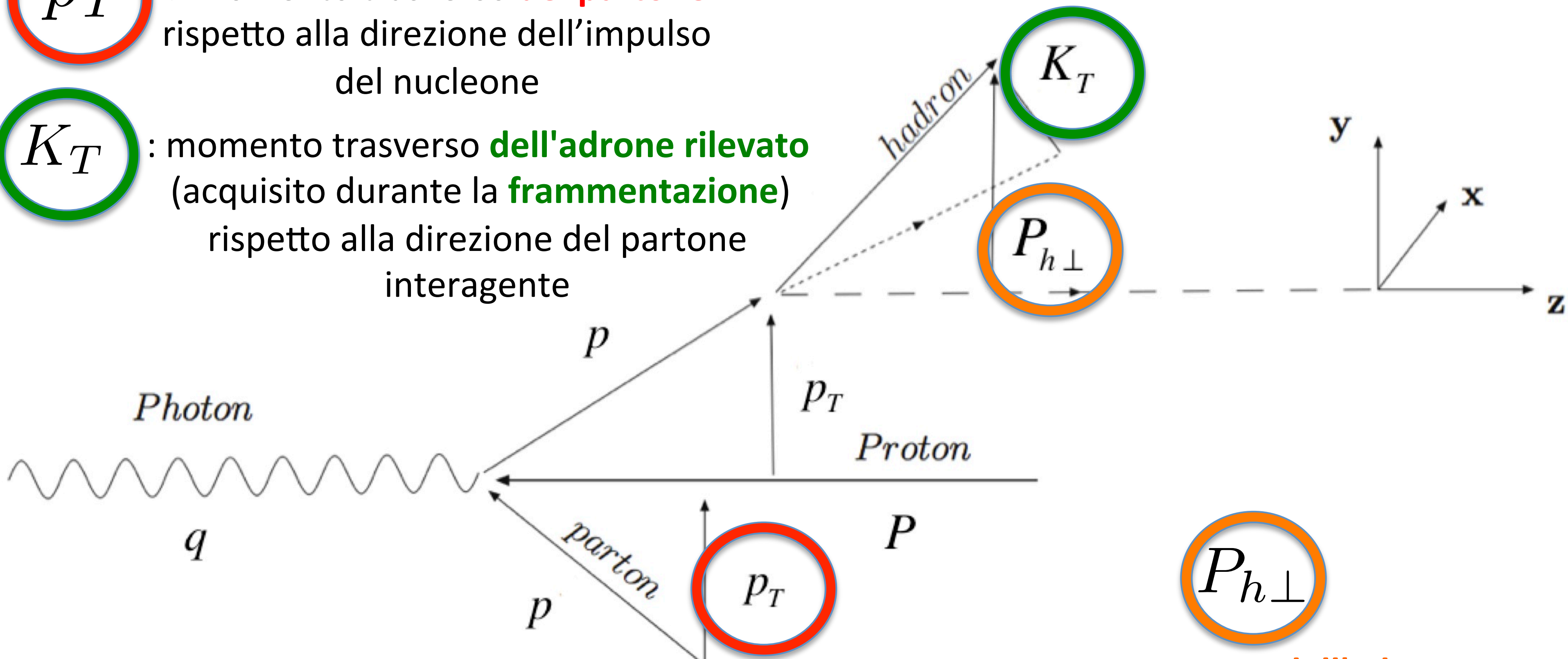
$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$



SIDIS

p_T : momento trasverso **del partone** rispetto alla direzione dell'impulso del nucleone

K_T : momento trasverso **dell'adrone rilevato** (acquisito durante la **frammentazione**) rispetto alla direzione del partone interagente



$P_{h\perp}$ momento trasverso **dell'adrone rilevato** rispetto alla direzione del fotone

SIDIS

Approssimazioni utilizzate

1. One-photon exchange

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$$\boxed{\begin{aligned} P_{h\perp}^2 &\ll Q^2 \\ p_T^2 &\ll Q^2 \end{aligned}}$$

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Approssimazioni utilizzate

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Cross-section:

$$\boxed{\frac{d^{(4)}\sigma}{dx\ dz\ dQ^2\ dP_{h\perp}^2} = \frac{\pi\alpha^2}{xQ^4}\left[1+\left(1-\frac{Q^2}{xs}\right)\right]F_{UU,T}(x,z,Q^2;P_{h\perp}^2)}$$

Unpolarized structure function

$F_{UU,T}$ e momenti trasversi

$$F_{UU,T}(x,z,Q^2;P_{h\perp}^2) = x\sum_a f_1^a(x,Q^2,p_T^2)\otimes D_1^a(z,Q^2,K_T^2)$$

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1) Dalla espressione della convoluzione si ricava la relazione tra i momenti trasversi

$$\boxed{\langle P_{h\perp}^2\rangle = z^2\langle p_T^2\rangle + \langle K_T^2\rangle}$$

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2) Ma come **parametrizzare** la parte TMD di queste funzioni?

Ipotesi gaussiana flavour-independent

Parametrizzazione gaussiana

$$f_1^a(x,Q^2,p_T^2) = f_1^a(x,Q^2) \cdot \frac{1}{\pi\langle p_T^2\rangle} \exp\left\{-\frac{p_T^2}{\langle p_T^2\rangle}\right\}$$

Parti TMD indipendenti dal flavour

$$D_1^a(z,Q^2,K_T^2) = D_1^a(z,Q^2) \cdot \frac{1}{\pi\langle K_T^2\rangle} \exp\left\{-\frac{K_T^2}{\langle K_T^2\rangle}\right\}$$

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NB: non c'è vera fattorizzazione tra parte collineare e parte TMD:

$$\langle p_T^2 \rangle = \langle p_T^2 \rangle (x, Q^2)$$

$$\langle K_T^2 \rangle = \langle K_T^2 \rangle (z, Q^2)$$

Ipotesi gaussiana flavour-independent

- È un **buon modello** per fittare i dati con

$$P_{h\perp}^2 \ll Q^2$$

Ipotesi gaussiana flavour-independent

- È un **buon modello** per fittare i dati con

$$P_{h\perp}^2 \ll Q^2$$

- Ma ha un notevole deficit:

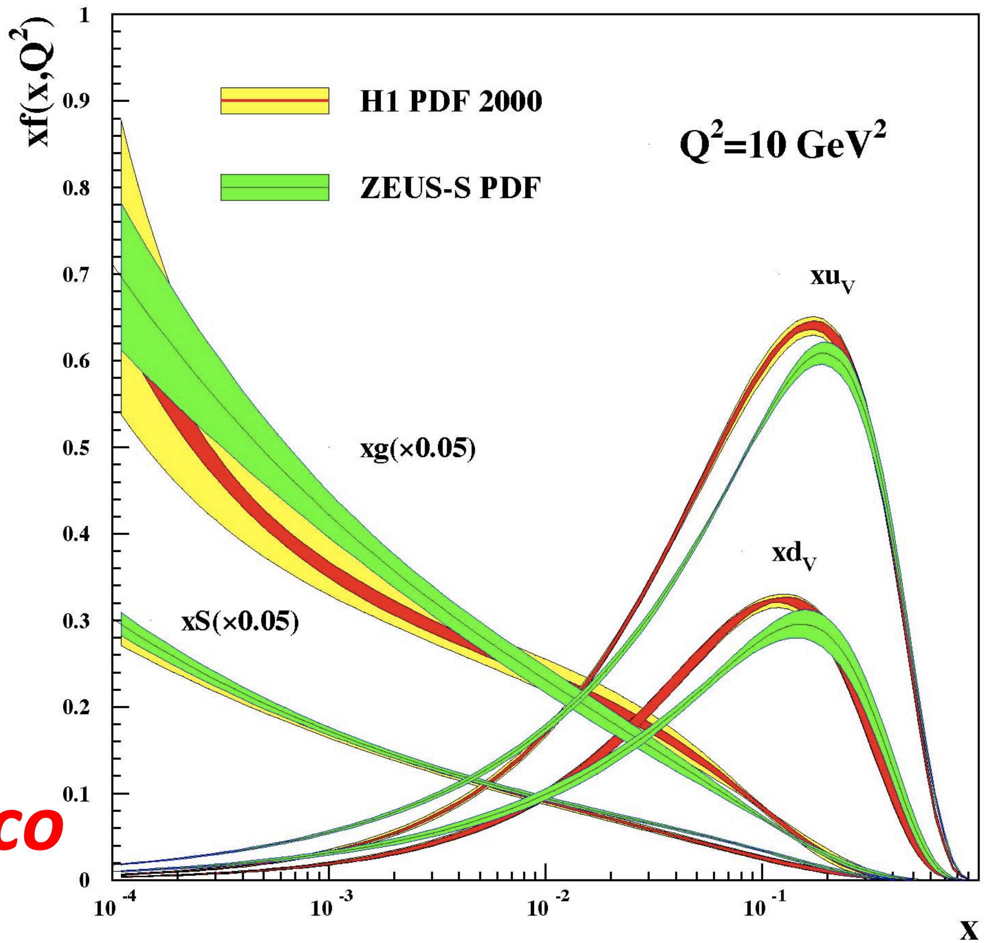
ASSENZA DELLA FLAVOUR DEPENDENCE

Ipotesi: flavour-dependence

Essendo le PDF collineari
FORTEMENTE
dipendenti dal flavour
ci aspettiamo che le loro
generalizzazioni 3-dim
siano altrettanto.

Un'ipotesi ***euristica***…

A cui vorremmo conferire
supporto fenomenologico

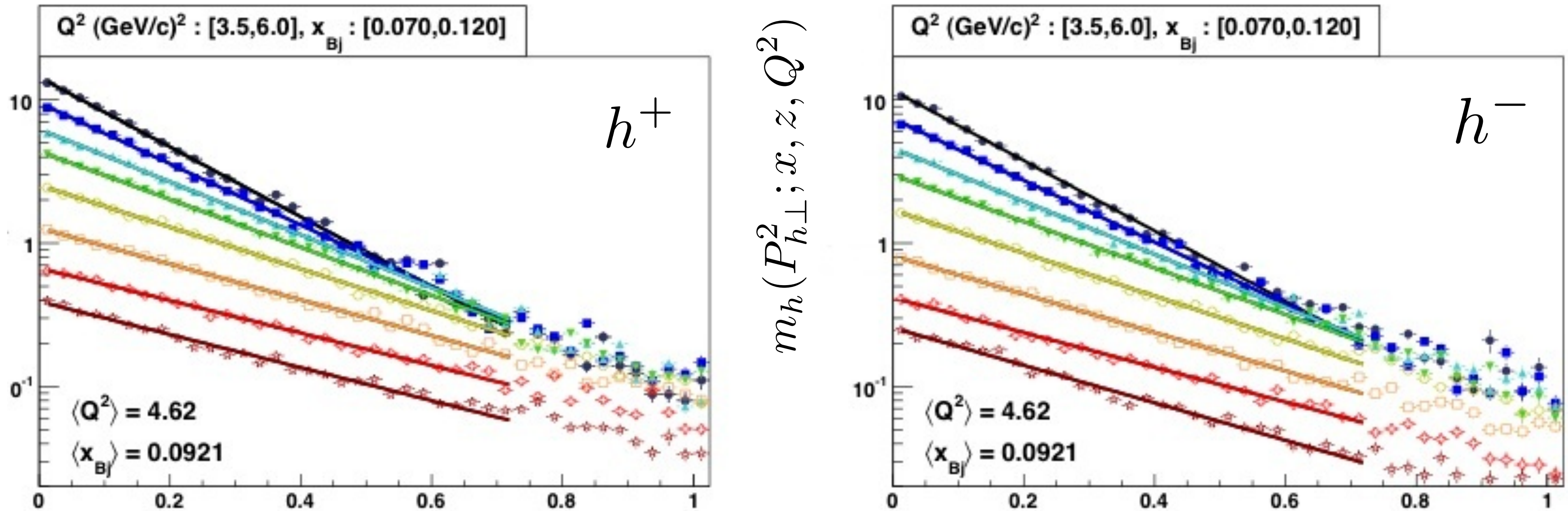


COMPASS DATA ANALYSIS

PROGETTO

Esplorazione della dipendenza dal flavour
delle TMDs non polarizzate
sfruttando gli ampli intervalli cinematici esplorati
e l'elevata statistica collezionata da COMPASS

COMPASS: fit di dati preliminari



$$m_h(P_{h\perp}^2; x, z, Q^2) = \frac{d^{(4)}\sigma^{SIDIS}}{d^{(2)}\sigma^{DIS}} = \mathcal{A}_\pm \exp\left\{ -\frac{P_{h\perp}^2}{\langle P_{h\perp}^2 \rangle_\pm} \right\}$$

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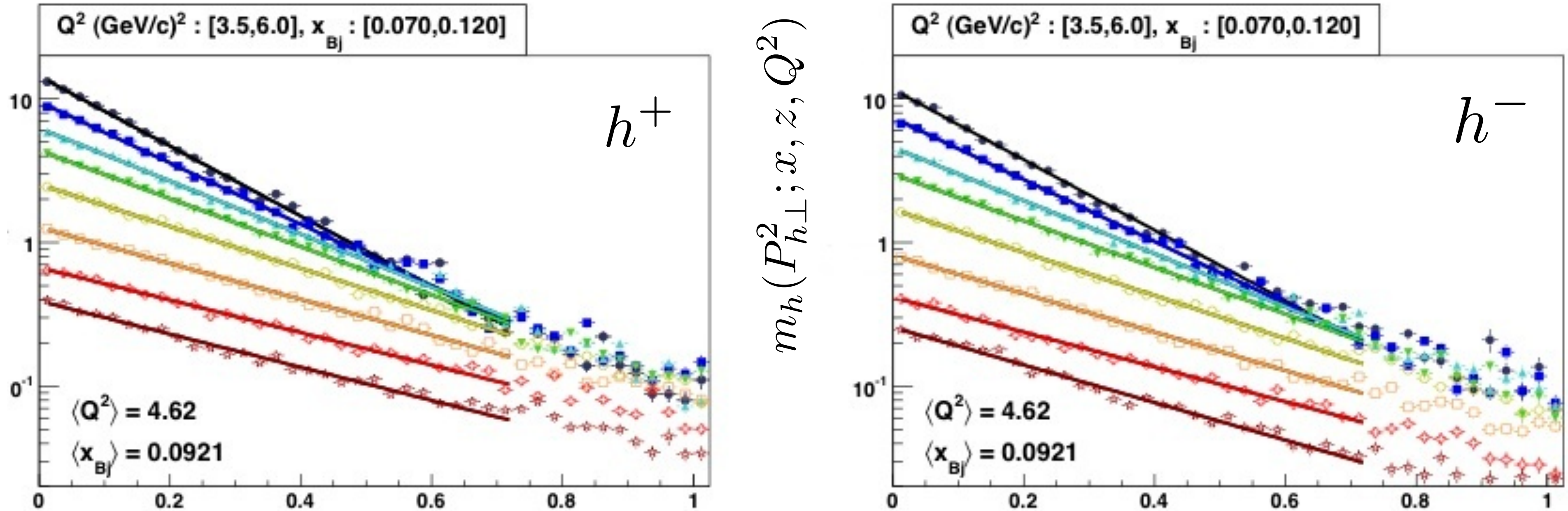

