Transverse structure of the nucleon
Part 4: Advanced topics

The gauge link

Need of a gauge link

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{ip\cdot\xi} \big\langle P,S \big| \, \bar{\psi}_j(0) \, \psi_i(\xi) \, \big| P,S \big\rangle$$

Need of a gauge link

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$$\psi(\xi) \to e^{i\alpha(\xi)} \, \psi(\xi)$$

Need of a gauge link

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Need of a gauge link

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$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)} \, U(\xi_1,\xi_2) \, e^{-i\alpha(\xi_2)}.$$

Need of a gauge link

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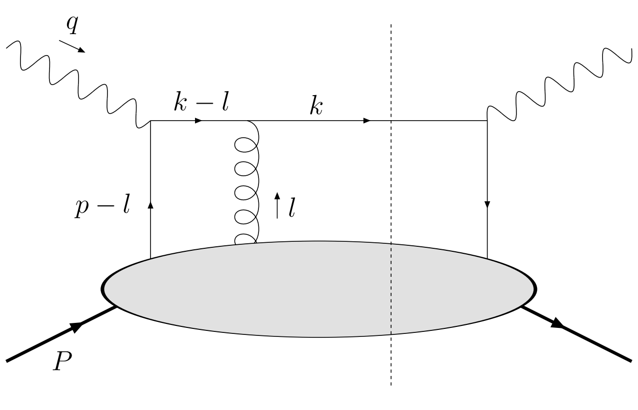
$$\psi(\xi) \to e^{i\alpha(\xi)}\, \psi(\xi)$$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{ip\cdot\xi} \langle P,S \big| \bar{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi) \big| P,S \rangle$$

$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)}\, U(\xi_1,\xi_2)\, e^{-i\alpha(\xi_2)}.$$

$$U_{[a,b]} = \mathcal{P} \exp\left[-ig \int_a^b d\eta^\mu \, A_\mu(\eta) \right]$$

Birth of the gauge link

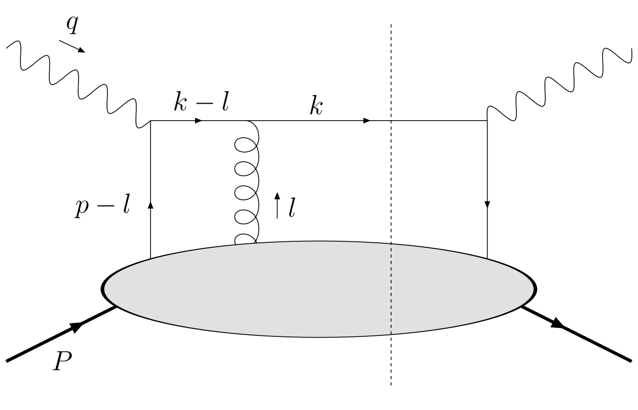


(a)

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \frac{\not k - \not l + m}{(k-l)^2 - m^2 + i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

Birth of the gauge link



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$$i\frac{\not k - \not l + m}{(k-l)^2 - m^2 + i\epsilon} \approx i\frac{k^-\gamma^+}{-2l^+k^- + i\epsilon} \approx \frac{i}{2}\,\frac{\gamma^+}{-l^+ + i\epsilon}$$

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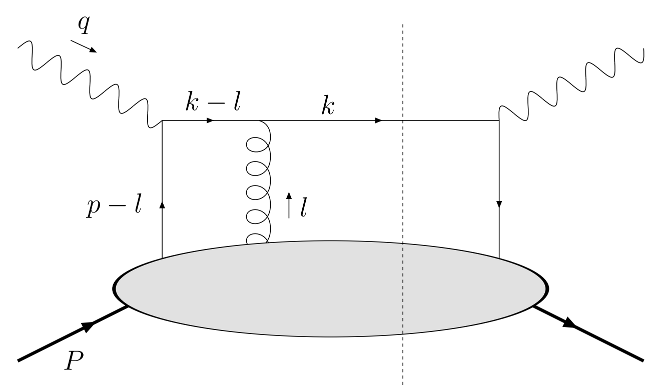
Birth of the gauge link



(a)

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \frac{\not{k}-\not{l}+m}{(k-l)^2-m^2+i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

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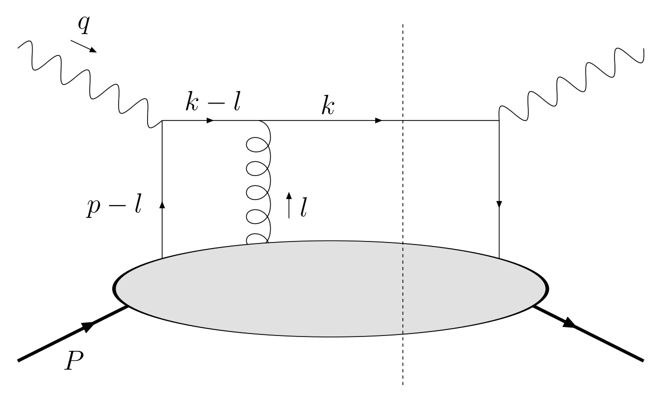
$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\frac{\gamma^-\gamma^+}{2}\gamma_\nu(ig)\frac{A^+(\eta)}{-l^++i\epsilon}\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

Birth of the gauge link



(a)

$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \, \frac{\not{k} - \not{l} + m}{(k-l)^2 - m^2 + i\epsilon} \, \gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

$$i \, \frac{\not{k} - \not{l} + m}{(k-l)^2 - m^2 + i\epsilon} \approx i \frac{k^-\gamma^+}{-2l^+k^- + i\epsilon} \approx \frac{i}{2} \, \frac{\gamma^+}{-l^+ + i\epsilon}$$

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$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0) \, \gamma_\mu\gamma^+ \, \gamma_\nu \, (-ig) \int_{\infty^-}^{\xi^-} \mathrm{d}\eta^- \; A^+(\eta) \; \psi(\xi)|P,S\rangle\Big|_{\eta^+=0;\; \boldsymbol{\eta}_T = \boldsymbol{\xi}_T}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

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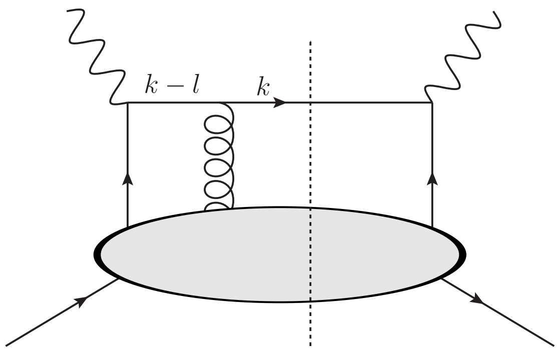
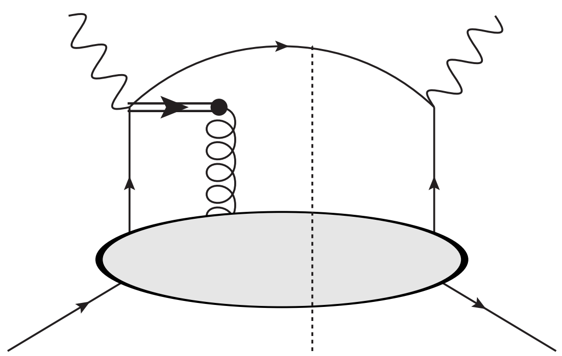


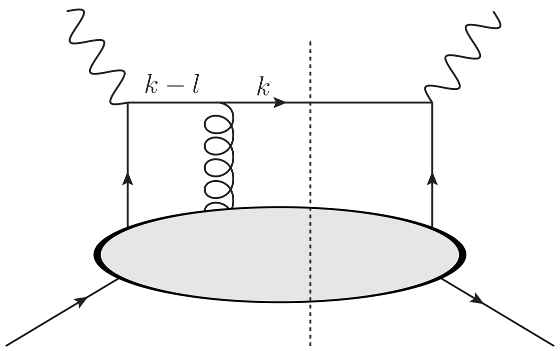
$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\eta^+=0;\;\boldsymbol{\eta}_T=\boldsymbol{\xi}_T}$$



$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2\,\frac{1}{2}\mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$

$$\Phi^{(a)}(x,S) \sim \left\langle P,S\right|\bar{\psi}(0)\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)\left|P,S\right\rangle$$

Birth of the gauge link



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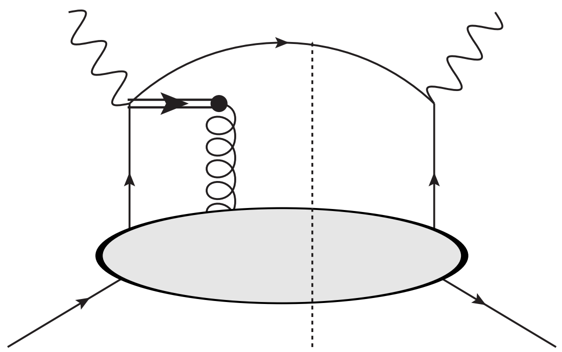

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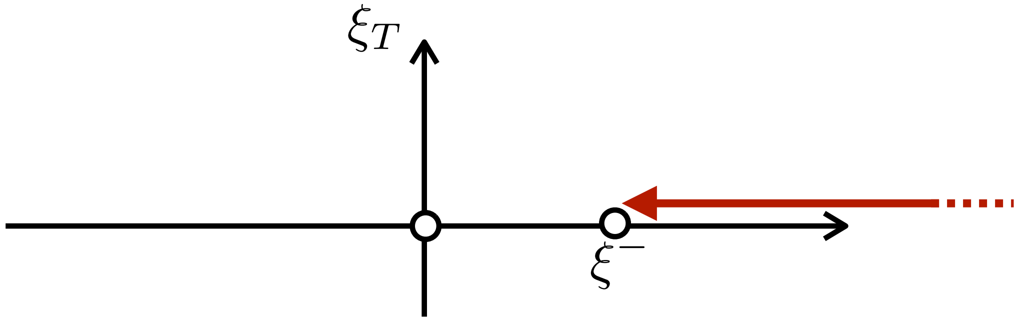
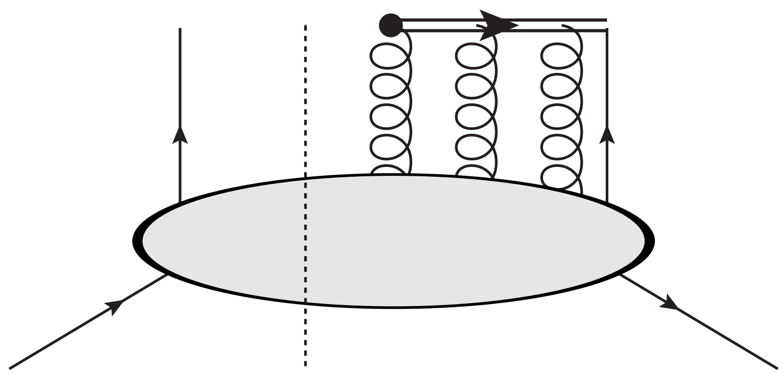
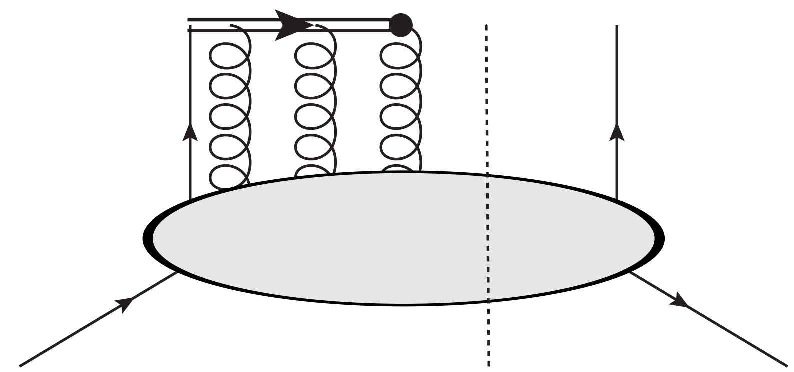


Shape of the gauge link

$$\Phi(x,S) \sim \langle P,S \,|\, \bar{\psi}(0)\, U_{[0,\infty^-]}\, U_{[\infty^-,\xi^-]} \psi(\xi) \,|\, P,S \rangle$$





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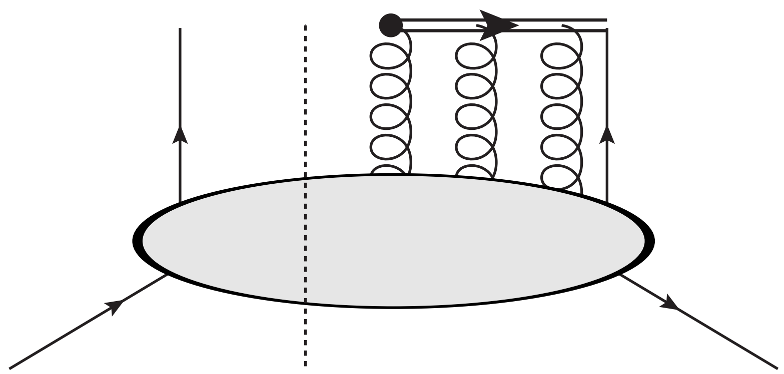