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Best-fit parameters

184 bin cinematici

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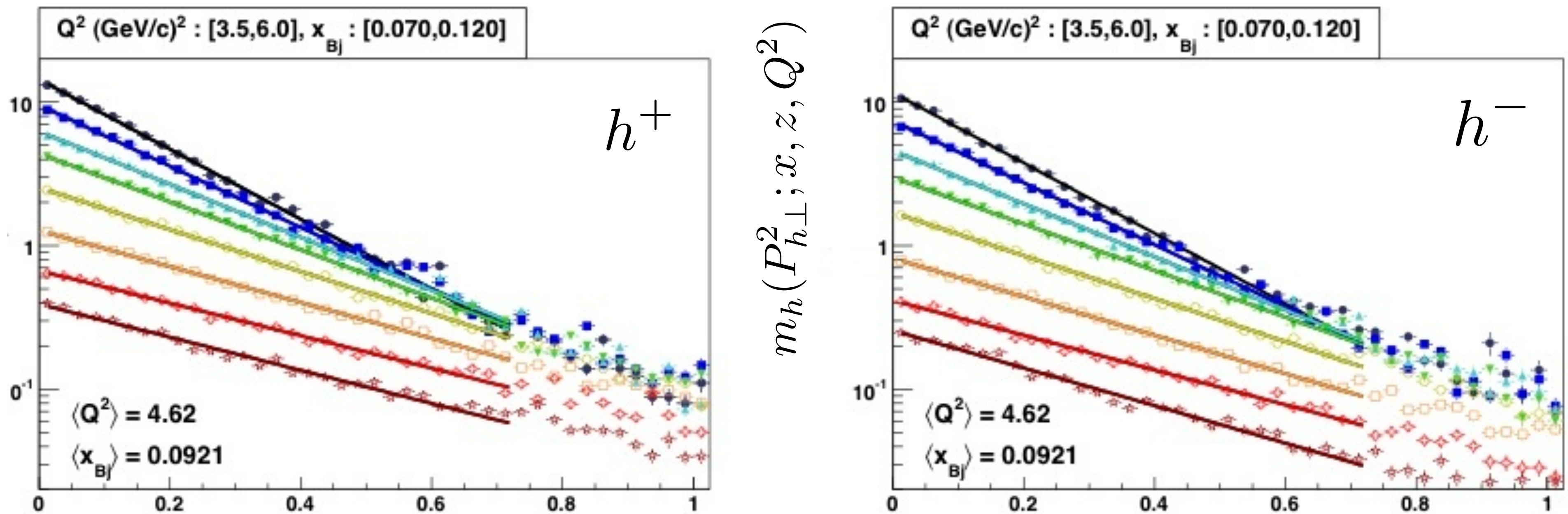

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Il carattere gaussiano riflette la flavour independence nell'ipotesi gaussiana

Best-fit parameters

184 bin cinematici

COMPASS: fit di dati preliminari

$1 < Q^2 < 10 \text{ GeV}^2$

Massa invariante al quadrato del fotone virtuale scambiato

$0.0045 < x < 0.12$

Frazione di impulso collineare portato dal partone colpito

$0.05 < z^2 < 0.56$

Frazione dell'energia del fotone virtuale portata dall'adrone frammentato

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Frazione dell'energia del fotone virtuale portata dall'adrone frammentato

Bin	$\langle z^2\rangle_{h+}$	$\langle P_T^2\rangle_{h+}$	A_{h+}	χ^2_{h+}/ndf	$\langle z^2\rangle_{h-}$	$\langle P_T^2\rangle_{h-}$	A_{h-}	χ^2_{h-}/ndf
81	0.05	0.21 ± 0.003	12.77 ± 0.046	7.6	0.05	0.21 ± 0.004	11.52 ± 0.045	5.2
82	0.08	0.24 ± 0.004	8.35 ± 0.034	5.7	0.08	0.23 ± 0.004	7.39 ± 0.033	3.9
83	0.11	0.26 ± 0.005	5.63 ± 0.027	2.3	0.11	0.25 ± 0.005	4.93 ± 0.026	4.2
84	0.14	0.28 ± 0.006	4.00 ± 0.023	2.6	0.14	0.27 ± 0.006	3.39 ± 0.021	3.4
85	0.20	0.30 ± 0.005	2.50 ± 0.013	2.4	0.20	0.28 ± 0.005	2.08 ± 0.012	5.4
86	0.30	0.31 ± 0.007	1.43 ± 0.009	4.6	0.30	0.29 ± 0.007	1.20 ± 0.009	4.0
87	0.42	0.31 ± 0.008	0.88 ± 0.007	5.7	0.42	0.28 ± 0.009	0.77 ± 0.007	7.0
88	0.56	0.27 ± 0.010	0.65 ± 0.007	11.0	0.56	0.23 ± 0.010	0.60 ± 0.007	11.2

Table C.9: Fit results for $0.018 < x_{Bj} < 0.025$ and $1 < Q^2 < 1.5$ $(\text{GeV/c})^2$. All variables are defined in the text except $A_{h^{+,-}}$, the fitted Gaussian amplitude. Only statistical error were used for the fit, this explains the high χ^2 in the low z intervals were the statistical error is very small.

PROCEDIMENTO

Fit gaussiani flavour-independent
delle molteplicità adroniche preliminari

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Set di pseudo-dati sperimentali
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Set di pseudo-dati sperimentali
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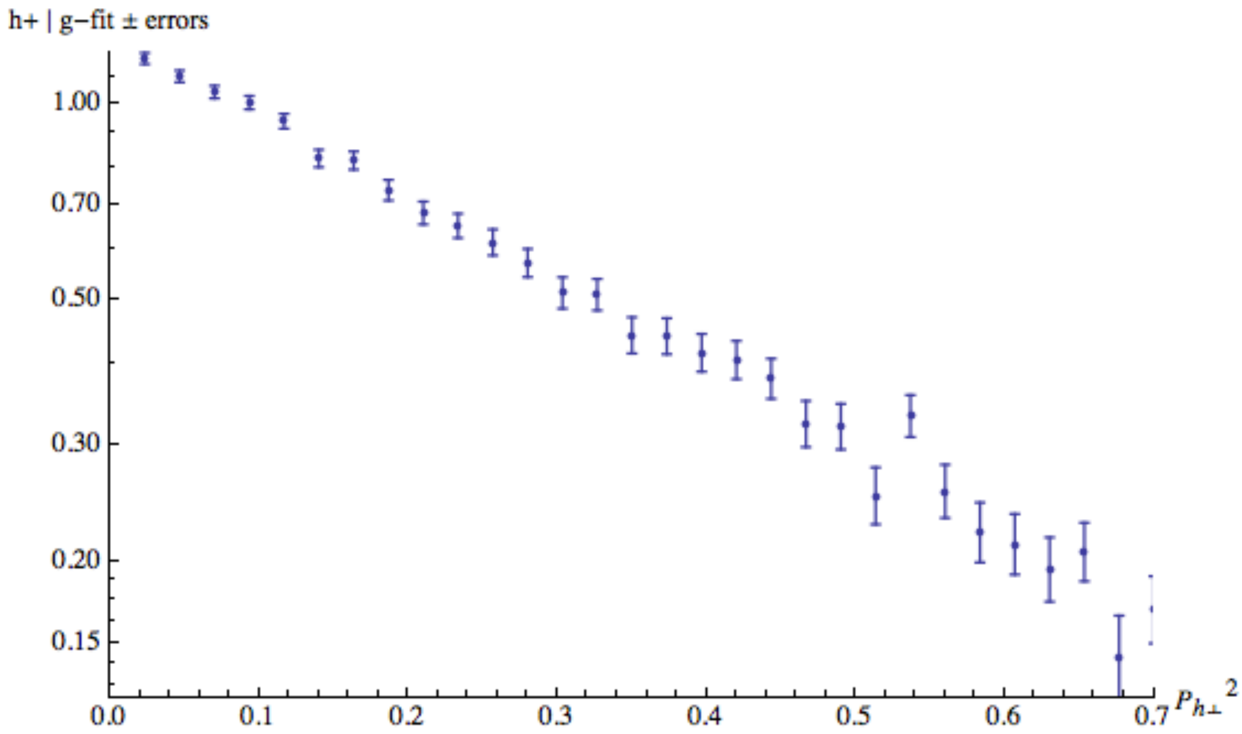
Fit flavour-dependent (non gaussiani)
degli pseudo-dati

METODO DELLE MOLTEPLICITA' RANDOM

$$m_{h_\pm}(P_{h\perp}^2;Y) = g_{C_\pm}(P_{h\perp}^2;Y) + \mathcal{N}(0,\mathcal{E}_\pm(P_{h\perp}^2;Y))$$



Rumore gaussiano con media nulla e deviazione standard

$$\mathcal{E}_\pm$$

calcolata mediante **propagazione degli errori** statistici sui parametri di best-fit di COMPASS

$$\mathcal{A}_\pm \ , \ \langle P_{h\perp}^2 \rangle_\pm$$

METODO DELLE MOLTEPLICITA' RANDOM

Deviazione standard totale

$$\mathcal{E}_{\pm}(P_{h\perp}^2;Y) = \sqrt{[\mathrm{b}\ \Delta g_{C_{\pm}}(P_{h\perp}^2;Y)]^2 + [\lambda\ g_{C_{\pm}}(P_{h\perp}^2;Y)]^2}$$

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componente statistica

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componente statistica

Propagazione degli errori sui valori di best-fit per $\mathcal{A}_{\pm}$ e $\langle P_{h\perp}^2 \rangle_{\pm}$

$$\Delta g_{C_\pm} = \sqrt{\left(\frac{\partial g_{C_\pm}}{\partial \mathcal{A}_\pm}\right)^2 \left(\Delta \mathcal{A}_\pm\right)^2 + \left(\frac{\partial g_{C_\pm}}{\partial \langle P_{h\perp}^2 \rangle_\pm}\right)^2 \left(\Delta \langle P_{h\perp}^2 \rangle_\pm\right)^2}$$

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componente statistica *componente sistematica*

Propagazione degli errori sui valori di best-fit per $\mathcal{A}_\pm$ e $\langle P_{h\perp}^2 \rangle_\pm$

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REPLICHE STATISTICHE

Il **rumore gaussiano** viene **replicato** M volte

- Per ottenere *M set di pseudo-dati*

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REPLICHE STATISTICHE

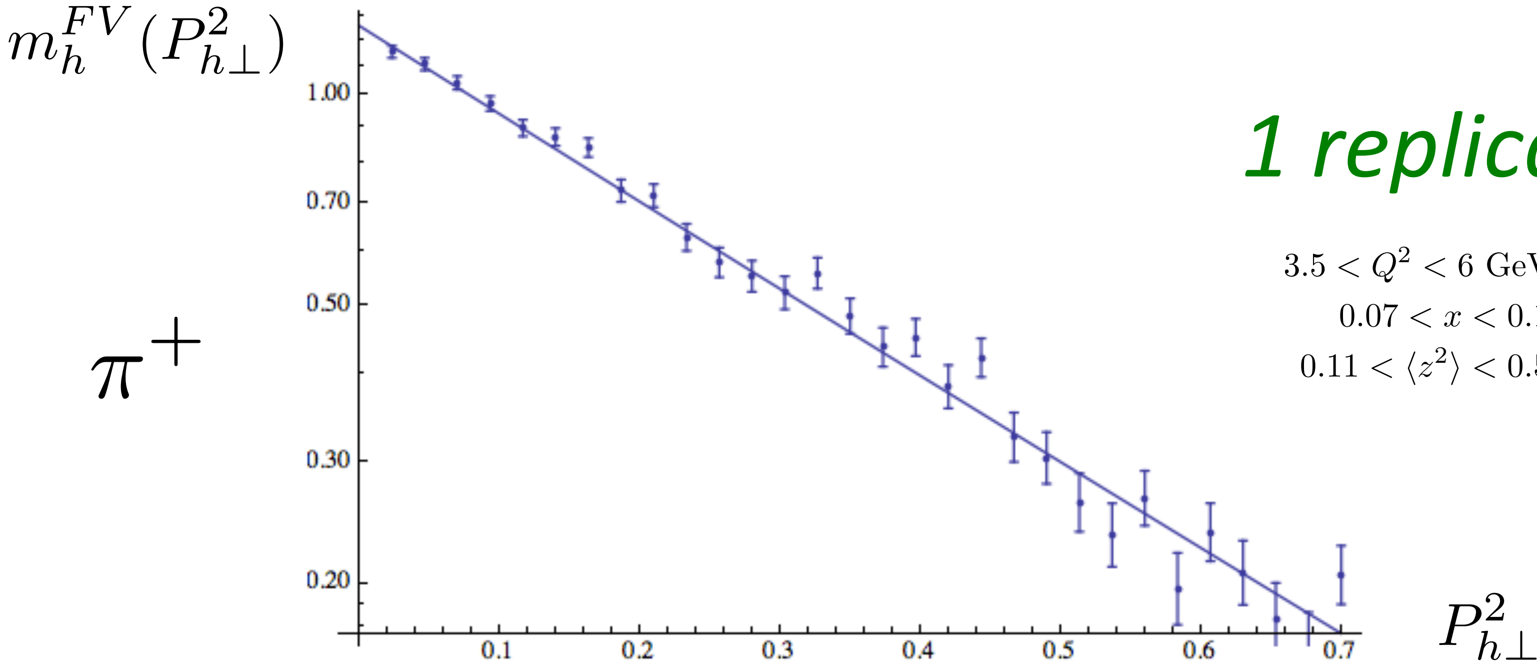
Il **rumore gaussiano** viene **replicato** M volte

- Per ottenere *M set di pseudo-dati*
- Consente di ottenere distribuzioni per i valori di best-fit
- È virtualmente possibile valutare il comportamento dei valori di best-fit per varie repliche dell'esperimento

Minimizzazione del χ^2

$$\chi^2(v) = \sum_{\pi^\pm} \sum_{P^2_{h\perp}, Y} \frac{[m_h(P^2_{h\perp}, Y) - m_h^{FV}(P^2_{h\perp}, Y, v)]^2}{\mathcal{E}(P^2_{h\perp}, Y)^2}$$

π^+



1 replica

$3.5 < Q^2 < 6 \text{ GeV}^2$

$0.07 < x < 0.12$

$0.11 < \langle z^2 \rangle < 0.56$

Minimizzazione del χ^2

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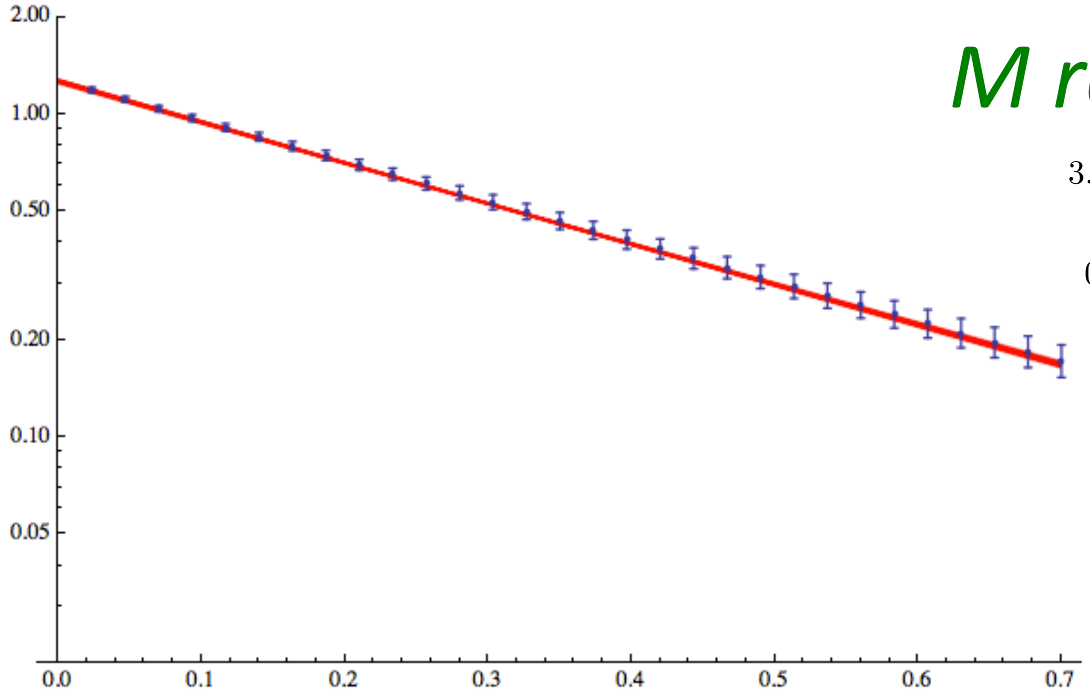
$m_h^{FV}(P_{h\perp}^2)$

π^+



$P_{h\perp}^2$

M repliche

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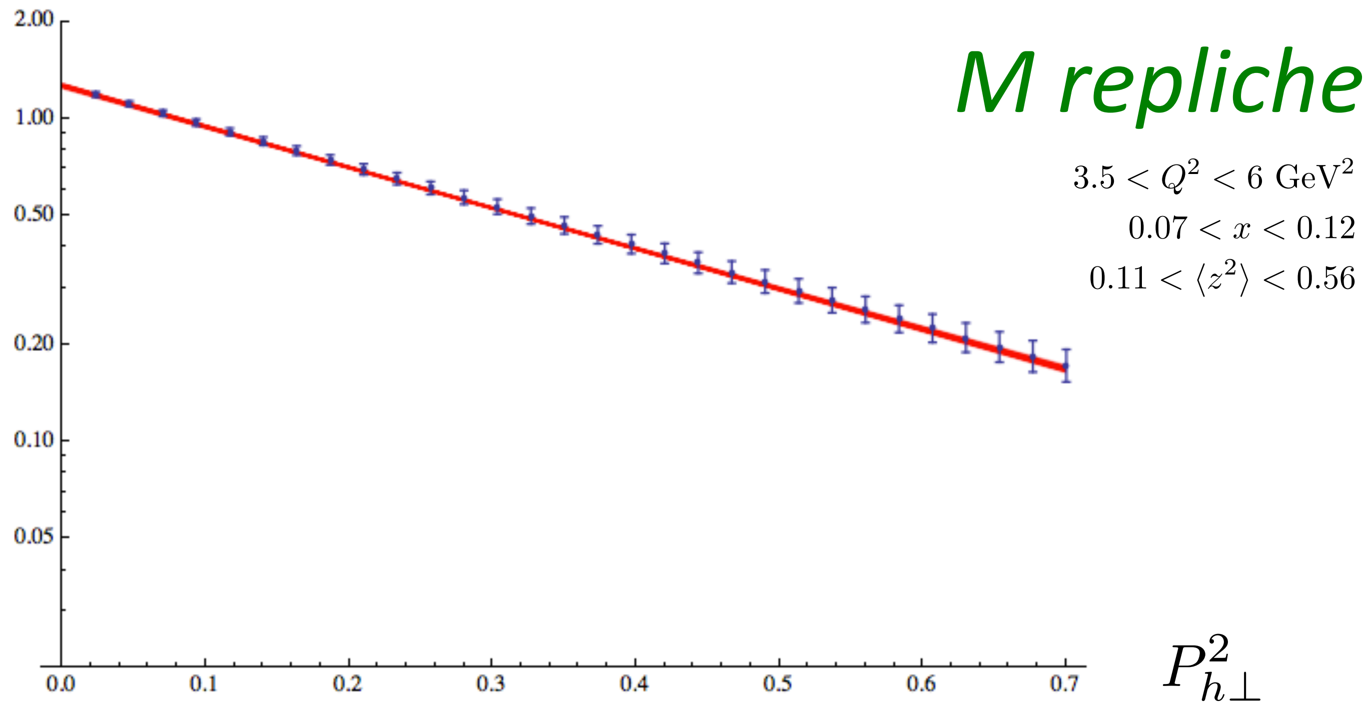
$m_h^{FV}(P^2_{h\perp})$

π^+



$P^2_{h\perp}$

FLAVOUR ANALYSIS

Molteplicità flavour-independent

$$m_{h_\pm}(P_{h\perp}^2; x, z, Q^2) = \pi \frac{\sum_a [e_a^2 f_1^a(x, Q^2) D_1^{a\to\pi^\pm}(z, Q^2)]}{\sum_a e_a^2 f_1^a(x, Q^2)} \boxed{\frac{e^{-\frac{P_{h\perp}^2}{z^2\langle p_T^2\rangle + \langle K_T^2\rangle}}}{\pi(z^2\langle p_T^2\rangle + \langle K_T^2\rangle)}}$$

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Generalizziamo questa espressione introducendo una gaussiana per ogni flavour

$$\langle p_{T,\mathrm{up}}^2\rangle \;,\; \langle p_{T,\mathrm{down}}^2\rangle \;,\; \langle p_{T,\mathrm{sea}}^2\rangle$$

(idem per i processi di frammentazione)

Isospin symmetry and charge-conjugation

- Funzioni di frammentazione simmetriche per simultaneo scambio di quark up e down (isospin symmetry) e coniugazione di carica dell'adrone rilevato (charge-conjugation)

$$D_1^{u\to\pi^+} = D_1^{d\to\pi^-}$$

$$D_1^{u\to\pi^-} = D_1^{d\to\pi^+}$$

Isospin symmetry and charge-conjugation

- Distinguiamo i processi di frammentazione in:

1. Favored:

$$D_1^{u\to\pi^+} = D_1^{d\to\pi^-} \doteq D_1^{\mathrm{fav}}$$
$$D_1^{\bar{u}\to\pi^-} = D_1^{\bar{d}\to\pi^+} \doteq D_1^{\mathrm{fav}}$$

2. Unfavored:

$$D_1^{u\to\pi^-} = D_1^{d\to\pi^+} \doteq D_1^{\mathrm{unf}}$$
$$D_1^{\bar{u}\to\pi^+} = D_1^{\bar{d}\to\pi^-} \doteq D_1^{\mathrm{unf}}$$

$$\pi^+(u\bar{d}) \;, \pi^-(\bar{u}d)$$

A seconda che Il partone
Che frammenta sia un quark di valenza dell'adrone
Rivelato o meno

π^+

$$
\begin{aligned}
m_{\pi^+}(x,z,Q^2,P_{h\perp}^2) = {} & \frac{\pi n}{\sum_a e_a^2 f_1^a(x,Q^2)} \cdot \\
& \cdot \Bigg\{ e_u^2 f_1^{u_v}(x,Q^2) D_1^{fav}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 u+F}}}{\pi(z^2 u+F)} + \\
& + e_d^2 f_1^{d_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 d+U}}}{\pi(z^2 d+U)} + \\
& + e_{st}^2 f_1^{st_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_u^2 f_1^{u-u_v}(x,Q^2) D_1^{fav}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+F}}}{\pi(z^2 s+F)} + \\
& + e_{\bar{u}}^2 f_1^{\bar{u}}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_d^2 f_1^{d-d_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_{\bar{d}}^2 f_1^{\bar{d}}(x,Q^2) D_1^{fav}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+F}}}{\pi(z^2 s+F)} + \\
& + e_{st}^2 f_1^{st-st_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_{\overline{st}}^2 f_1^{\overline{st}}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} \Bigg\} ,
\end{aligned}
$$

π^-

$$
\begin{aligned}
m_{\pi^-}(x,z,Q^2,P_{h\perp}^2) = {} & \frac{\pi n}{\sum_a e_a^2 f_1^a(x,Q^2)} \cdot \\
& \cdot \Bigg\{ e_u^2 f_1^{u_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 u+U}}}{\pi(z^2 u+U)} + \\
& + e_d^2 f_1^{d_v}(x,Q^2) D_1^{fav}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 d+F}}}{\pi(z^2 d+F)} + \\
& + e_{st}^2 f_1^{st_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_u^2 f_1^{u-u_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_{\bar{u}}^2 f_1^{\bar{u}}(x,Q^2) D_1^{fav}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+F}}}{\pi(z^2 s+F)} + \\
& + e_d^2 f_1^{d-d_v}(x,Q^2) D_1^{fav}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+F}}}{\pi(z^2 s+F)} + \\
& + e_{\bar{d}}^2 f_1^{\bar{d}}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_{st}^2 f_1^{st-st_v}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} + \\
& + e_{\overline{st}}^2 f_1^{\overline{st}}(x,Q^2) D_1^{unf}(z,Q^2) \frac{e^{-\frac{-P_{h\perp}^2}{z^2 s+U}}}{\pi(z^2 s+U)} \Bigg\} .
\end{aligned}
$$

MOMENTI TRASVERSI E CINEMATICA

$$\lim_{\substack{x\to 0\\ x\to 1}} \langle p^2_{T,a} \rangle (x, Q^2) = 0 \qquad \lim_{\substack{z\to 0\\ z\to 1}} \langle K^2_{T,j} \rangle (z, Q^2) = 0$$

MOMENTI TRASVERSI E CINEMATICA

$$\lim_{\substack{x\to 0\\ x\to 1}} \langle p_{T,a}^2 \rangle (x, Q^2) = 0 \qquad \lim_{\substack{z\to 0\\ z\to 1}} \langle K_{T,j}^2 \rangle (z, Q^2) = 0$$

TMD evolution

$$\langle p_{T,a}^2 \rangle (x, Q^2) \sim 1 + \ln \frac{Q^2}{Q_0^2}$$

$$\langle K_{T,j}^2 \rangle (z, Q^2) \sim 1 + \ln \frac{Q^2}{Q_0^2}$$

MOMENTI TRASVERSI E CINEMATICA

$$\langle p_{T,a}^2\rangle(x,Q^2) = N_a\, x^{\alpha_a}(1-x)^{\beta_a}\left[1+\ln\left(\frac{Q^2}{Q_0^2}\right)\right]$$

Parametri di best-fit

$$Q_0^2 = 1\ \mathrm{GeV}^2$$

$$\langle K_{T,j}^2\rangle(z,Q^2) = N_j\, z^{\alpha_j}(1-z)^{\beta_j}\left[1+\ln\left(\frac{Q^2}{Q_0^2}\right)\right]$$

ANALISI GLOBALE

Fit combinato su 84 bin cinematici forniti da COMPASS, per π^+ e π^-

$$1.5 < Q^2 < 10 \ \mathrm{GeV}^2$$

$$0.012 < x < 0.12$$

$$0.08 < \langle z^2 \rangle < 0.56$$

ANALISI GLOBALE

Fit combinato su 84 bin cinematici forniti da COMPASS, per π^+ e π^-

$184 \rightarrow 84$ bin

selection criterion
$\chi^2/\mathrm{dof}_{\,Raj} < 4$

$$1.5 < Q^2 < 10 \text{ GeV}^2$$

$$0.012 < x < 0.12$$

$$0.08 < \langle z^2 \rangle < 0.56$$

ANALISI GLOBALE

Fit combinato su 84 bin cinematici forniti da COMPASS, per π^+ e π^-

$184 \rightarrow 84$ bin

$$\boxed{\begin{array}{c}\text{selection criterion} \\ \chi^2/\text{dof}_{\ Raj} < 4\end{array}}$$

$$1.5 < Q^2 < 10 \text{ GeV}^2$$

$$0.012 < x < 0.12$$

$$0.08 < \langle z^2 \rangle < 0.56$$

Errore statistico: **raddoppiato** rispetto al valore calcolato da COMPASS

ANALISI GLOBALE

Fit combinato su 84 bin cinematici forniti da COMPASS, per π^+ e π^-

$184 \rightarrow 84$ bin

$$\boxed{\begin{array}{c}\text{selection criterion} \\ \chi^2/\text{dof}_{\,Raj} < 4\end{array}}$$

$$1.5 < Q^2 < 10 \text{ GeV}^2$$

$$0.012 < x < 0.12$$

$$0.08 < \langle z^2 \rangle < 0.56$$

Errore statistico: **raddoppiato** rispetto al valore calcolato da COMPASS

Si introduce un errore sistematico pari al **10%** del valore della molteplicità

ANALISI GLOBALE

Fit combinato su 84 bin cinematici forniti da COMPASS, per π^+ e π^-

$184 \rightarrow 84$ bin

$$\boxed{\begin{array}{l}\text{selection criterion} \\ \quad \chi^2/\text{dof}_{\ Raj} < 4\end{array}}$$

$$1.5 < Q^2 < 10 \text{ GeV}^2$$

$$0.012 < x < 0.12$$

$$0.08 < \langle z^2 \rangle < 0.56$$

Errore statistico: **raddoppiato** rispetto al valore calcolato da COMPASS

Si introduce un errore sistematico pari al **10%** del valore della molteplicità

$$\mathcal{E}_{\pm}(P_{h\perp}^2;Y) = \sqrt{[\mathrm{b}\,\Delta g_{C_\pm}(P_{h\perp}^2;Y)]^2 + [\lambda\, g_{C_\pm}(P_{h\perp}^2;Y)]^2}$$

$$\boxed{\begin{array}{r}\mathrm{b} = 2 \\ \lambda = 0.1\end{array}}$$

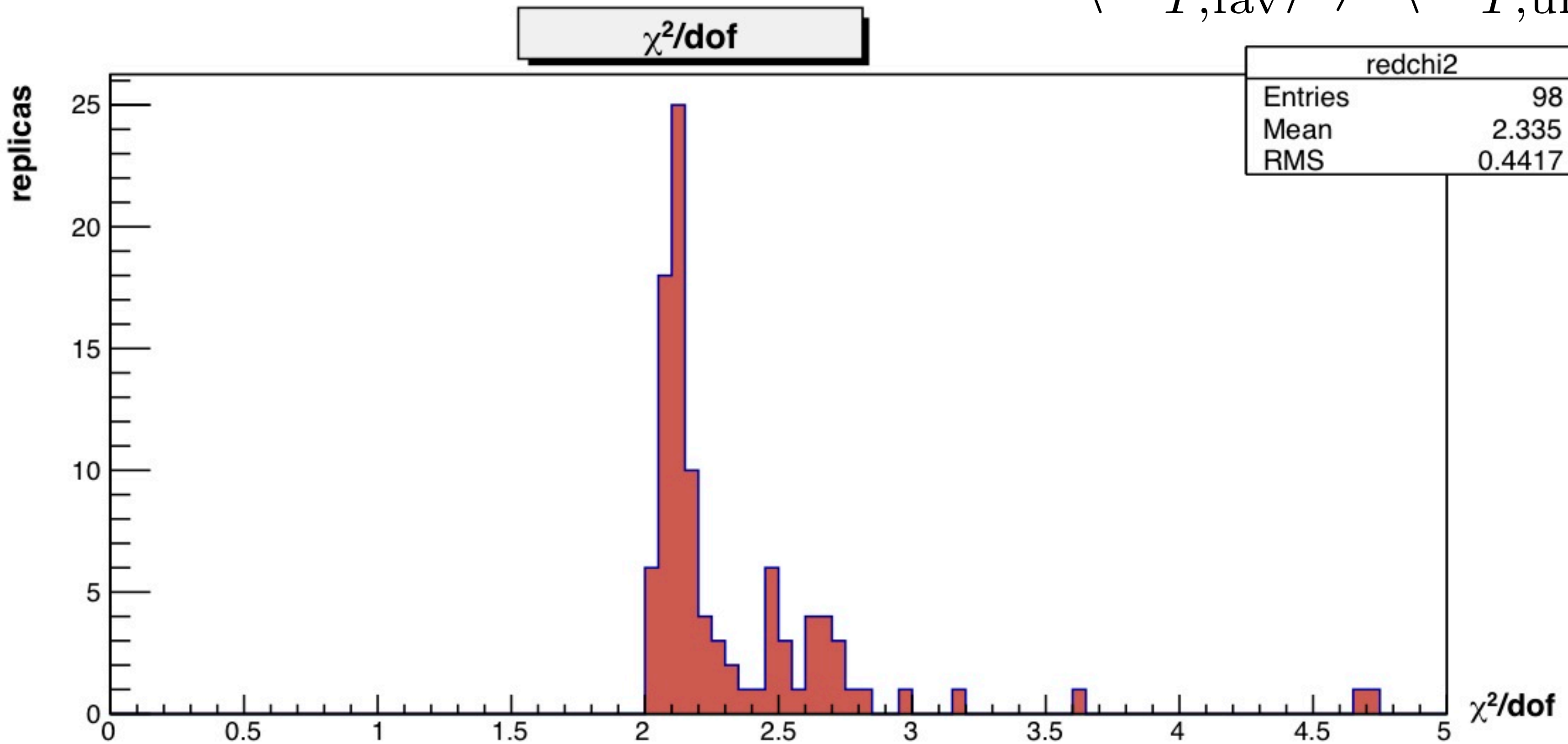
RISULTATI

TEST DI FLAVOUR-DEPENDENCE

$$\langle \chi^2/dof \rangle = 2.335$$

$$\langle p^2_{T,\mathrm{up}} \rangle \neq \langle p^2_{T,\mathrm{down}} \rangle \neq \langle p^2_{T,\mathrm{sea}} \rangle$$

$$\langle K^2_{T,\mathrm{fav}} \rangle \neq \langle K^2_{T,\mathrm{unf}} \rangle$$