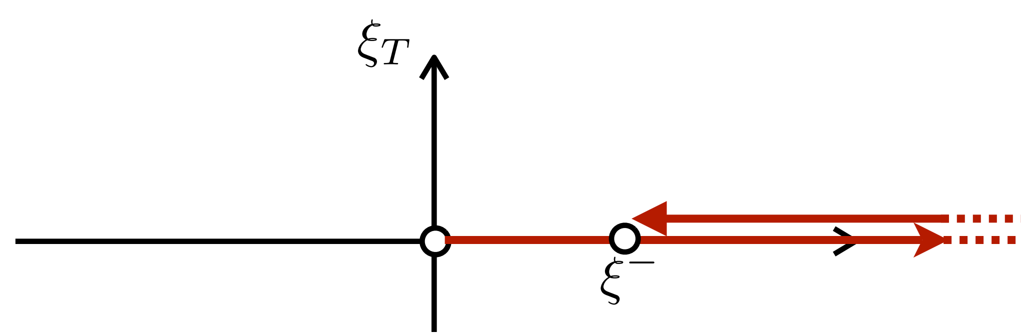

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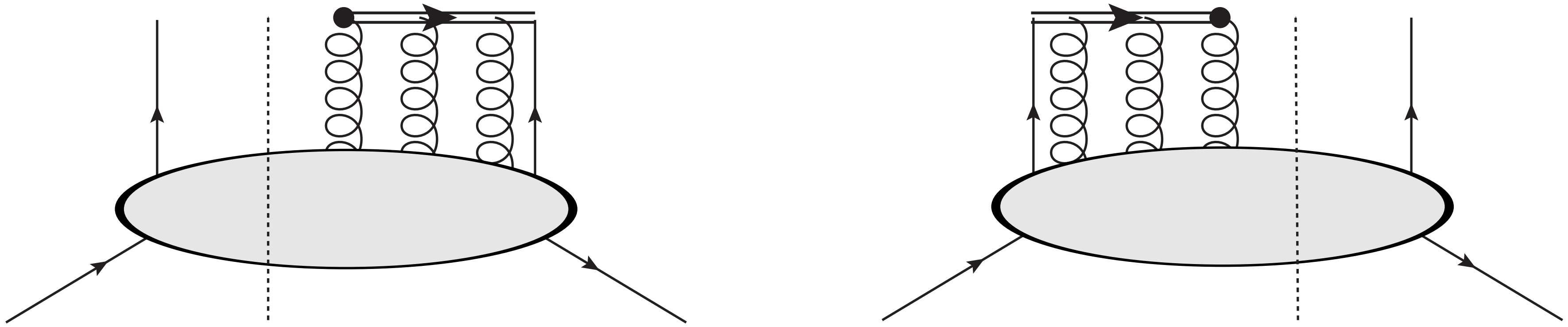
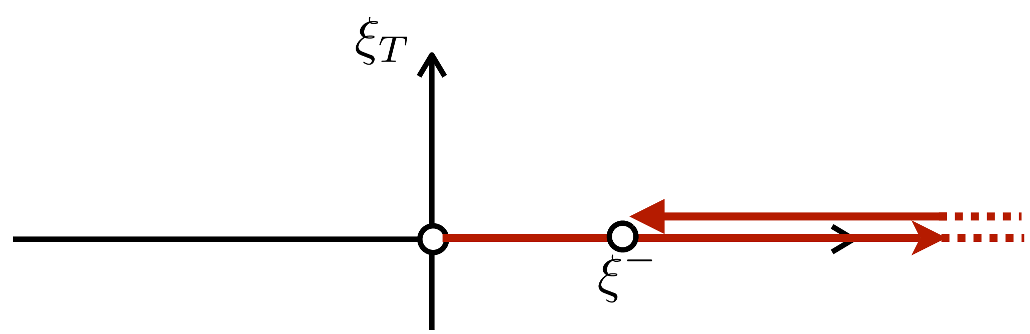
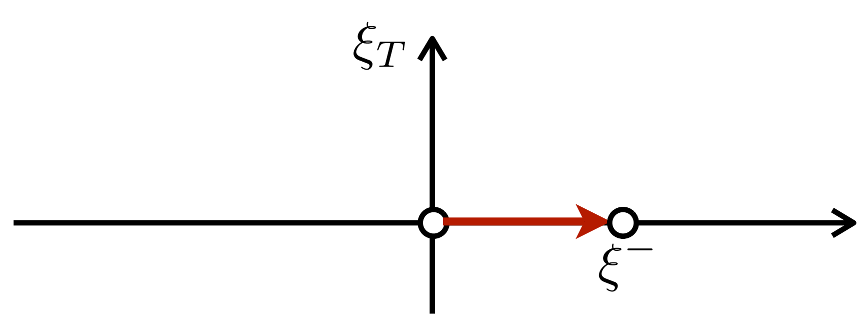
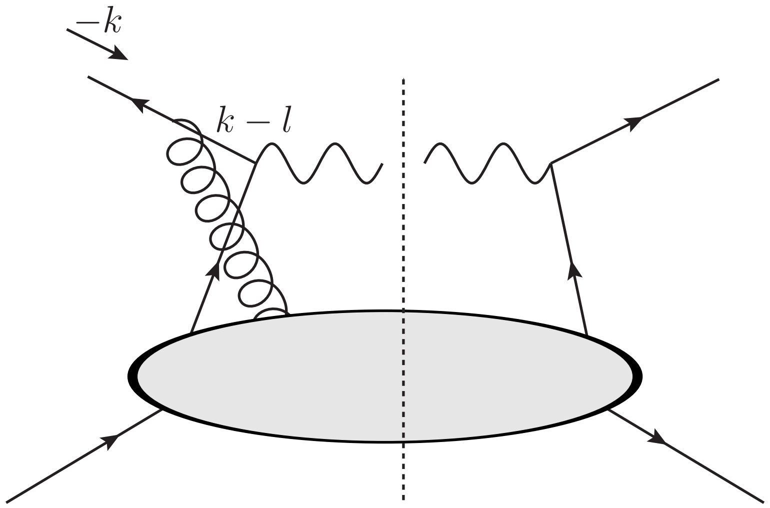






Gauge link in Drell-Yan



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Collins, PLB 536 (02)

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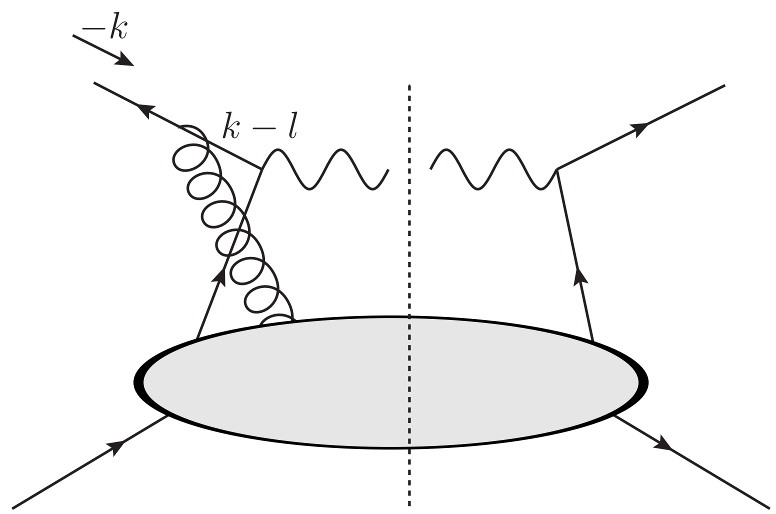

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Collins, PLB 536 (02)

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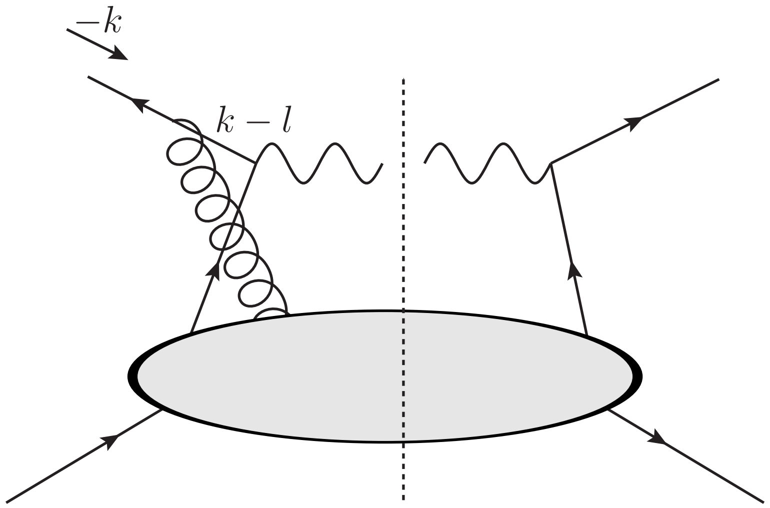

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$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{-\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\eta^+=0;\;\boldsymbol{\eta}_T=\boldsymbol{\xi}_T}$$

Collins, PLB 536 (02)

Gauge link for TMDs

$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{8\pi^3} \, e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

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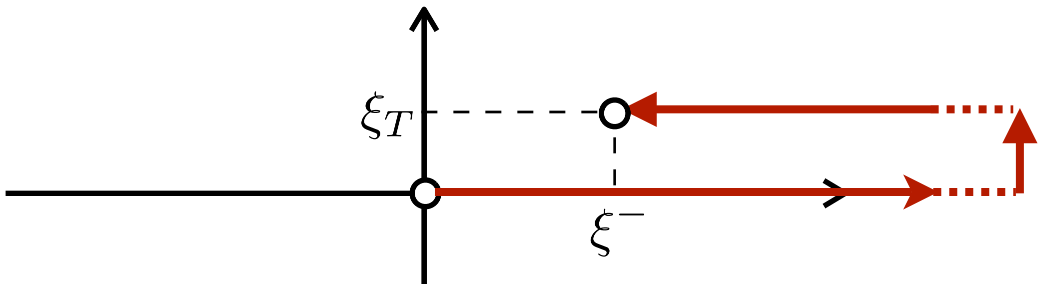
SIDIS



$U_{[+]}$

Gauge link for TMDs

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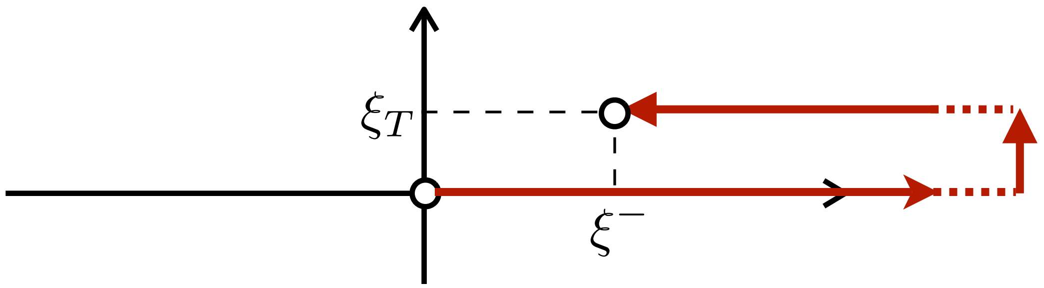
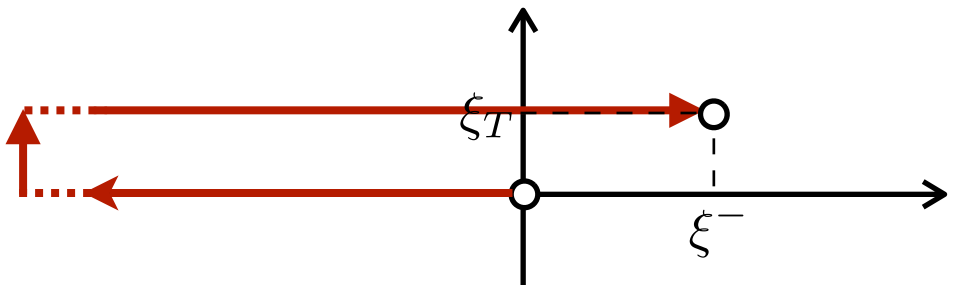
SIDIS



$U_{[+]}$

Drell-Yan



$U_{[-]}$

Gauge link for TMDs

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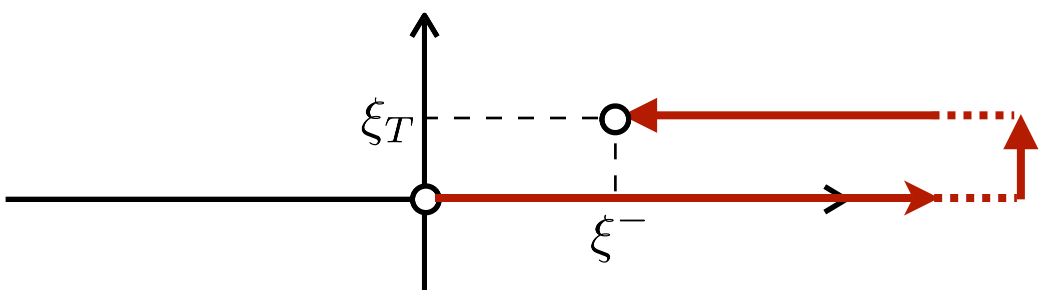
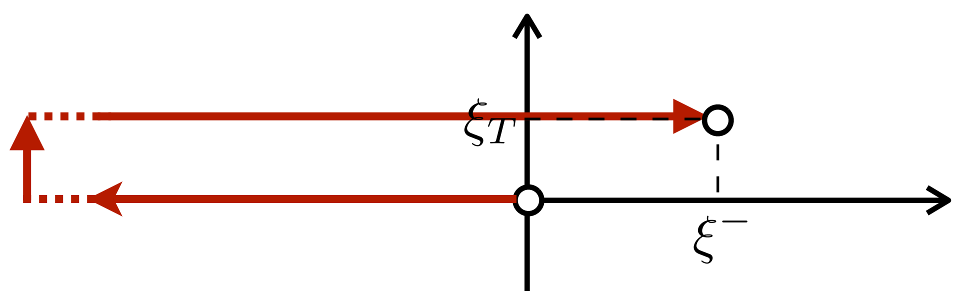
SIDIS



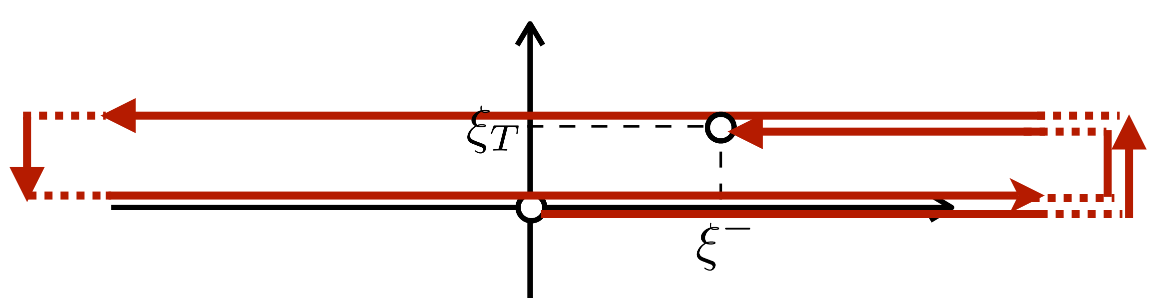
$U_{[+]}$

Drell-Yan



$U_{[-]}$

pp to hadrons



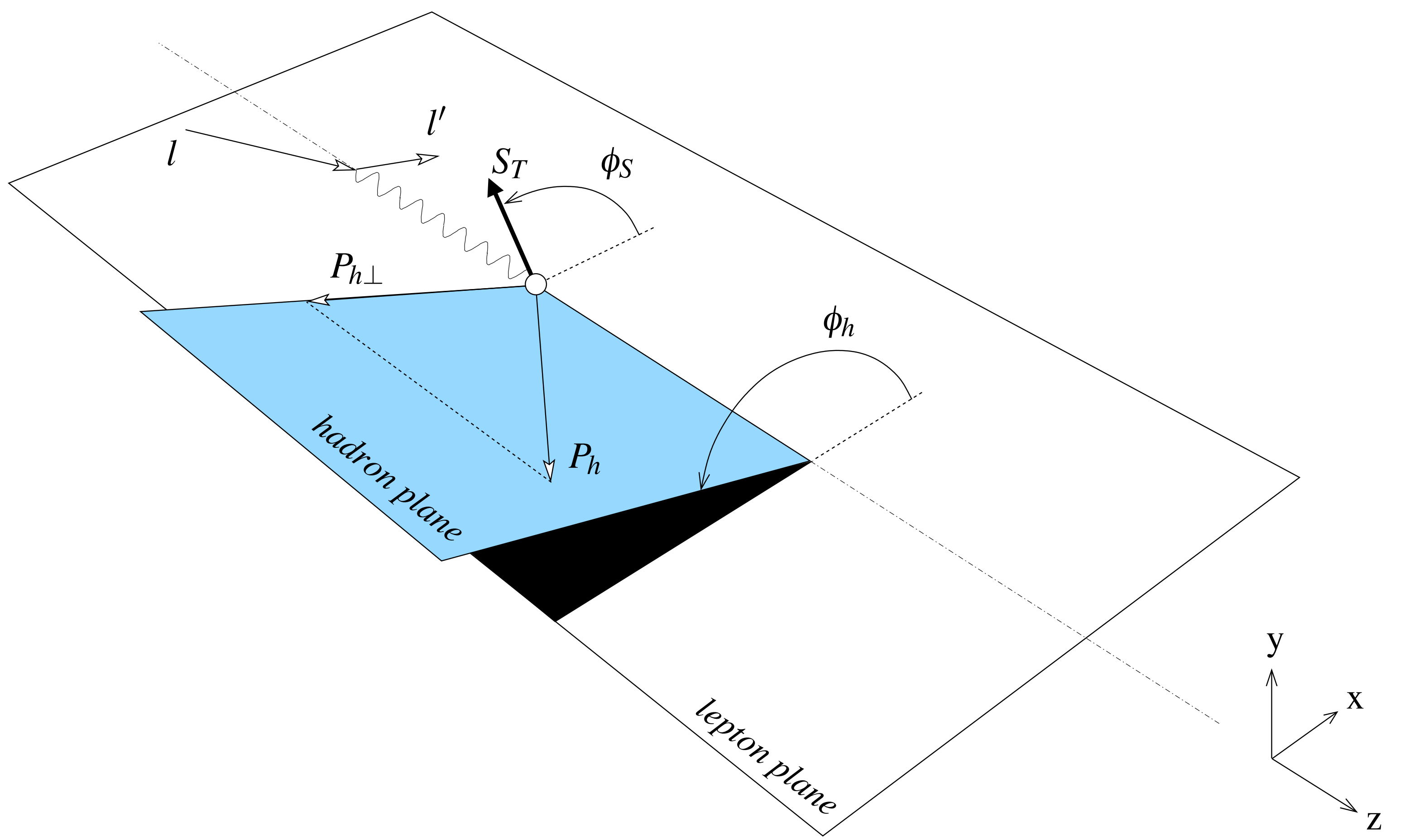
$U_{[\square]} U_{[+]}$

+ several others

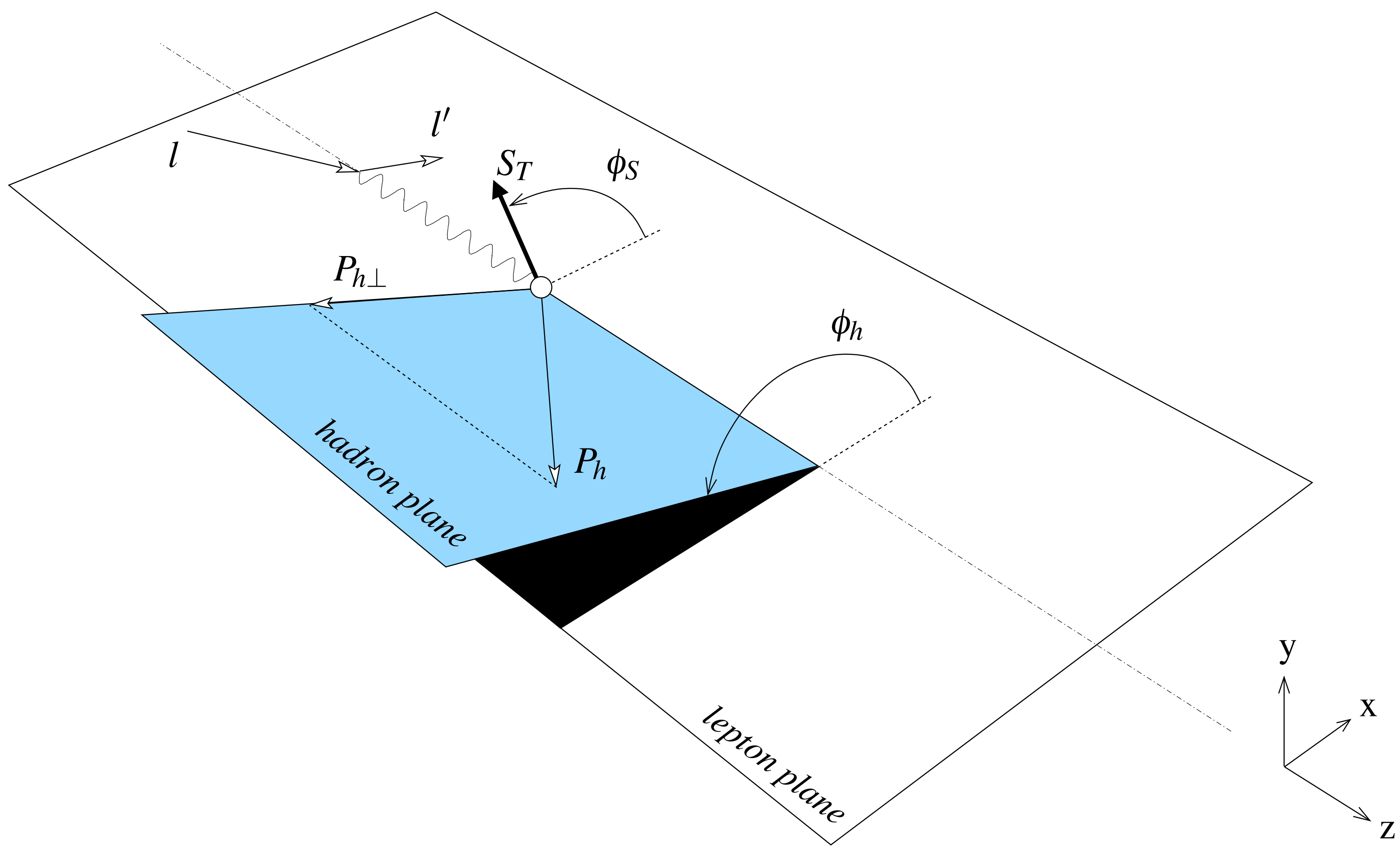
Bomhof, Mulders, Pijlman, PLB 596 (04)

High and low transverse momentum

SIDIS once again

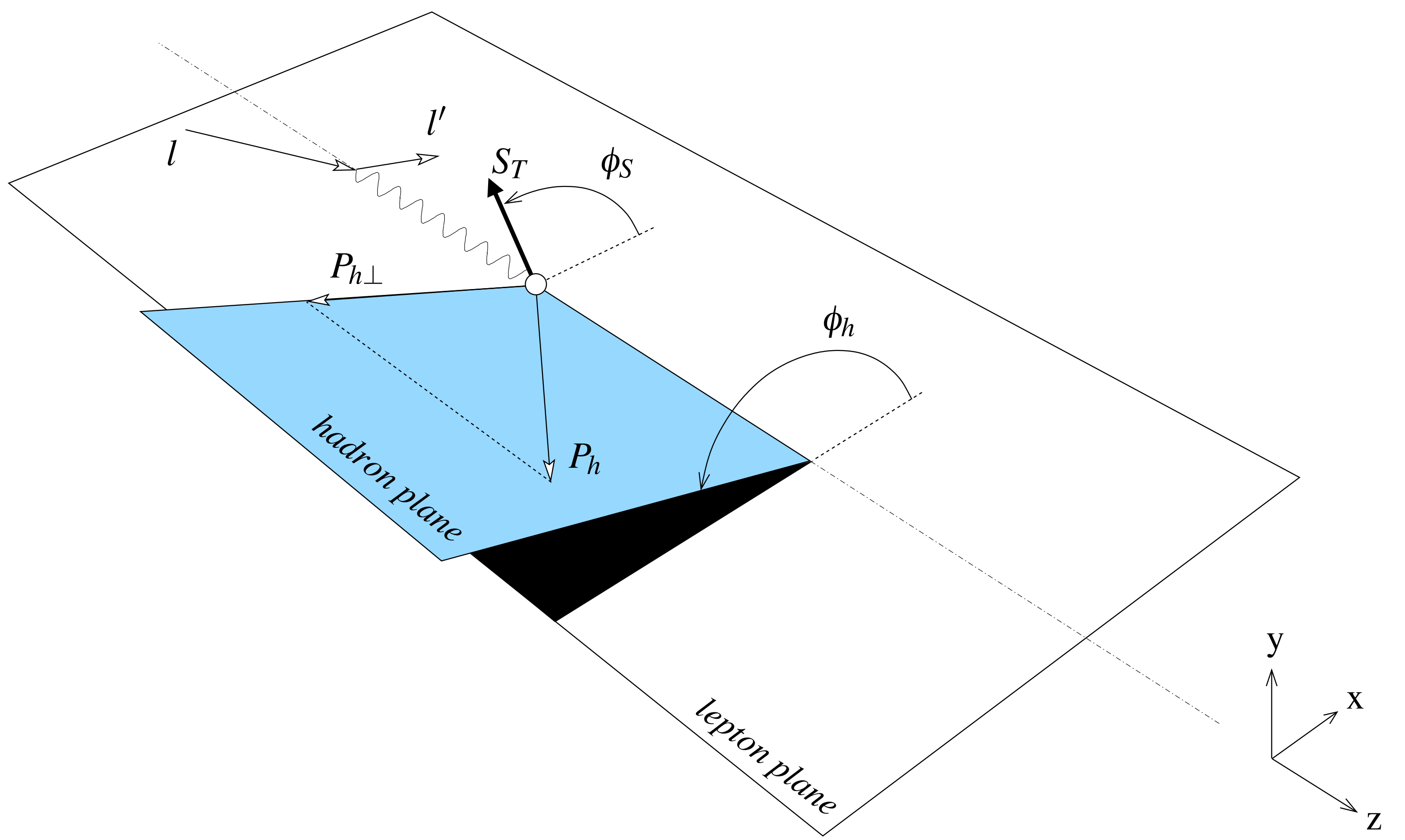


SIDIS once again



$$Q = \text{photon virtuality}$$
$$M = \text{hadron mass}$$
$$P_{h\perp} = \text{hadron transverse momentum}$$