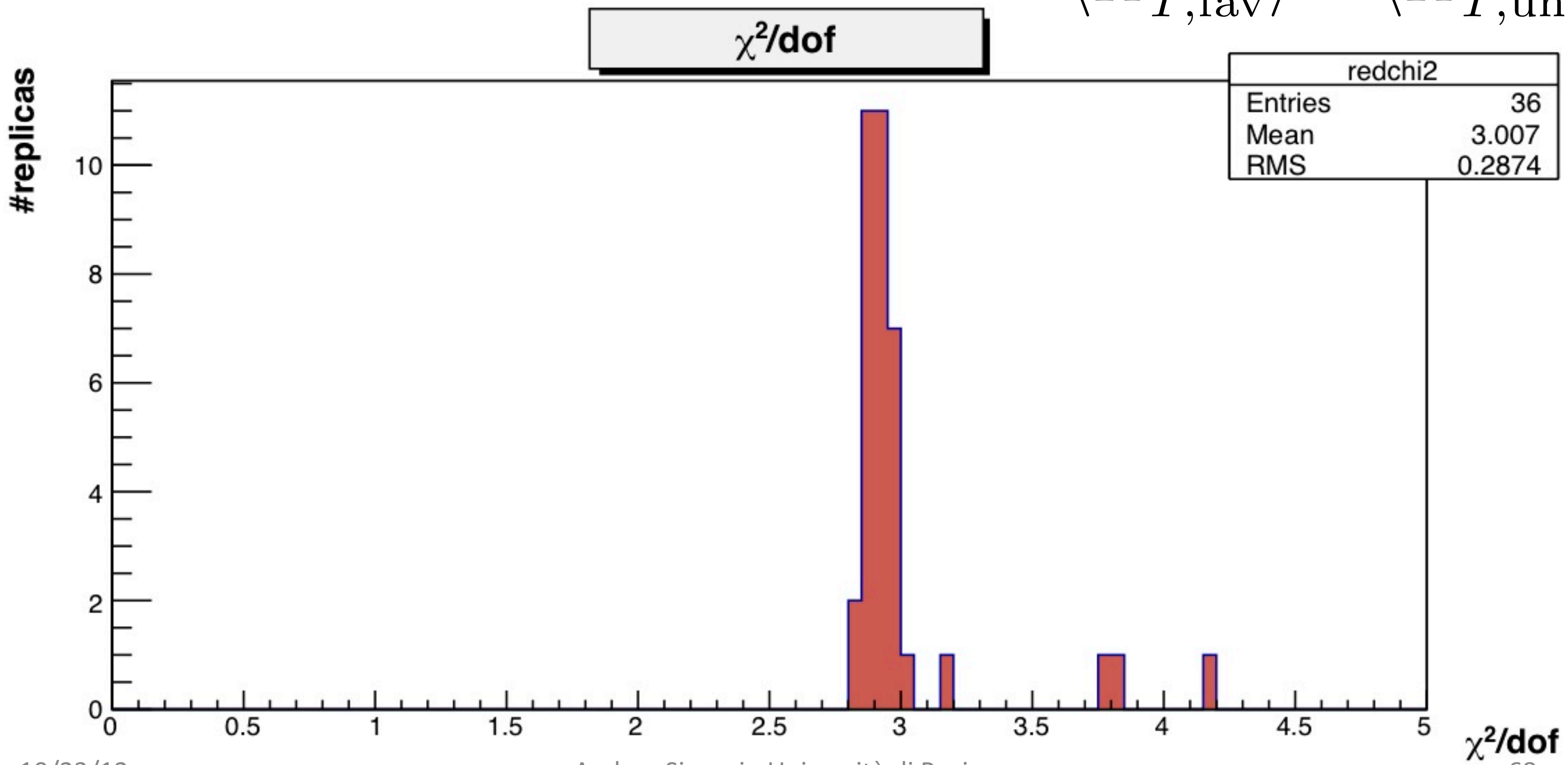


TEST DI FLAVOUR-INDEPENDENCE

$$\langle \chi^2/dof \rangle = 3.007$$

$$\langle p^2_{T,\mathrm{up}} \rangle = \langle p^2_{T,\mathrm{down}} \rangle = \langle p^2_{T,\mathrm{sea}} \rangle$$

$$\langle K^2_{T,\mathrm{fav}} \rangle = \langle K^2_{T,\mathrm{unf}} \rangle$$



TEST DI FLAVOUR-DEPENDENCE

$$\boxed{\text{sea} \approx \text{up} > \text{down}}$$

Comportamento globale in x e Q²

$$\boxed{2.5 < Q^2 < 3.5 \text{ GeV}^2 \quad , \quad 0.04 < x < 0.07}$$

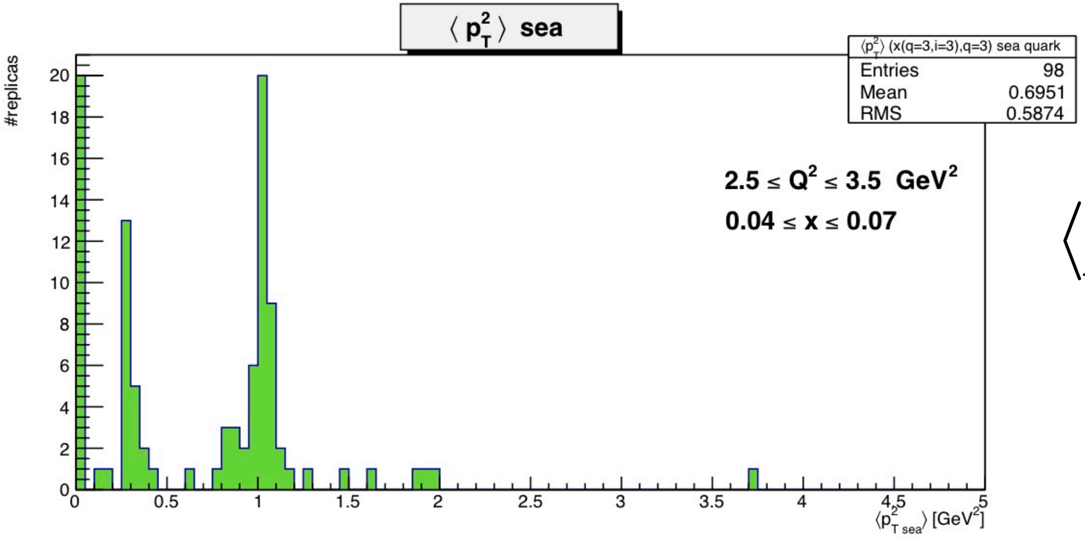
$$\langle p^2_{T,\text{up}} \rangle = 0.60 \pm 0.53 \ \text{ GeV}^2$$

$$\langle p^2_{T,\text{down}} \rangle = 0.56 \pm 0.99$$

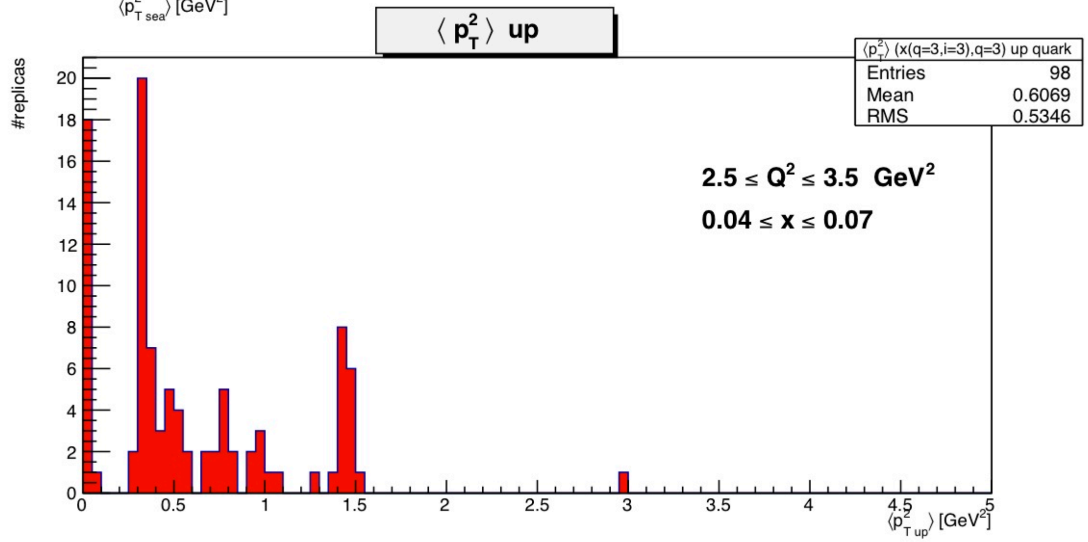
$$\langle p^2_{T,\text{sea}} \rangle = 0.69 \pm 0.58$$

Valori medi
ed errori statistici

TEST DI FLAVOUR-DEPENDENCE

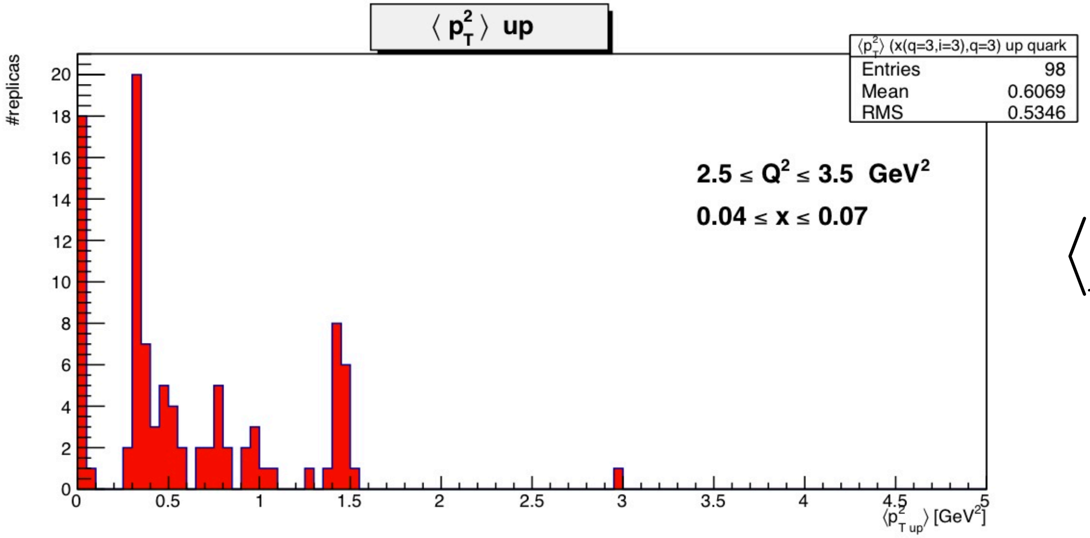


$$\langle p^2_{T,\mathrm{sea}}\rangle = 0.69 \pm 0.58$$
$$\mathrm{GeV}^2$$

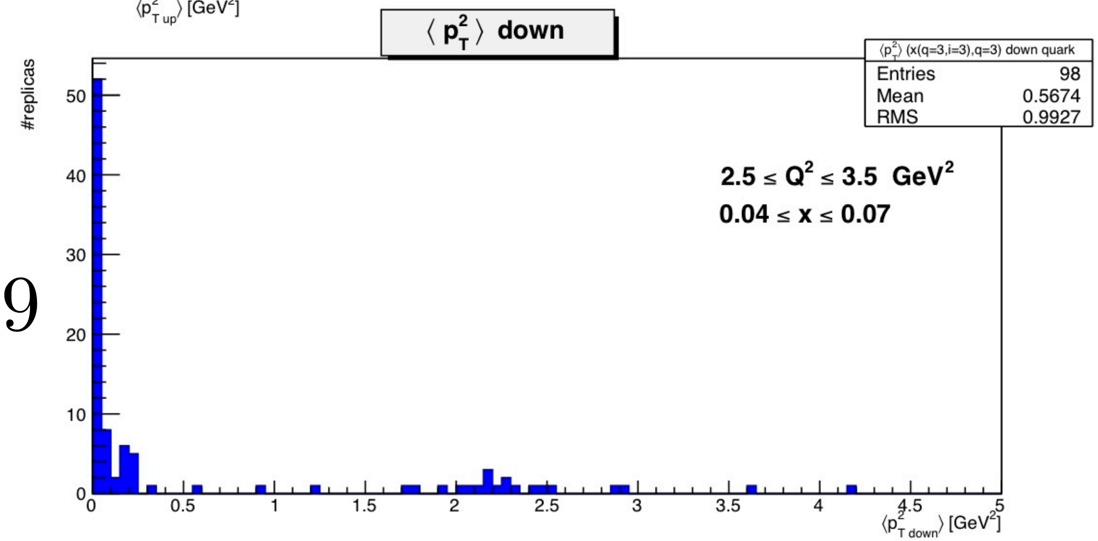


$$\langle p^2_{T,\mathrm{up}}\rangle = 0.60 \pm 0.53$$
$$\mathrm{GeV}^2$$

TEST DI FLAVOUR-DEPENDENCE



$$\langle p^2_{T,\mathrm{up}} \rangle = 0.60 \pm 0.53$$
$$\mathrm{GeV}^2$$



$$\langle p^2_{T,\mathrm{down}} \rangle = 0.56 \pm 0.99$$
$$\mathrm{GeV}^2$$

TEST DI FLAVOUR-DEPENDENCE

$$\boxed{\text{sea} \approx \text{up} > \text{down}}$$

Comportamento globale in x e Q²

$$\boxed{3.5 < Q^2 < 6 \text{ GeV}^2 \;,\; 0.07 < x < 0.12}$$

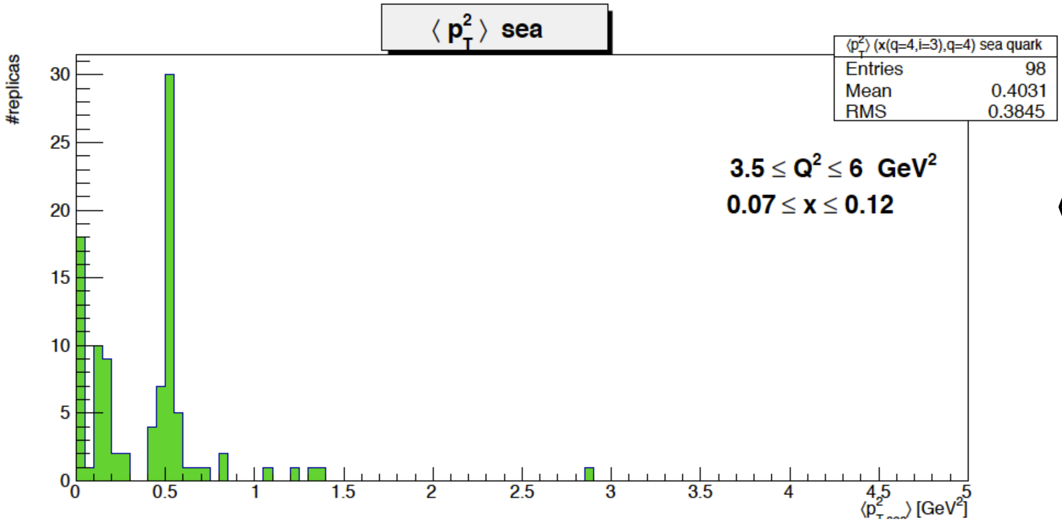
$$\langle p^2_{T,\text{up}} \rangle = 0.45 \pm 0.29 \;\; \text{GeV}^2$$

$$\langle p^2_{T,\text{down}} \rangle = 0.36 \pm 0.50$$

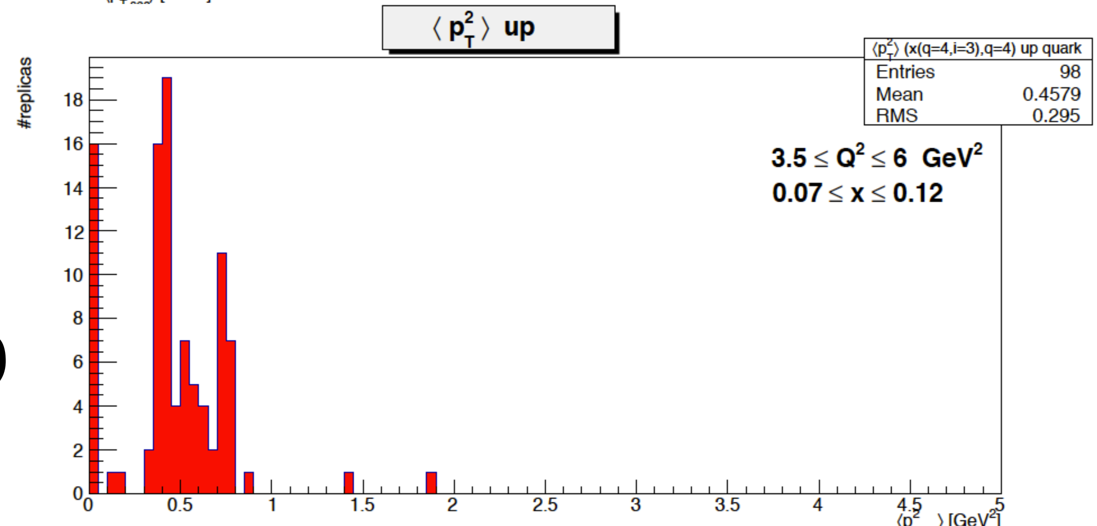
$$\langle p^2_{T,\text{sea}} \rangle = 0.40 \pm 0.38$$

Valori medi
ed errori statistici

TEST DI FLAVOUR-DEPENDENCE

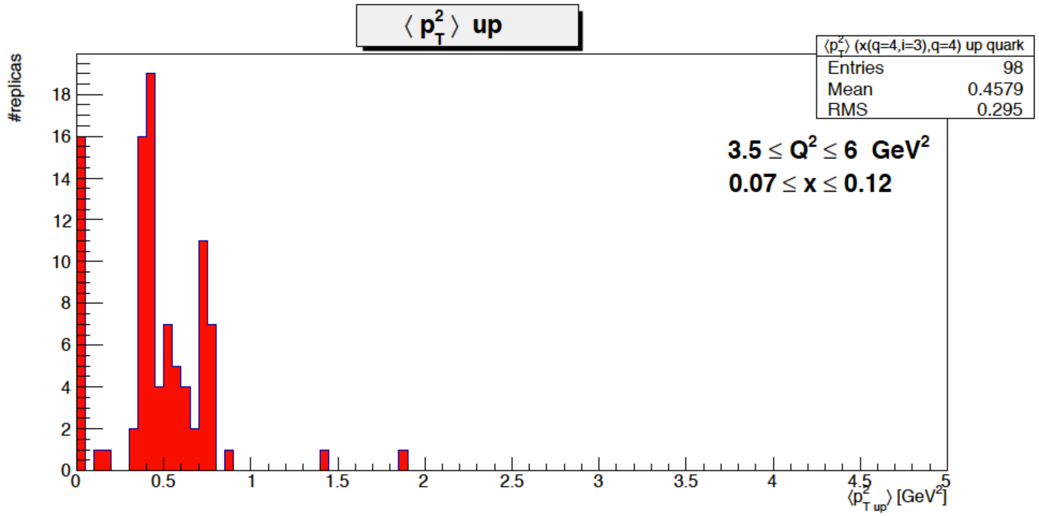


$$\langle p^2_{T,\mathrm{sea}} \rangle = 0.40 \pm 0.38 \ \mathrm{GeV}^2$$

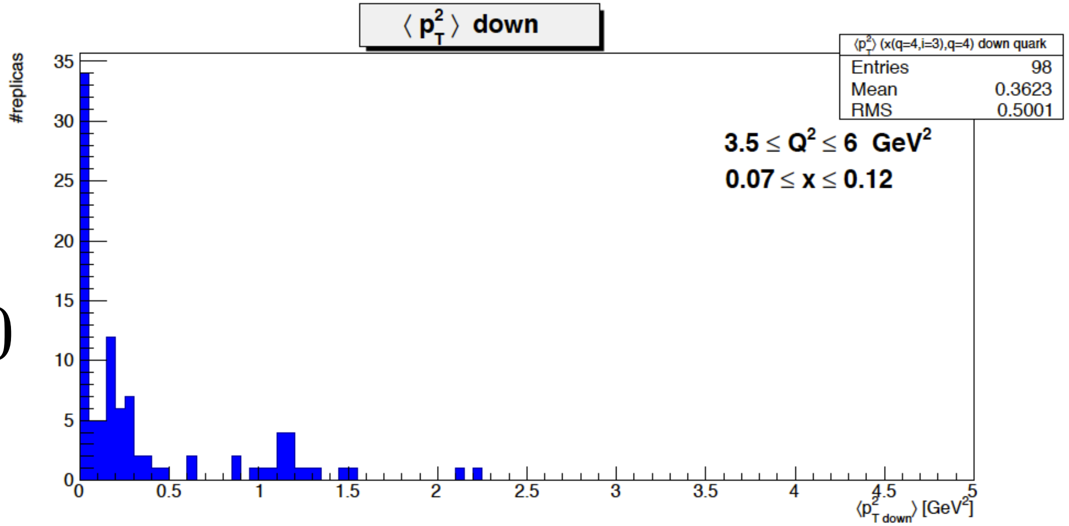


$$\langle p^2_{T,\mathrm{up}} \rangle = 0.45 \pm 0.29 \ \mathrm{GeV}^2$$

TEST DI FLAVOUR-DEPENDENCE



$$\langle p^2_{T,\mathrm{up}} \rangle = 0.45 \pm 0.29$$



$$\langle p^2_{T,\mathrm{down}} \rangle = 0.36 \pm 0.50$$

TEST DI FLAVOUR-DEPENDENCE

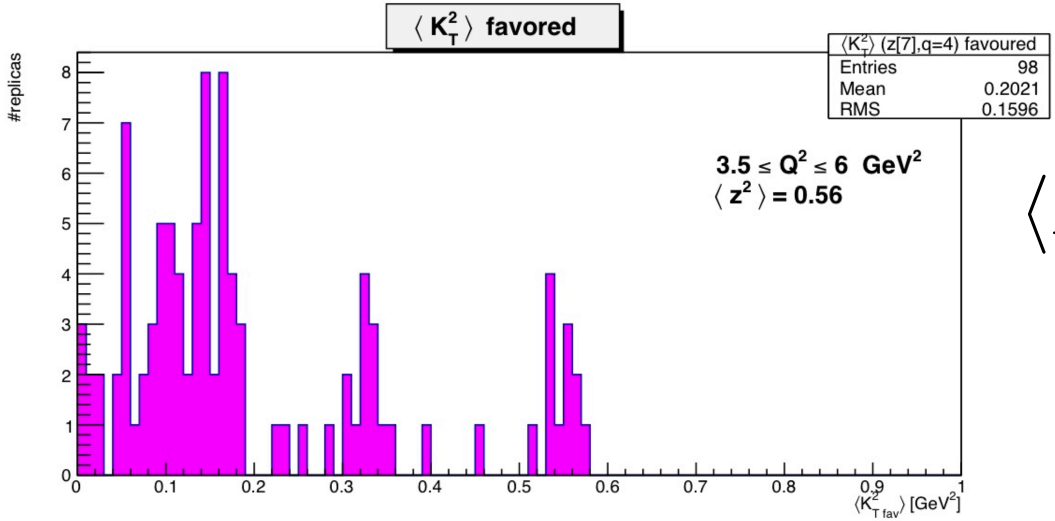
$$\langle K^2_{T,\mathrm{fav}}\rangle < \langle K^2_{T,\mathrm{unf}}\rangle \quad \text{for low } z \text{ values}$$

$$\langle K^2_{T,\mathrm{fav}}\rangle > \langle K^2_{T,\mathrm{unf}}\rangle \quad \text{for high } z \text{ values}$$

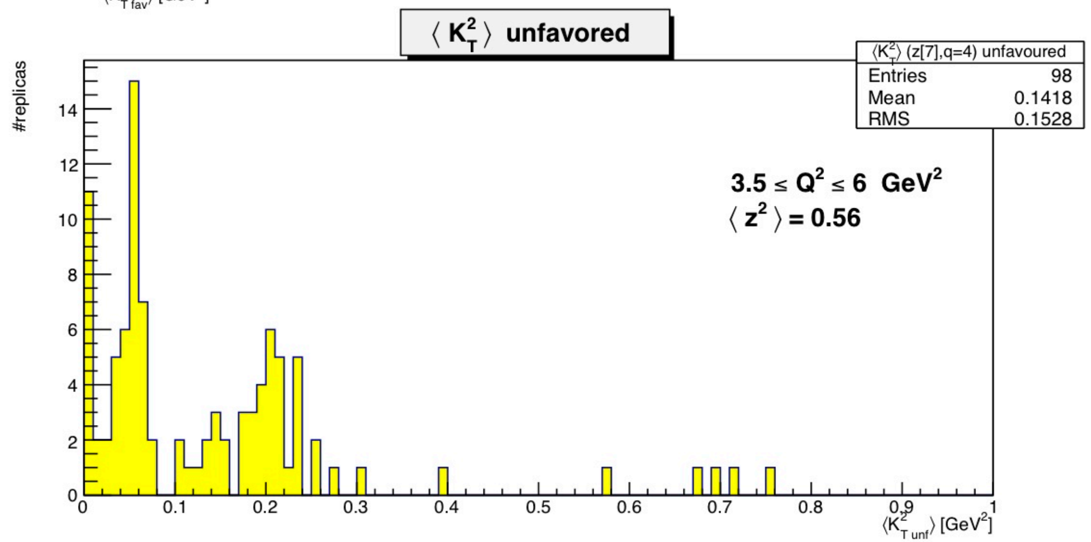
$\langle z^2\rangle$	$\langle K^2_{T,fav}\rangle$ [GeV^2]	$\langle K^2_{T,unf}\rangle$ [GeV^2]
0.08	0.19 ± 0.08	0.21 ± 0.05
0.14	0.24 ± 0.07	0.23 ± 0.10
0.56	0.20 ± 0.15	0.14 ± 0.15

Table 4.7: Mean values and standard deviations for $\langle K_T^2\rangle(z, Q^2)$ for different z values, with $3.5 < Q^2 < 6$.

TEST DI FLAVOUR-DEPENDENCE



$$\langle K^2_{T,\mathrm{fav}} \rangle = 0.20 \pm 0.15$$
$$\mathrm{GeV}^2$$



$$\langle K^2_{T,\mathrm{unf}} \rangle = 0.14 \pm 0.15$$
$$\mathrm{GeV}^2$$

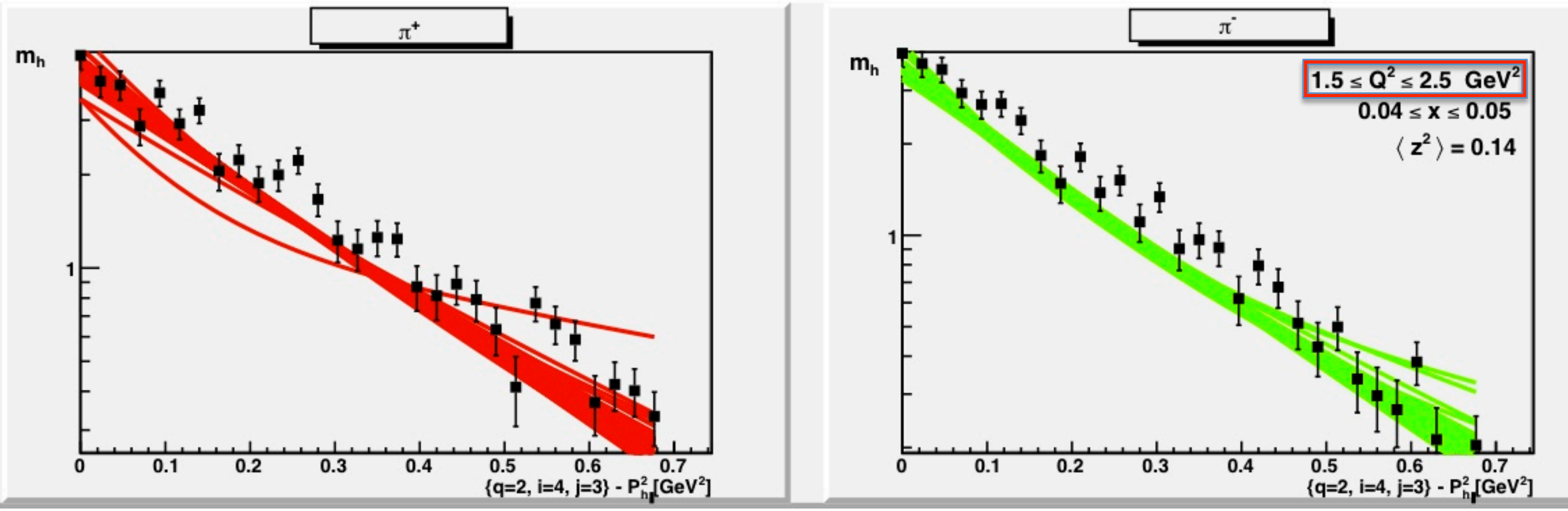
TEST DI FLAVOUR-DEPENDENCE

Criterio di selezione sulle repliche:

$$\chi^2/dof < 5$$

$$\langle p^2_{T,\mathrm{up}} \rangle \neq \langle p^2_{T,\mathrm{down}} \rangle \neq \langle p^2_{T,\mathrm{sea}} \rangle$$

$$\langle K^2_{T,\mathrm{fav}} \rangle \neq \langle K^2_{T,\mathrm{unf}} \rangle$$