SIDIS once again



Q = photon virtuality

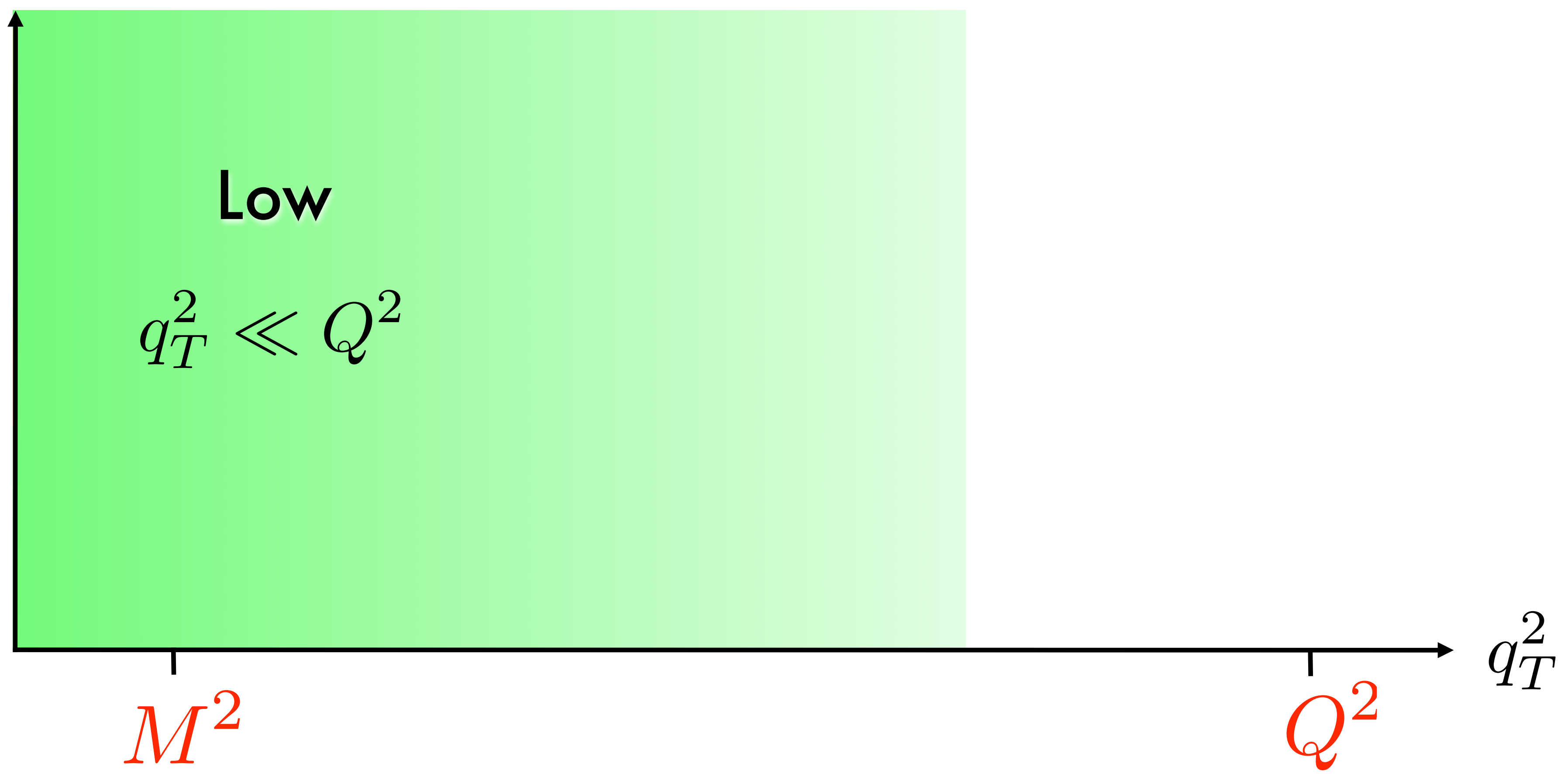
M = hadron mass

$P_{h\perp}$ = hadron transverse momentum

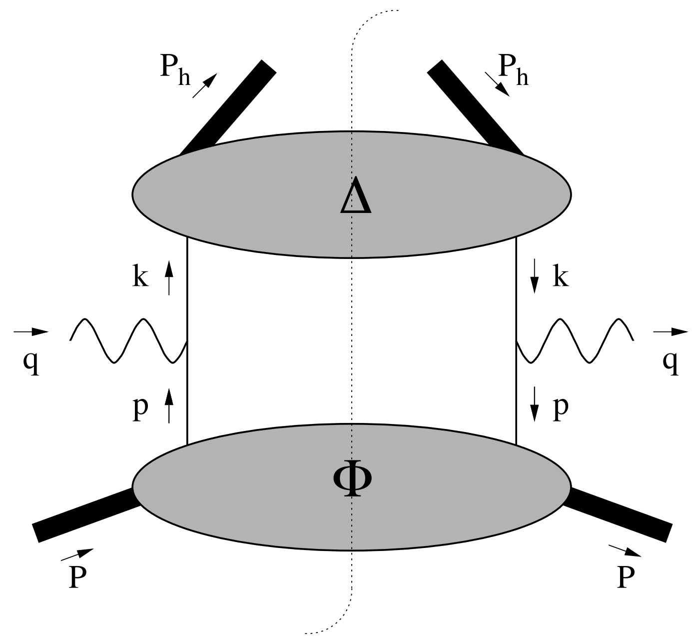
$$q_T^2 \approx P_{h\perp}^2 / z^2$$

Low and high transverse momentum

AB, D. Boer, M. Diehl, P.J. Mulders, JHEP 08 (08)



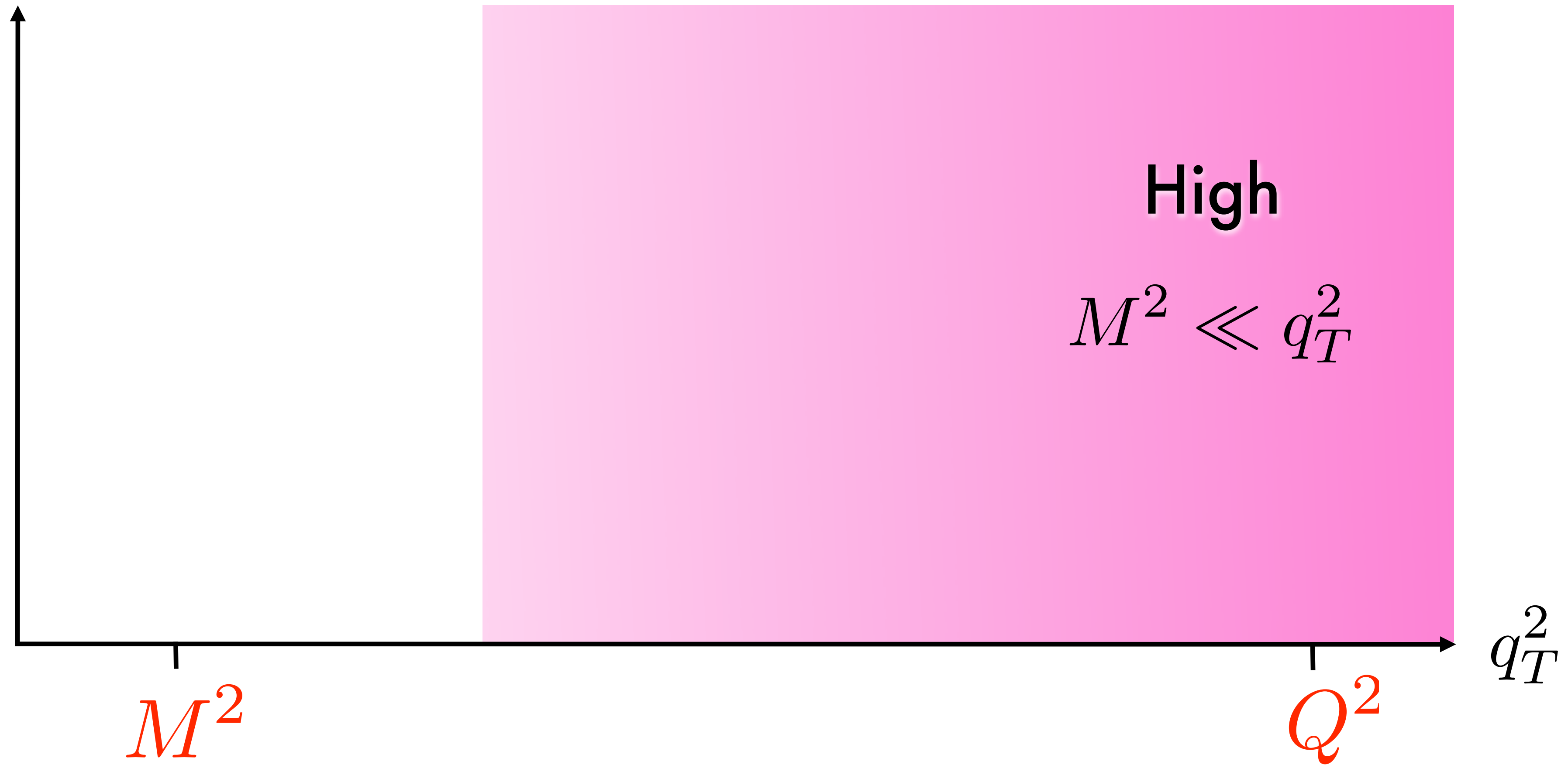
Example of low-transverse momentum result



$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big]$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\, e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

Low and high transverse momentum



Example of high-transverse momentum result

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}} \int_z^1 \frac{\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right)$$

$$\times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to q g)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to g q)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g \to q \bar{q})} \right]$$

Example of high-transverse momentum result

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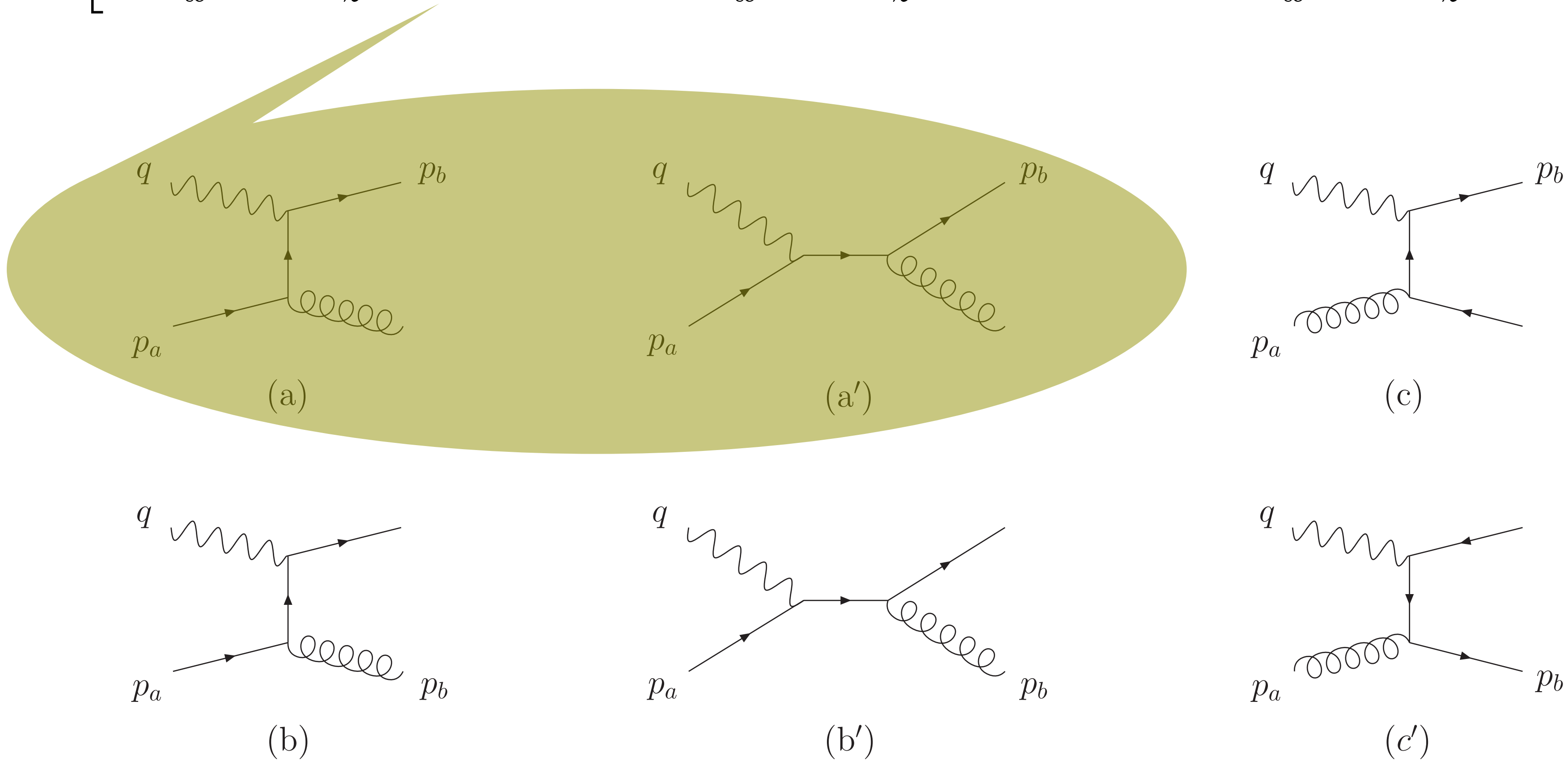

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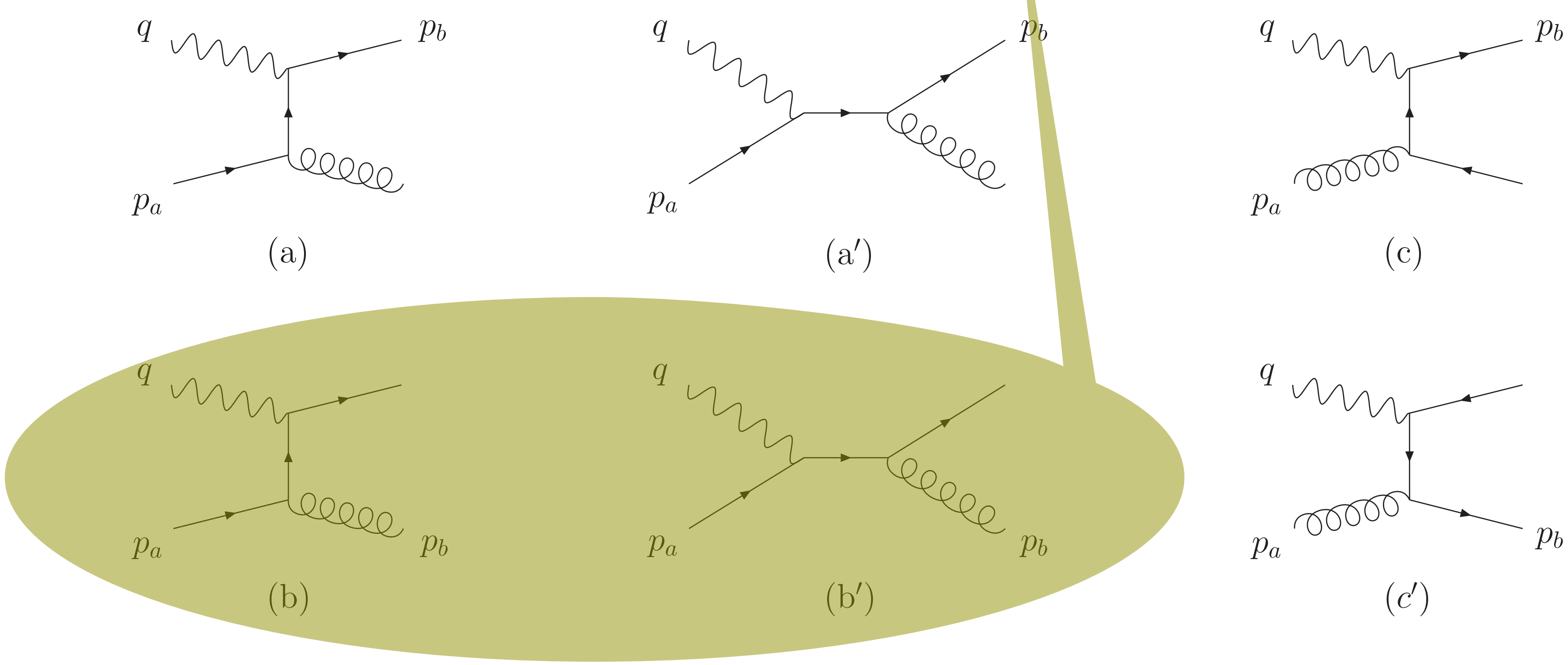
$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}} \int_z^1 \frac{\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right)$$

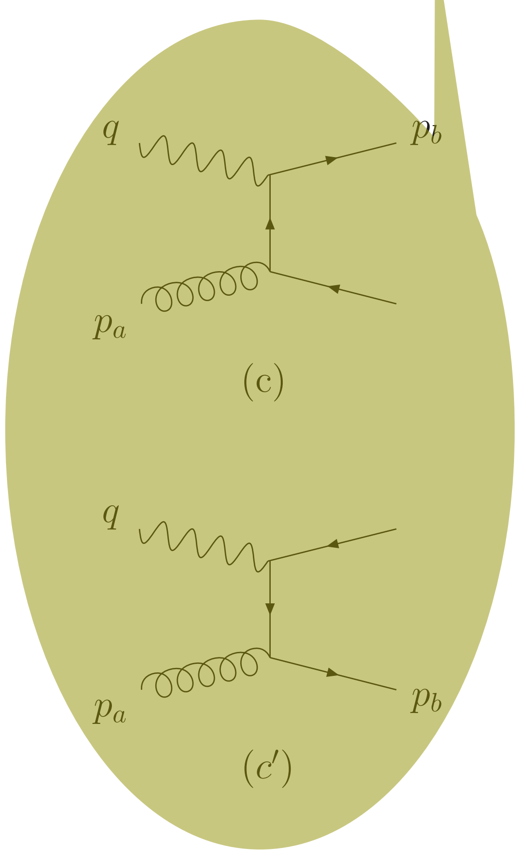
$$\times \left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})} \right]$$



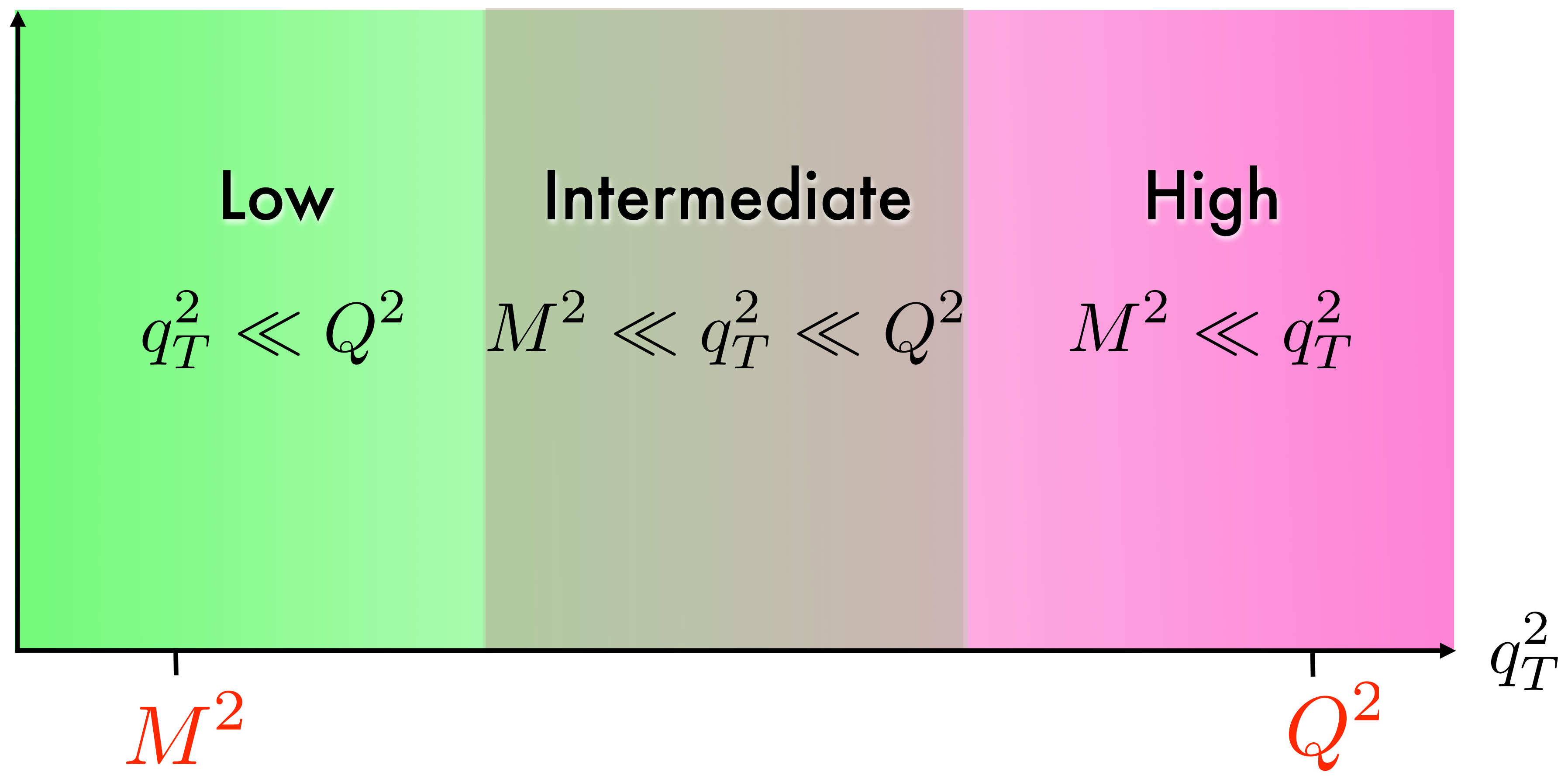
Example of high-transverse momentum result

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}}\int_z^1 \frac{\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right)$$

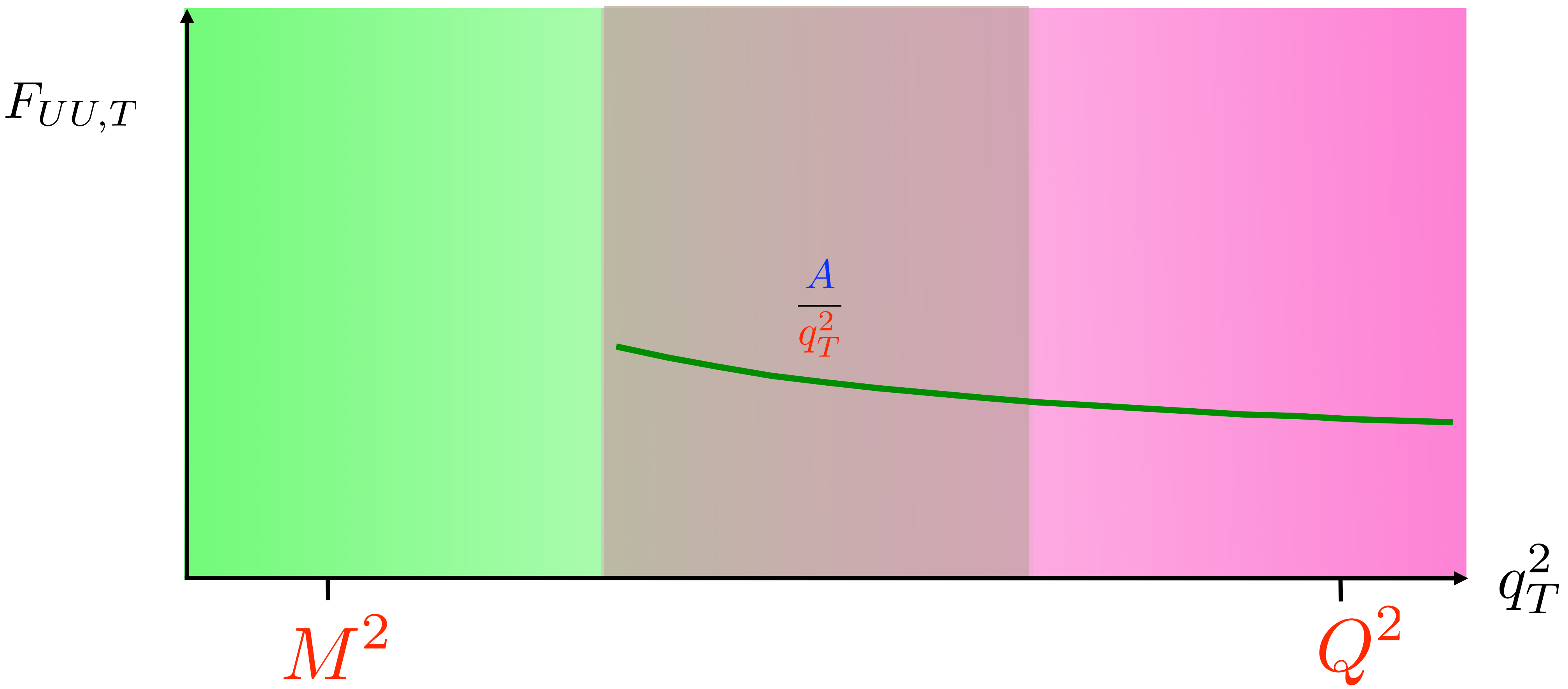
$$\times\left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$