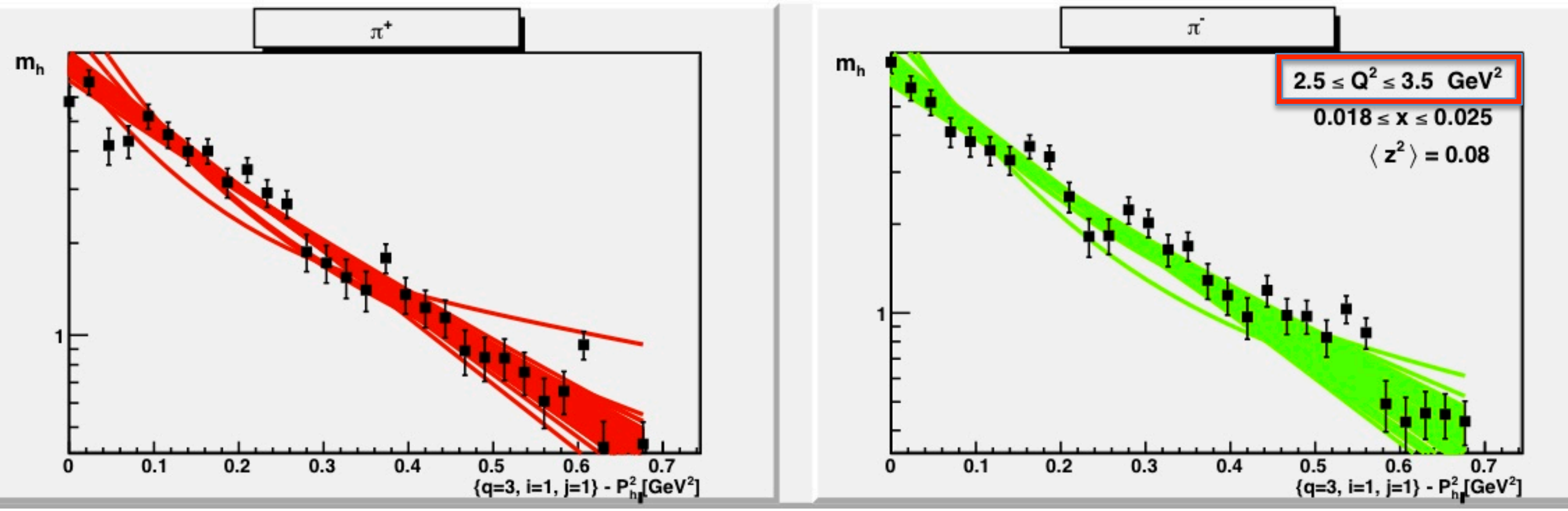
TEST DI FLAVOUR-DEPENDENCE

Criterio di selezione sulle repliche:

$$\chi^2/dof < 5$$

$$\langle p^2_{T,\mathrm{up}} \rangle \neq \langle p^2_{T,\mathrm{down}} \rangle \neq \langle p^2_{T,\mathrm{sea}} \rangle$$

$$\langle K^2_{T,\mathrm{fav}} \rangle \neq \langle K^2_{T,\mathrm{unf}} \rangle$$



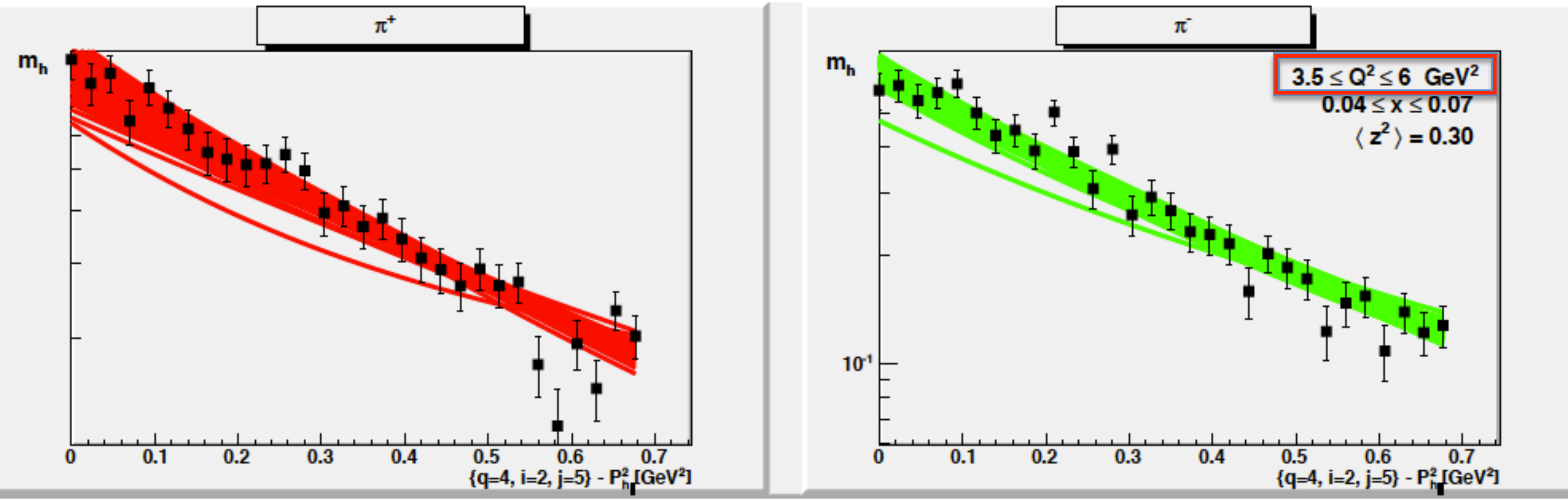
TEST DI FLAVOUR-DEPENDENCE

Criterio di selezione sulle repliche:

$$\chi^2/dof < 5$$

$$\langle p^2_{T,\mathrm{up}} \rangle \neq \langle p^2_{T,\mathrm{down}} \rangle \neq \langle p^2_{T,\mathrm{sea}} \rangle$$

$$\langle K^2_{T,\mathrm{fav}} \rangle \neq \langle K^2_{T,\mathrm{unf}} \rangle$$



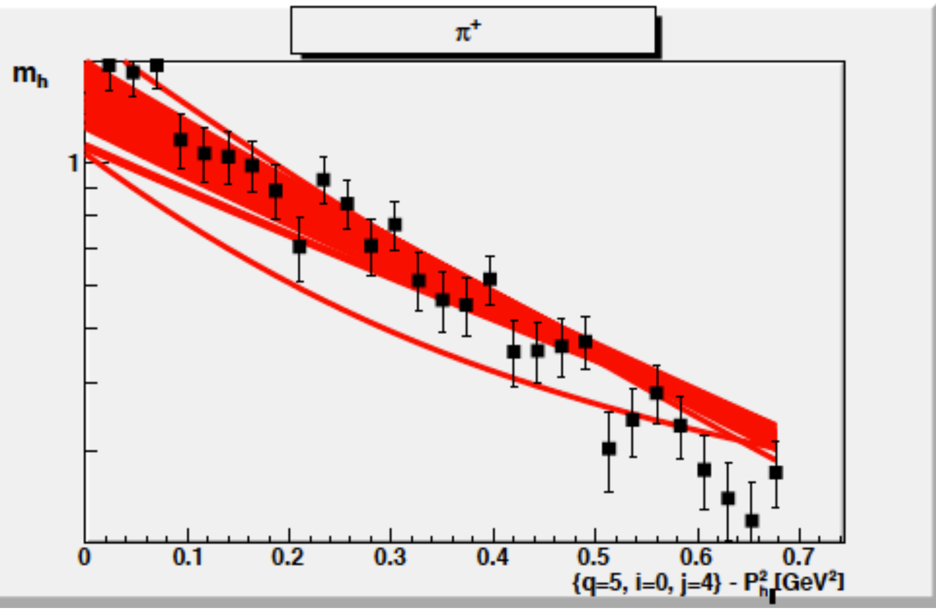
TEST DI FLAVOUR-DEPENDENCE

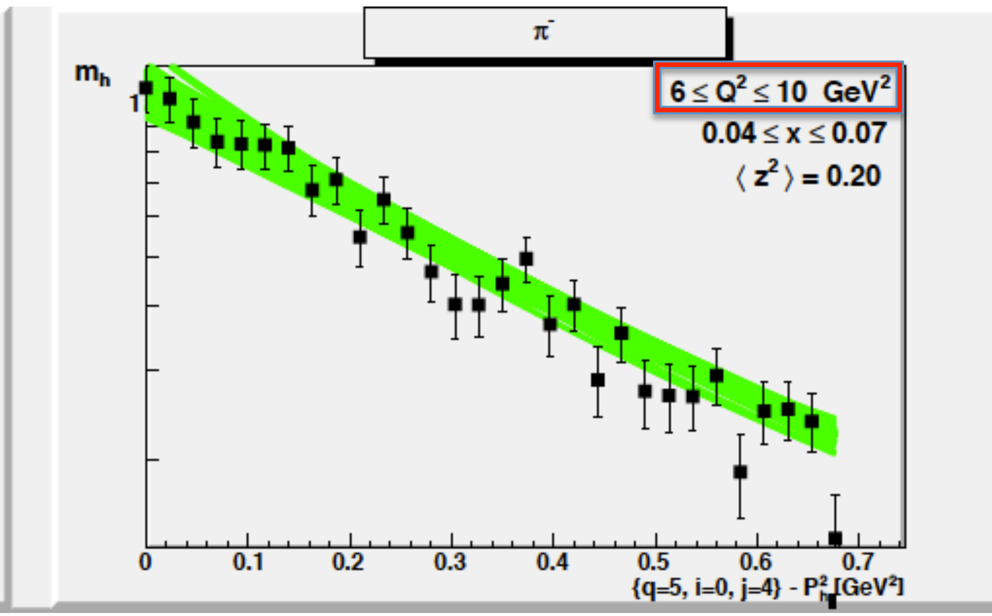
Criterio di selezione sulle repliche:

$$\chi^2/dof < 5$$

$$\langle p_{T,\text{up}}^2 \rangle \neq \langle p_{T,\text{down}}^2 \rangle \neq \langle p_{T,\text{sea}}^2 \rangle$$

$$\langle K_{T,\text{fav}}^2 \rangle \neq \langle K_{T,\text{unf}}^2 \rangle$$

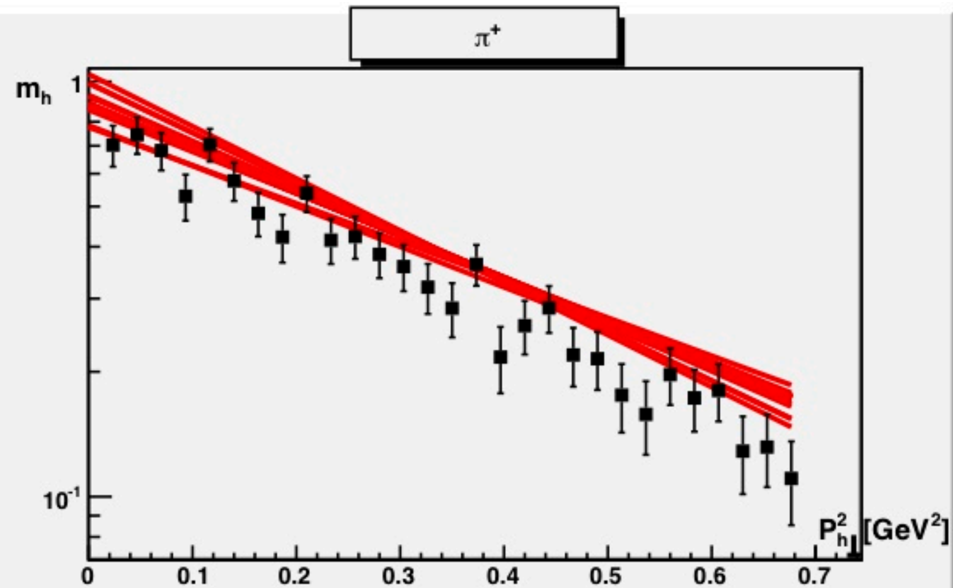


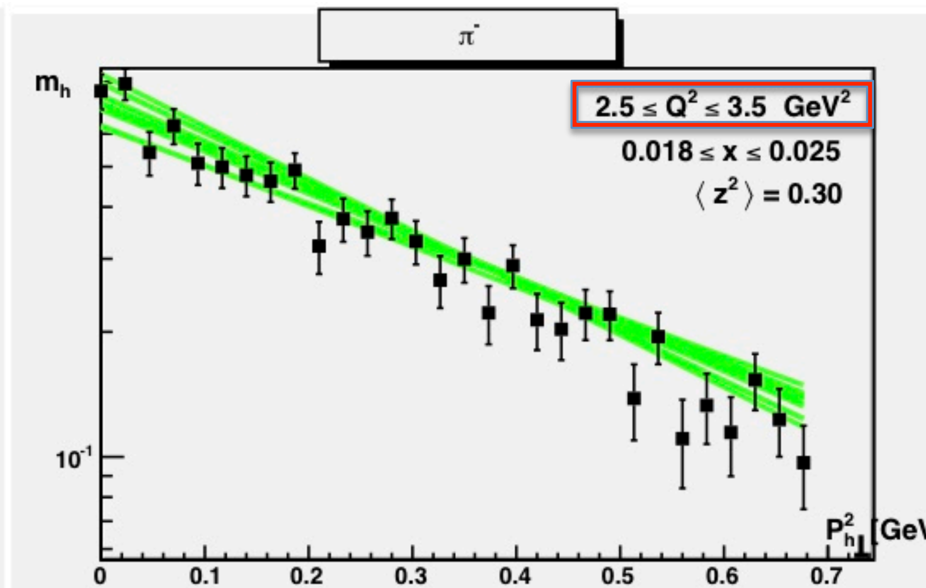


TEST DI FLAVOUR-INDEPENDENCE

Si fittano due set di *dati differenti* con la *stessa funzione*

$$\langle p_{T,\mathrm{up}}^2 \rangle = \langle p_{T,\mathrm{down}}^2 \rangle = \langle p_{T,\mathrm{sea}}^2 \rangle$$
$$\langle K_{T,\mathrm{fav}}^2 \rangle = \langle K_{T,\mathrm{unf}}^2 \rangle$$