Low and high transverse momentum

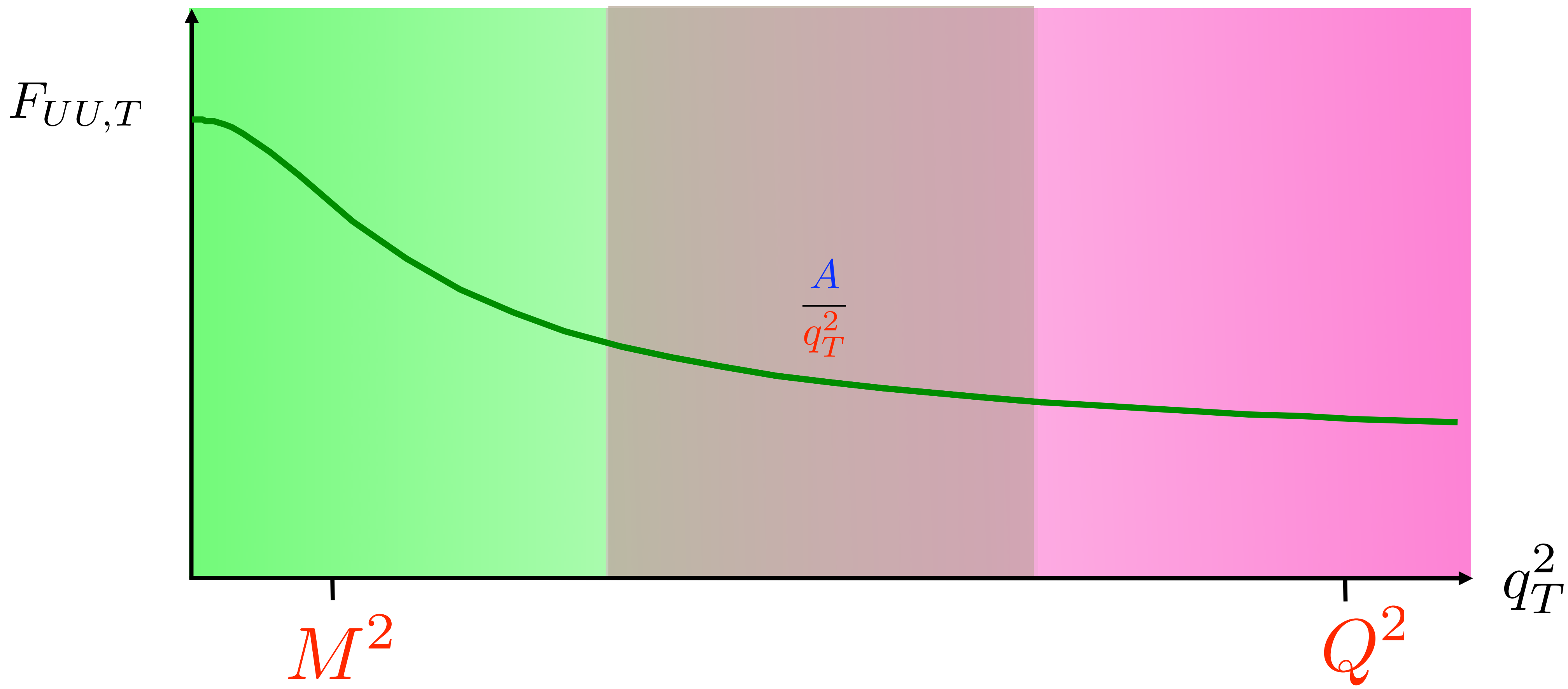


$F_{UU,T}$ structure function



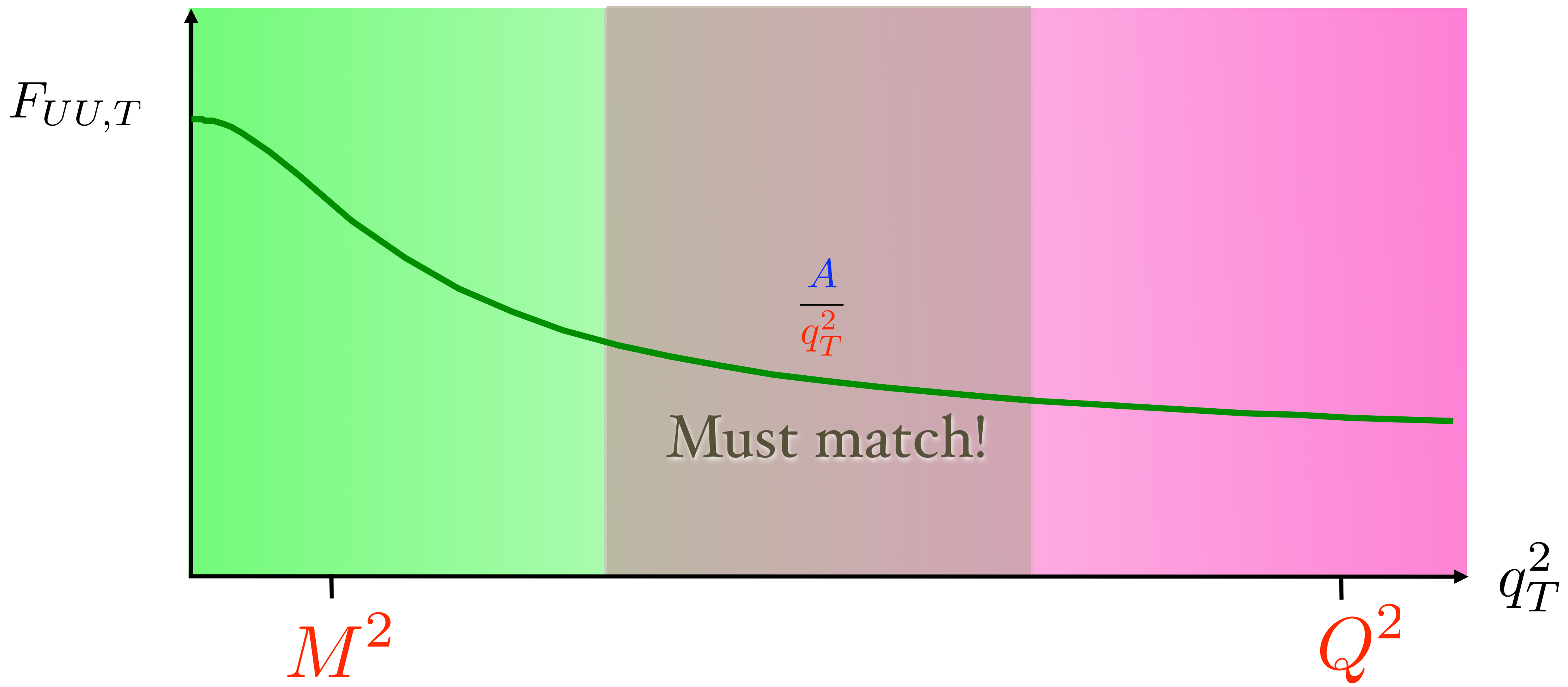
Collins, Soper, Sterman, NPB250 (85)

$F_{UU,T}$ structure function



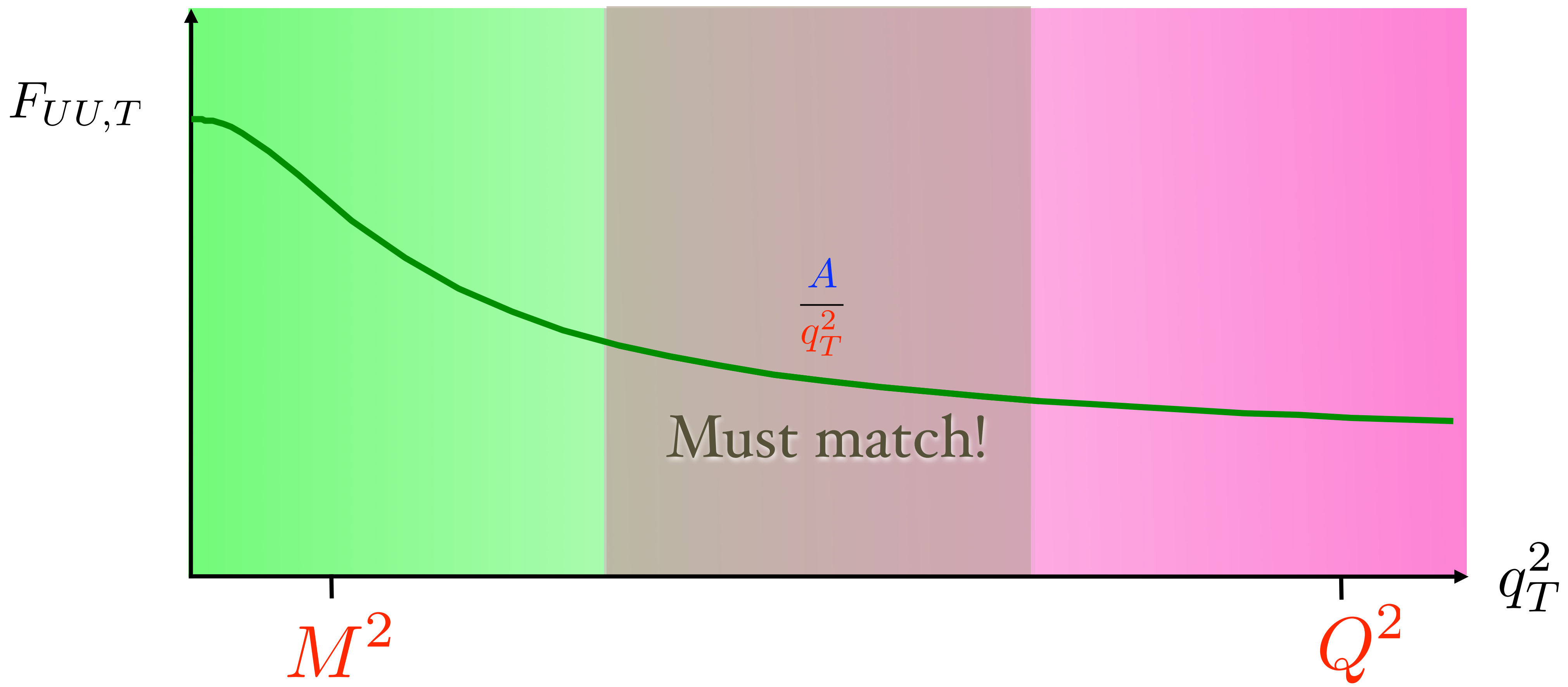
Collins, Soper, Sterman, NPB250 (85)

$F_{UU,T}$ structure function



Collins, Soper, Sterman, NPB250 (85)

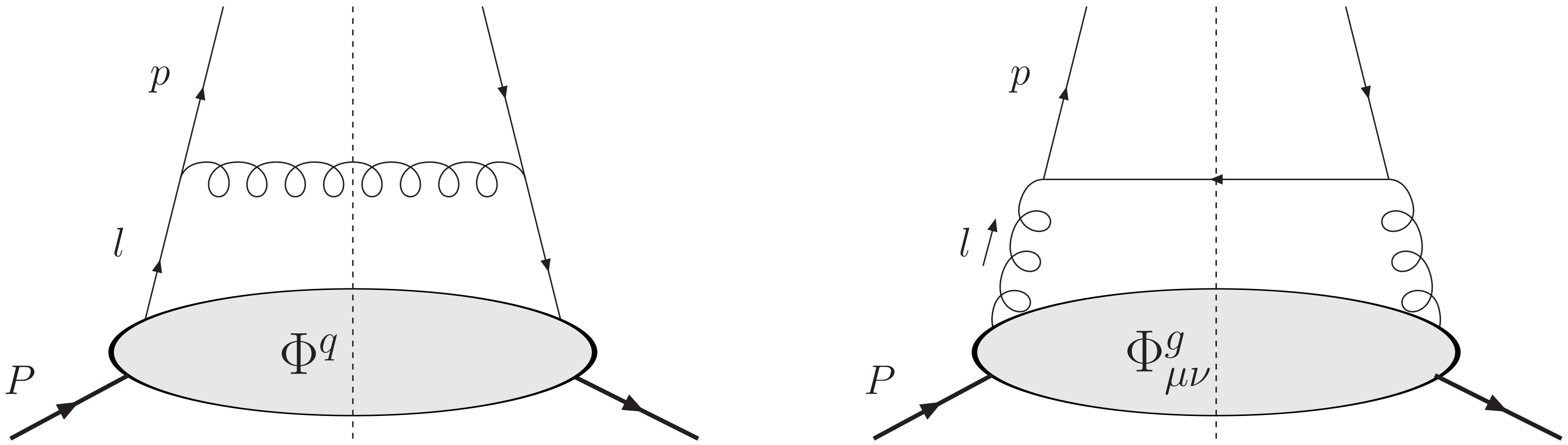
$F_{UU,T}$ structure function



The leading high-q_T part is just the "tail" of the leading low-q_T part

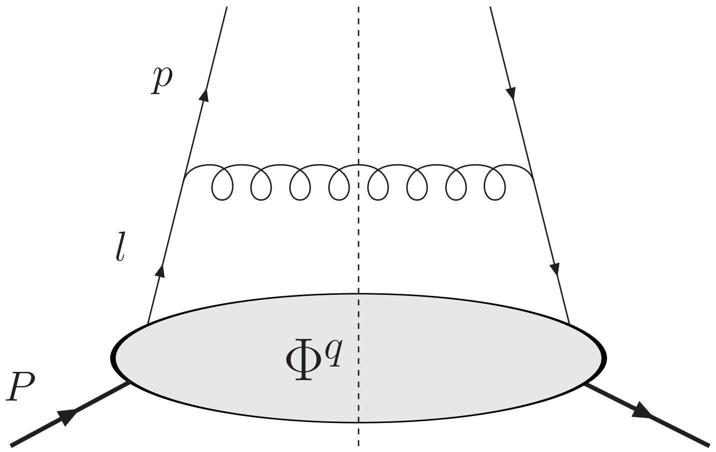
Collins, Soper, Sterman, NPB250 (85)

Perturbative corrections to TMDs

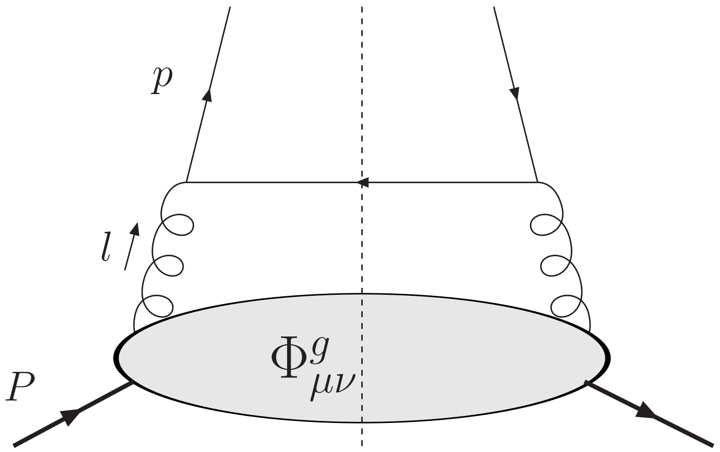


(a) (b)

Perturbative corrections to TMDs



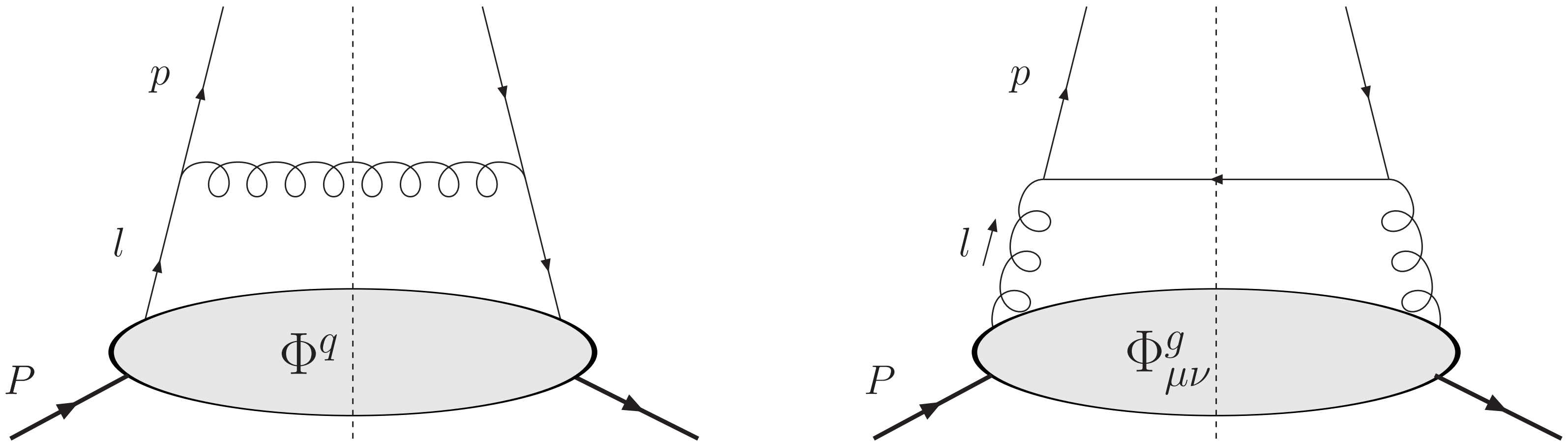
(a)



(b)

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

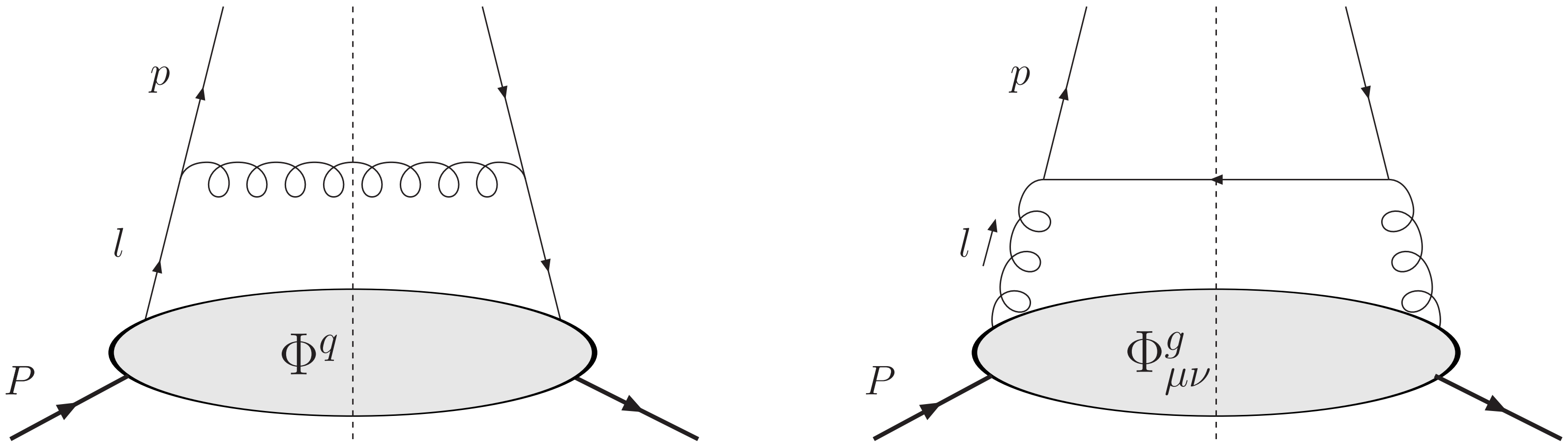
Perturbative corrections to TMDs



(a) (b)

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

Perturbative corrections to TMDs



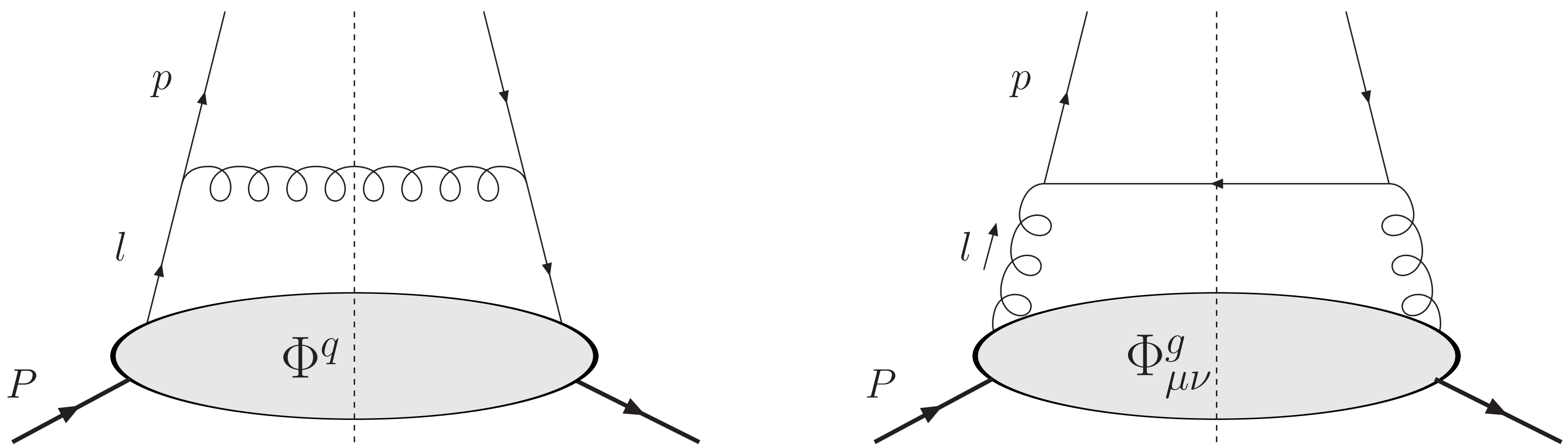
(a) (b)

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \left(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\right)(x)\right],$$

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[f_1^a(x)\,D_1^a(z)\,L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\left(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\right)(z)\right.$$
$$\left. + \left(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\right)(x)\,D_1^a(z)\right]$$

where $L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$

Perturbative corrections to TMDs



(a) (b)

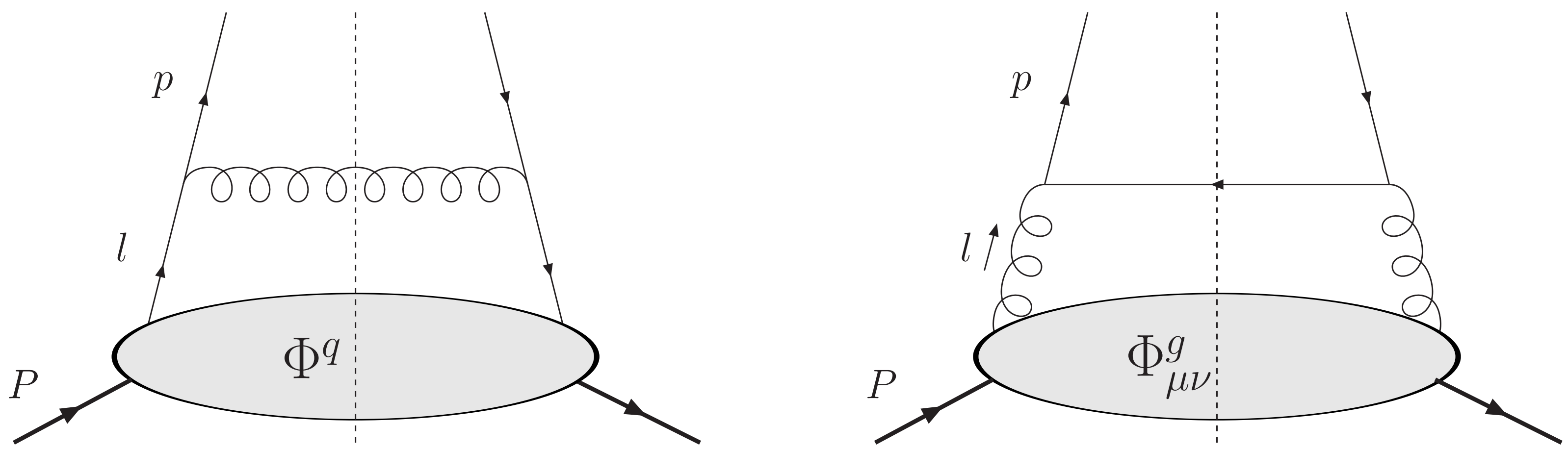
$$f_1^q(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[f_1^a(x)\,D_1^a(z)\,L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\big)(z)\right.$$
$$\left. + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\,D_1^a(z)\right]$$

DGLAP splitting functions

where $L\!\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$

Perturbative corrections to TMDs



(a) (b)

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \left(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\right)(x)\right],$$

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[f_1^a(x)\,D_1^a(z)\,L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\left(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\right)(z)\right.$$
$$\left. + \left(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\right)(x)\,D_1^a(z)\right]$$

Large log, needs resummation

DGLAP splitting functions

where $L\!\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$

Other TMDs

$$x f^{\perp} \sim \frac{1}{\boldsymbol{p}_T^2}\, \alpha_s\, \mathcal{F}\big[f_1\big]\,,$$

$$\ldots$$

$$f_{1T}^{\perp} \sim \frac{M^2}{\boldsymbol{p}_T^4}\, \alpha_s\, \mathcal{F}\big[f_{1T}^{\perp(1)}, \ldots\big]\,,$$

$$\ldots$$

$$x f_L^{\perp} \sim \frac{1}{\boldsymbol{p}_T^2}\, \alpha_s^2\, \mathcal{F}\big[g_1\big]\,,$$

$$\ldots$$

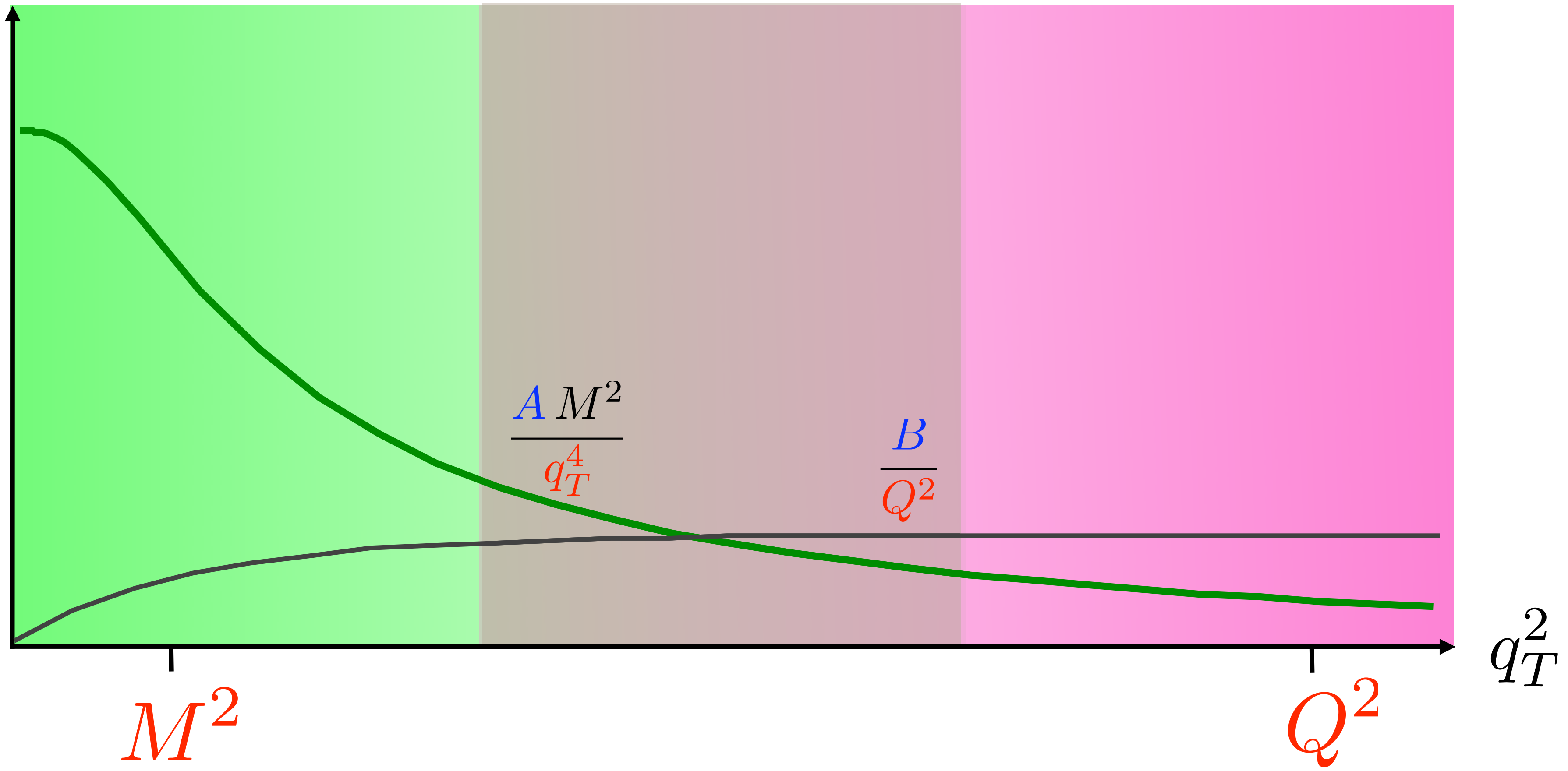
$$h_{1T}^{\perp} \sim \frac{M^2}{\boldsymbol{p}_T^4}\, \alpha_s^2\, \mathcal{F}\big[h_1\big]\,,$$

$$\ldots$$

AB, D. Boer, M. Diehl, P.J. Mulders, JHEP 08 (08)

Expected mismatch

The leading terms in the two expansions
CANNOT and MUST not match!