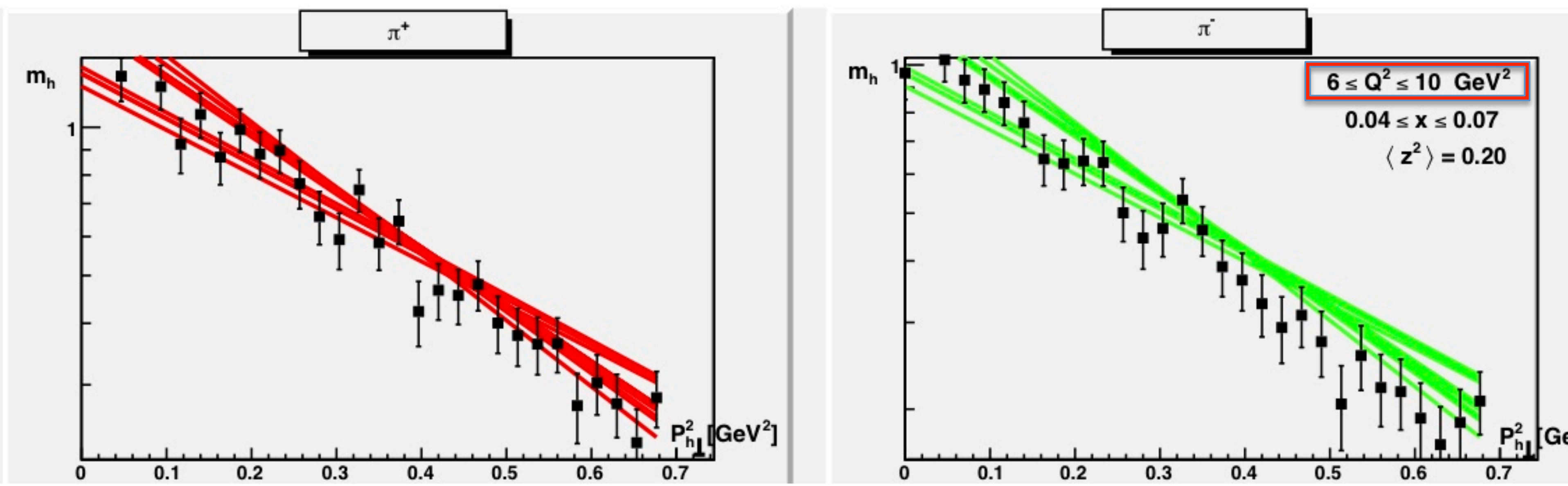




TEST DI FLAVOUR-INDEPENDENCE

Si fittano due set di *dati differenti* con la *stessa funzione*

$$\langle p^2_{T,\mathrm{up}} \rangle = \langle p^2_{T,\mathrm{down}} \rangle = \langle p^2_{T,\mathrm{sea}} \rangle$$
$$\langle K^2_{T,\mathrm{fav}} \rangle = \langle K^2_{T,\mathrm{unf}} \rangle$$



TEST DI FLAVOUR-INDEPENDENCE

$$\boxed{\text{sea} = \text{up} = \text{down} \ , \ \ \text{fav} = \text{unf}}$$

$$\boxed{3.5 < Q^2 < 6 \ \text{GeV}^2 \ , \ 0.07 < x < 0.12 \ , \ \langle z^2 \rangle = 0.11}$$

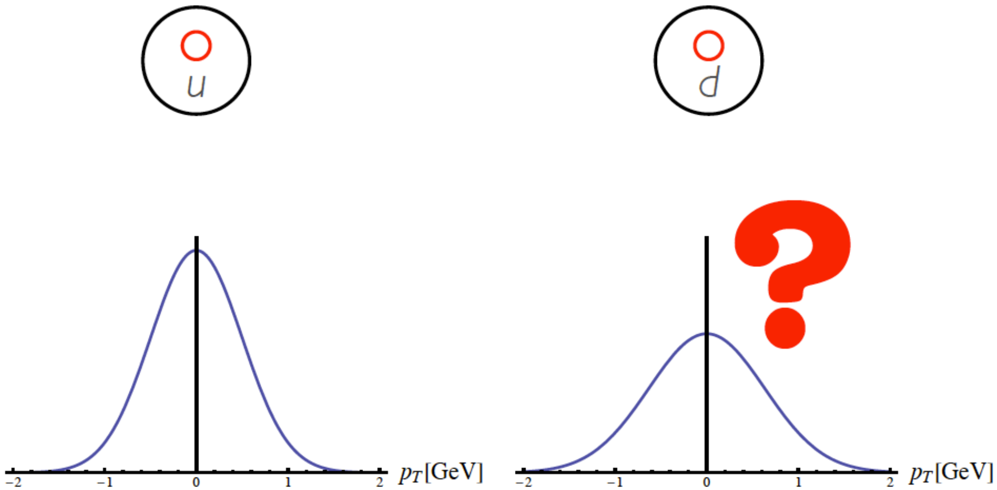
$$\langle p_T^2 \rangle = 0.76 \pm 0.15$$

Valori medi
ed errori statistici

$$\langle K_T^2 \rangle = 0.16 \pm 0.03$$

ALTRE ANALISI

JEFFERSON LAB



Jefferson Lab

"Transverse momentum dependence of semi-inclusive pion production."
Physics Letters B 665 (2008) 20–25

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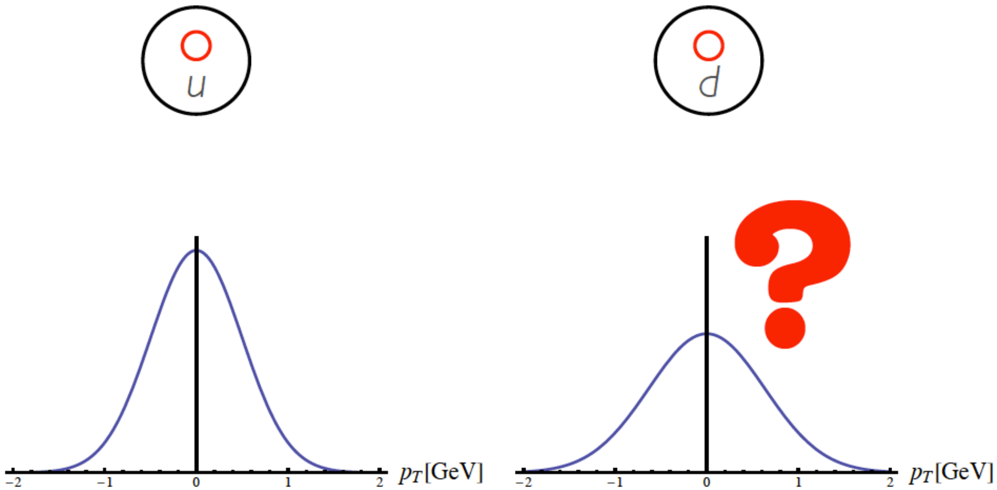

"Transverse momentum dependence of semi-inclusive pion production."
Physics Letters B 665 (2008) 20–25

COMPASS

$$1.5 < Q^2 < 10\ \mathrm{GeV}^2$$
$$0.012 < x < 0.12$$
$$0.08 < \langle z^2 \rangle < 0.56$$
$$P_{h\perp}^2 < 0.7\ \mathrm{GeV}^2$$

Jefferson Lab

$$2 < Q^2 < 4\ \mathrm{GeV}^2$$
$$0.2 < x < 0.5$$
$$0.3 < z < 1$$
$$P_{h\perp}^2 < 0.2\ \mathrm{GeV}^2$$

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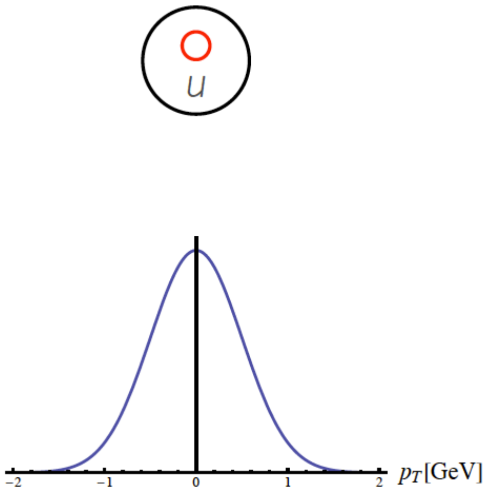

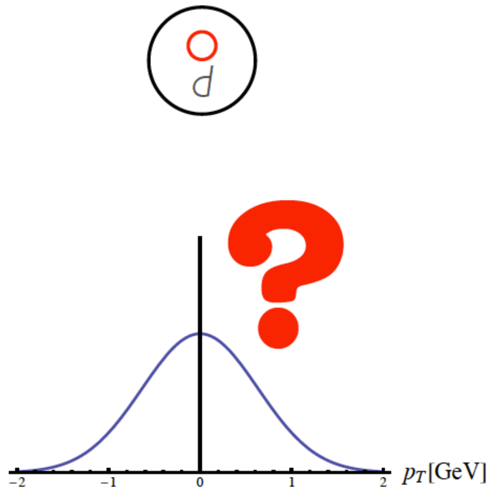

"Transverse momentum dependence of semi-inclusive pion production."
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$$\boxed{\langle p_{T,\mathrm{up}}^2 \rangle = -0.01 \pm 0.04 \ \mathrm{GeV}^2}$$

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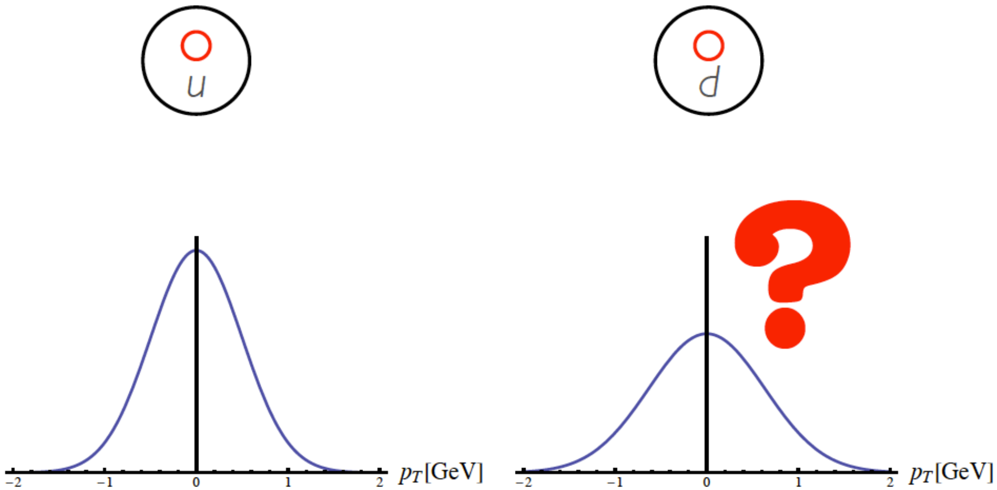

"Transverse momentum dependence of semi-inclusive pion production."
Physics Letters B 665 (2008) 20–25

$$\boxed{\begin{aligned} \langle p_{T,\mathrm{up}}^2 \rangle &= -0.01 \pm 0.04\ \mathrm{GeV}^2 \\ \langle p_{T,\mathrm{down}}^2 \rangle &= 0.22 \pm 0.13\ \mathrm{GeV}^2 \end{aligned}}$$

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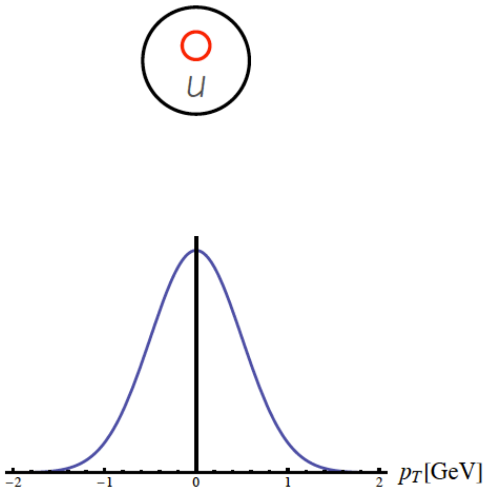

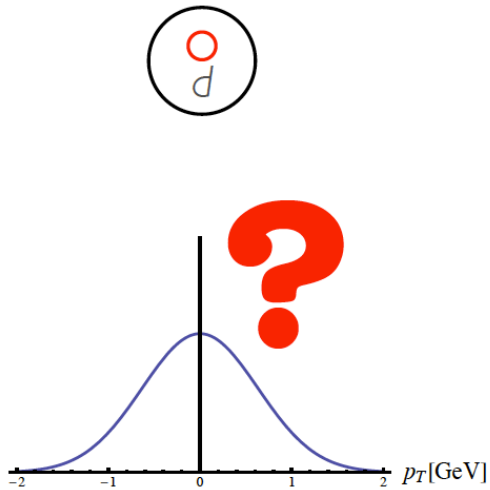

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"Transverse momentum dependence of semi-inclusive pion production."
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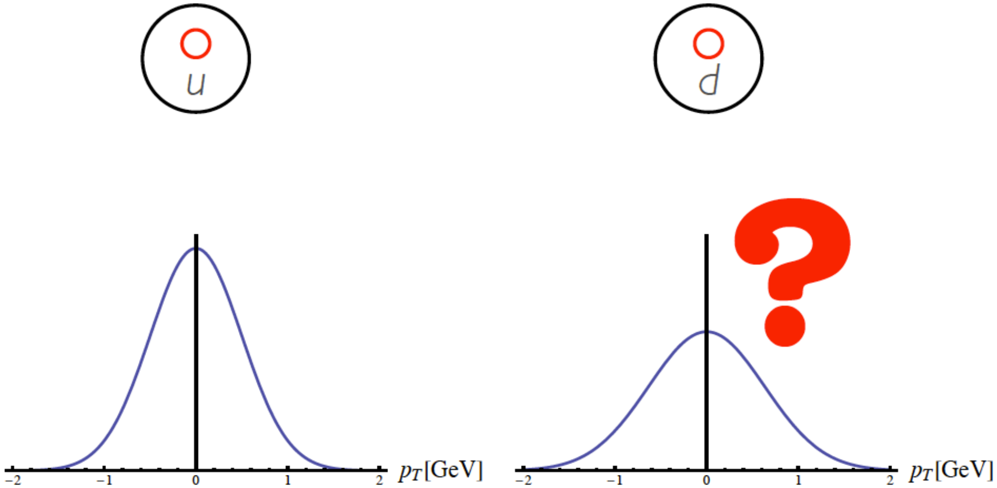
$$\boxed{\begin{aligned}\langle p^2_{T,\mathrm{up}}\rangle &= -0.01 \pm 0.04 \ \mathrm{GeV}^2 \\ \langle p^2_{T,\mathrm{down}}\rangle &= 0.22 \pm 0.13 \ \mathrm{GeV}^2\end{aligned}}$$

Incompatibili in una deviazione standard

LATTICE QCD



QCD su reticolo

"Exploring quark transverse momentum distributions with lattice QCD."
Phys.Rev. D83 (2011) 094507

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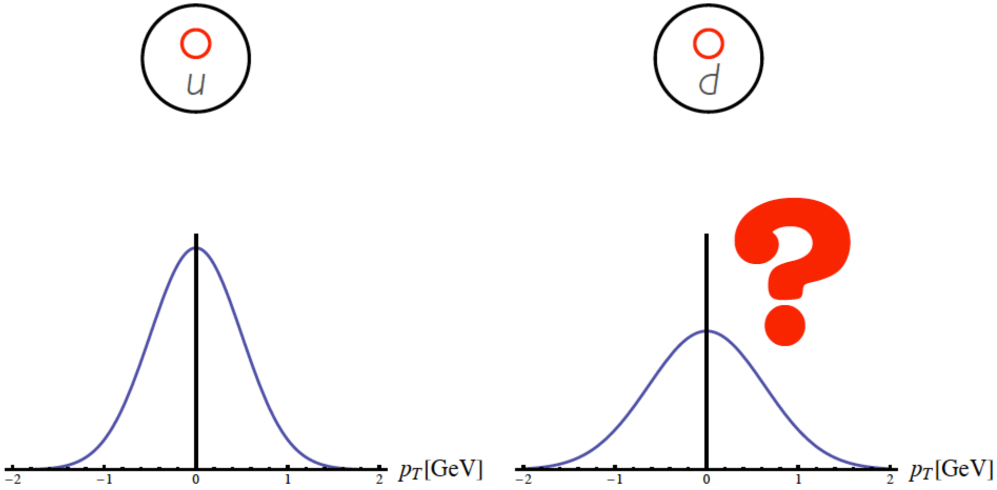

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1. **Valori integrati su x**

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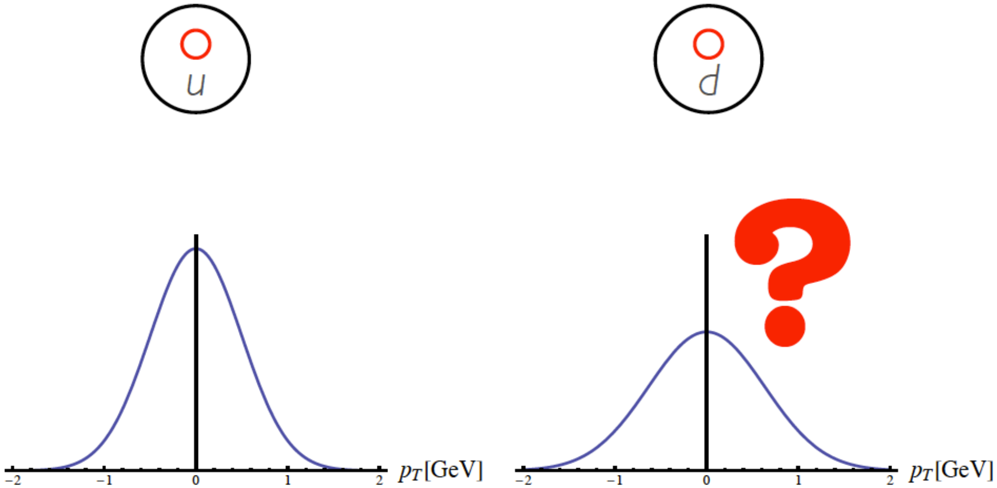

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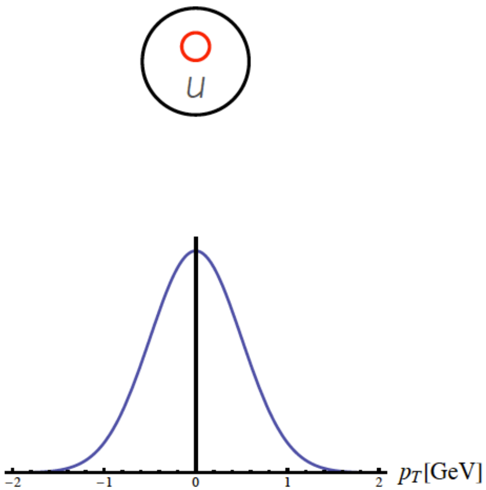
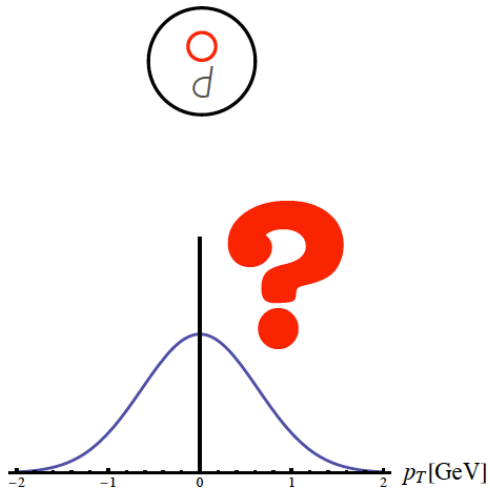
1. **Valori integrati su x**

2. **La dipendenza dalla scala energetica Q^2 non è definita**

LATTICE QCD





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$$\boxed{\langle p_{T,\mathrm{up}} \rangle = 0.394 \pm 0.004 \pm 0.027 \ \mathrm{GeV}}$$

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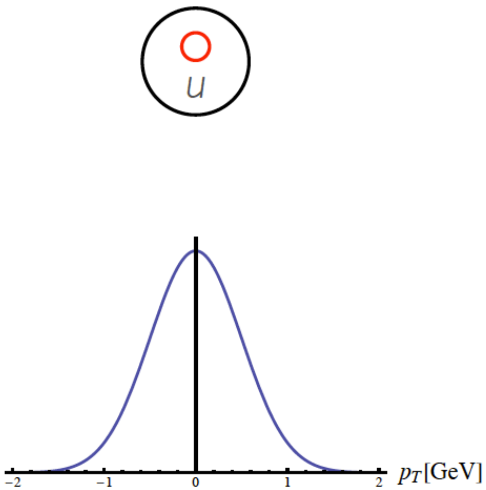

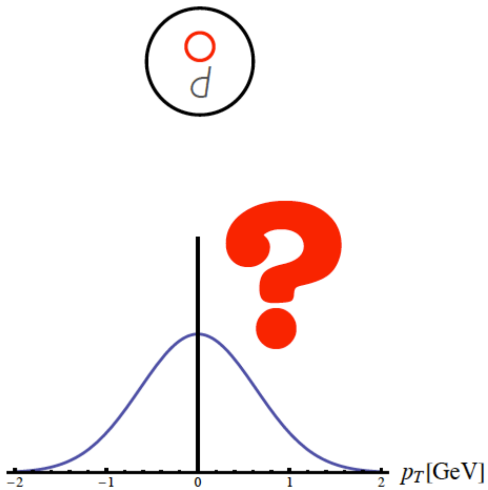

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$$\langle p_{T,\mathrm{up}} \rangle = 0.394 \pm 0.004 \pm 0.027 \ \mathrm{GeV}$$

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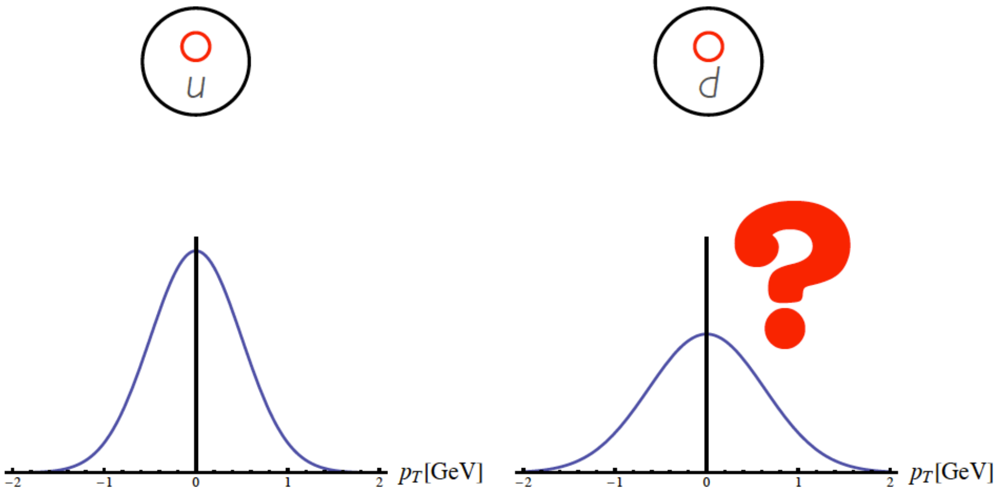

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Compatibili in una deviazione standard

JLAB & LATTICE vs COMPASS

I risultati della presente e delle precedenti analisi sono in ***disaccordo****, ma… :*

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3. Per il momento il fit è basato sui **fit gaussiani** dei **dati preliminari** di COMPASS

PROGETTI FUTURI

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- Jefferson Lab: dati con alta luminosità

PROGETTI FUTURI

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Quark del mare e gluoni ← $\boxed{\text{very high } s \Longrightarrow \text{very low } x}$

CONCLUSIONI

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- L’ipotesi gaussiana con flavour-dependence fitta meglio i dati dello stesso modello flavour-independent

- Il valore medio del momento quadratico medio del quark up è più grande di quello relativo al quark down

- Introducendo la flavour-dependence la molteplicità adronica (ovvero la funzione di struttura non polarizzata) perde il carattere gaussiano

Backup slides

Short-FUTURE PLANS

- Implementare criterio che stabilisca un limite inferiore per i valori di $\langle p_T^2 \rangle$, $\langle K_T^2 \rangle$

- Previsione per il deuterio

- Inserire correttamente l'effetto delle equazioni di evoluzione TMD

JEFFERSON LAB

x_B	$\langle p^2_{T,up}\rangle$ [GeV^2]	$\langle p^2_{T,down}\rangle$ [GeV^2]	$\langle p^2_{T,sea}\rangle$ [GeV^2]
$0.018-0.025$	1.09 ± 1.28	0.32 ± 1.04	1.46 ± 1.09
$0.025-0.04$	0.94 ± 0.99	0.81 ± 1.59	1.21 ± 0.90
$0.07-0.12$	0.45 ± 0.29	0.36 ± 0.50	0.40 ± 0.38

$\langle z^2\rangle$	$\langle K^2_{T,fav}\rangle$ [GeV^2]	$\langle K^2_{T,unf}\rangle$ [GeV^2]
0.08	0.19 ± 0.08	0.21 ± 0.05
0.14	0.24 ± 0.07	0.23 ± 0.10
0.56	0.20 ± 0.15	0.14 ± 0.15

$$3.5 < Q^2 < 6\ \mathrm{GeV}^2$$

COMPASS

$$\langle p^2_{T,up}\rangle = -0.01\pm0.04\ \mathrm{GeV}^2$$
$$\langle p^2_{T,down}\rangle = 0.22\pm0.13\ \mathrm{GeV}^2$$

$$\langle K^2_{T,fav}\rangle = 0.23\pm0.04\ \mathrm{GeV}^2$$
$$\langle K^2_{T,unf}\rangle = 0.19\pm0.04\ \mathrm{GeV}^2$$

Jefferson Lab

$$2 < Q^2 < 4\ \mathrm{GeV}^2$$
$$0.2 < x < 0.5$$
$$0.3 < z < 1$$
$$P^2_{h\perp} < 0.2\ \mathrm{GeV}^2$$

LATTICE QCD

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QCD su reticolo

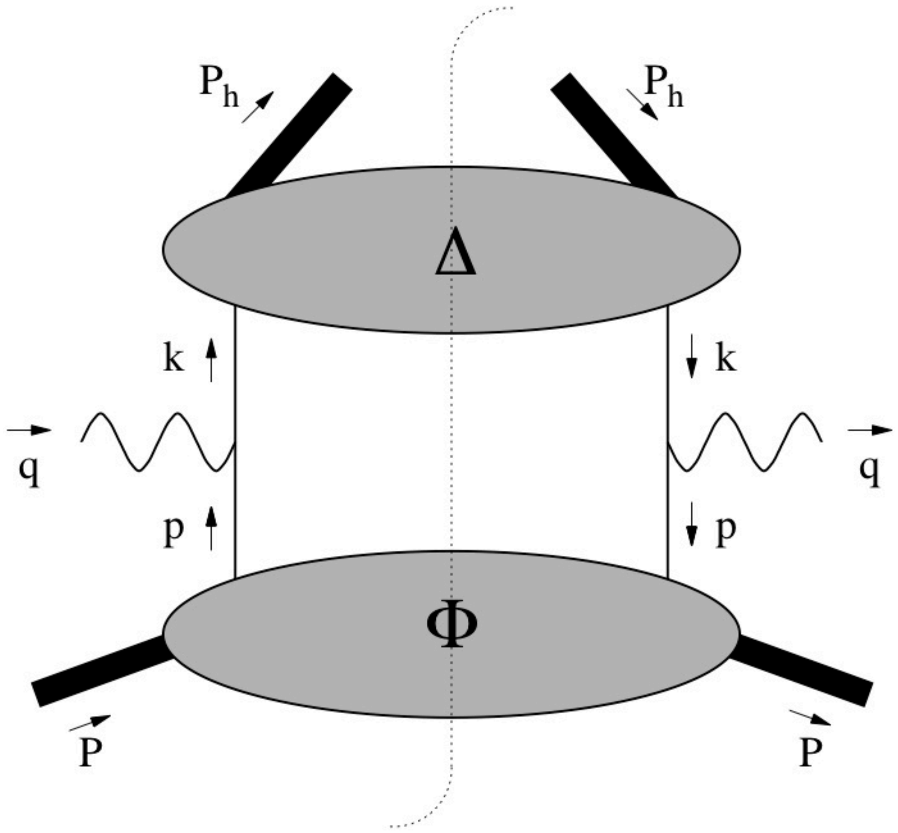
Teoria: TMDs in SIDIS

$$d^{(n)}\sigma \sim W^{\mu\nu} L_{\mu\nu}$$

Tensore adronico

Tensore leptonico

Interpretazione attraverso il Parton model del tensore adronico



Funzione "soft"
Che descrive il processo di adronizzazione

$$\text{parton} \to \text{hadron}$$

Funzione "soft" che descrive
La densità di presenza dei partoni nel nucleone

$$\text{hadron} \to \text{parton}$$

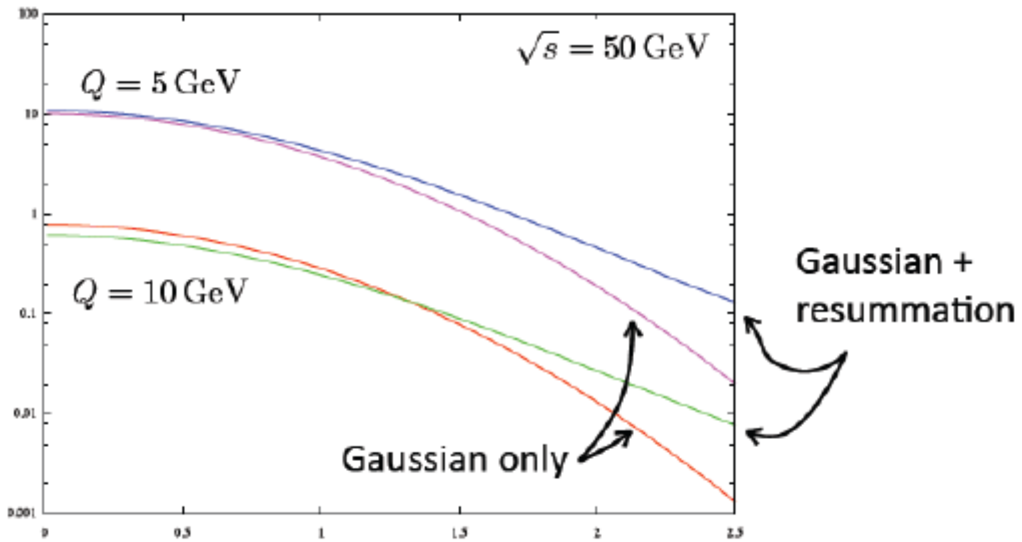
SIDIS at COMPASS

cinematica

- x : frazione di impulso collineare dell'adrone genitore trasportata dal partone
- z : frazione di energia del fotone virtuale portata dall'adrone frammentante
- Q^2 : massa invariante al quadrato del fotone virtuale scambiato

Flavour independent Gaussian ansatz

- pT resummation effects



Flavour independent Gaussian ansatz

- Orbital angular momentum effects

$$f_1(x, p_T^2) = |\psi_{s-\mathrm{wave}}|^2 + |\psi_{p-\mathrm{wave}}|^2 + \ldots$$

$$\text{At low } p_T^2 \ , |\psi_{p-\mathrm{wave}}|^2 \sim p_T^2$$

Because of their non-vanishing value in the limit of zero transverse momentum Gaussian TMDs would agree only with s-wave functions

Flavour independent Gaussian ansatz

- Flavour dependence