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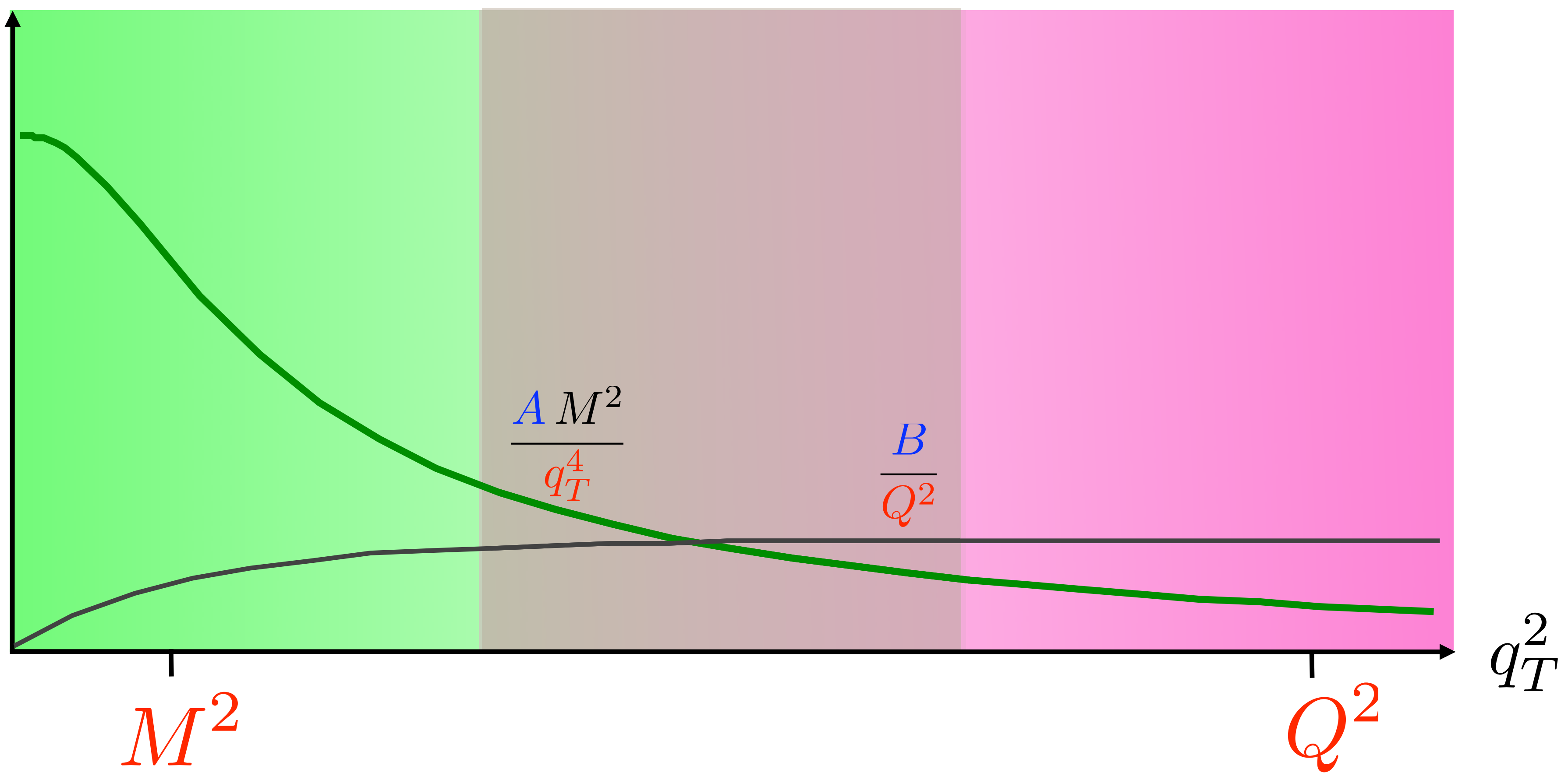
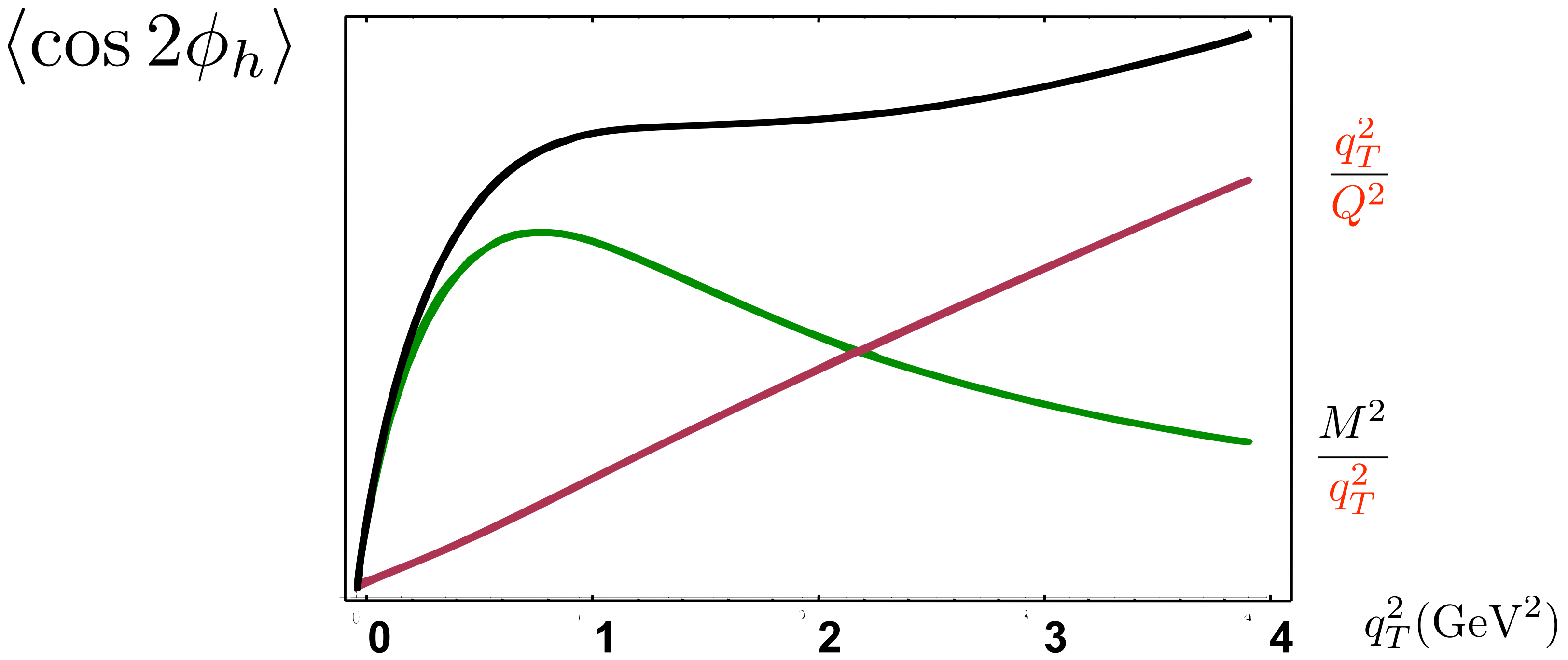


Two distinct mechanisms are involved

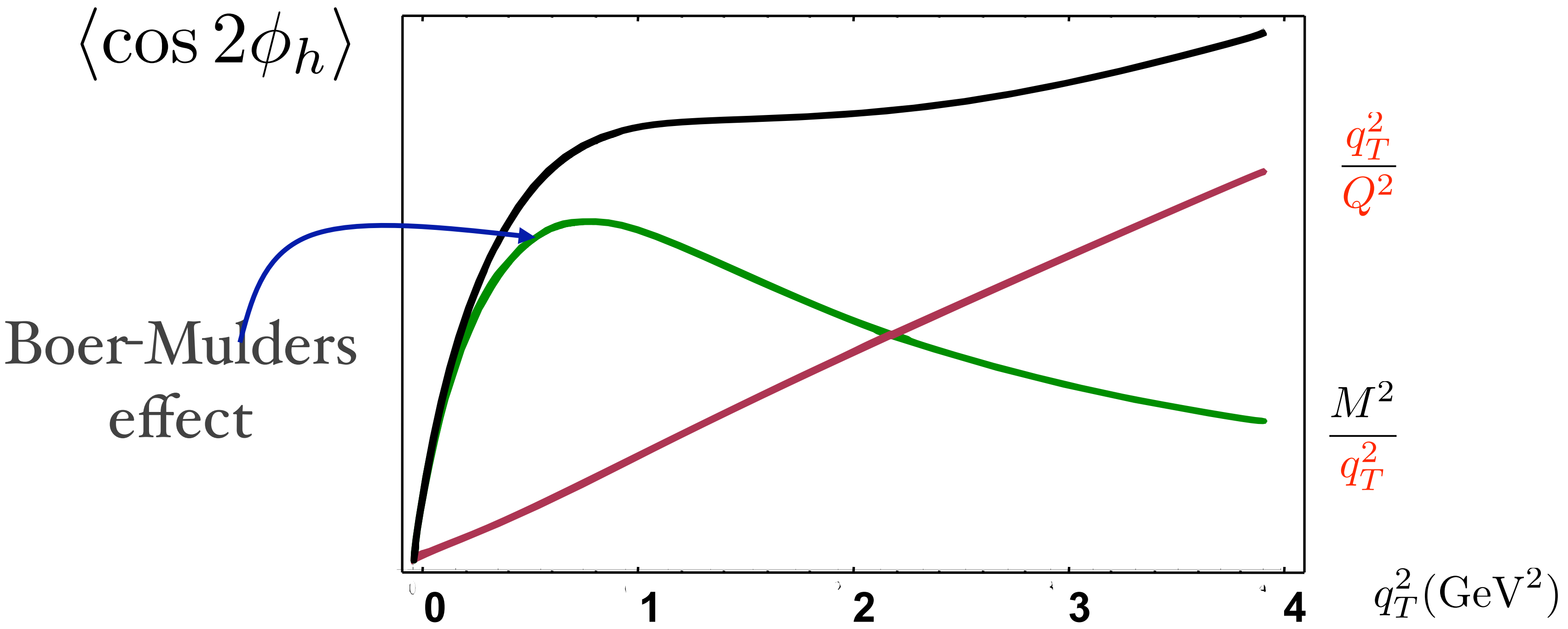
Cos 2$\boldsymbol{\Phi}$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



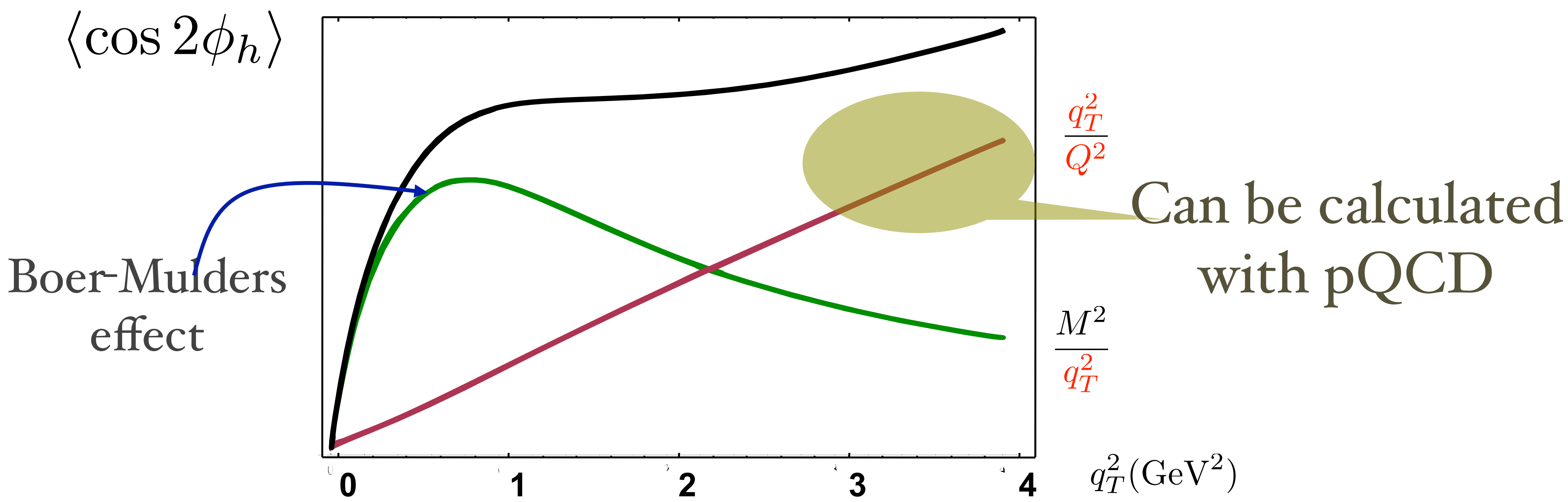
Cos 2$\boldsymbol{\Phi}$ asymmetry

see also Barone, Prokudin, Ma 0804.3024



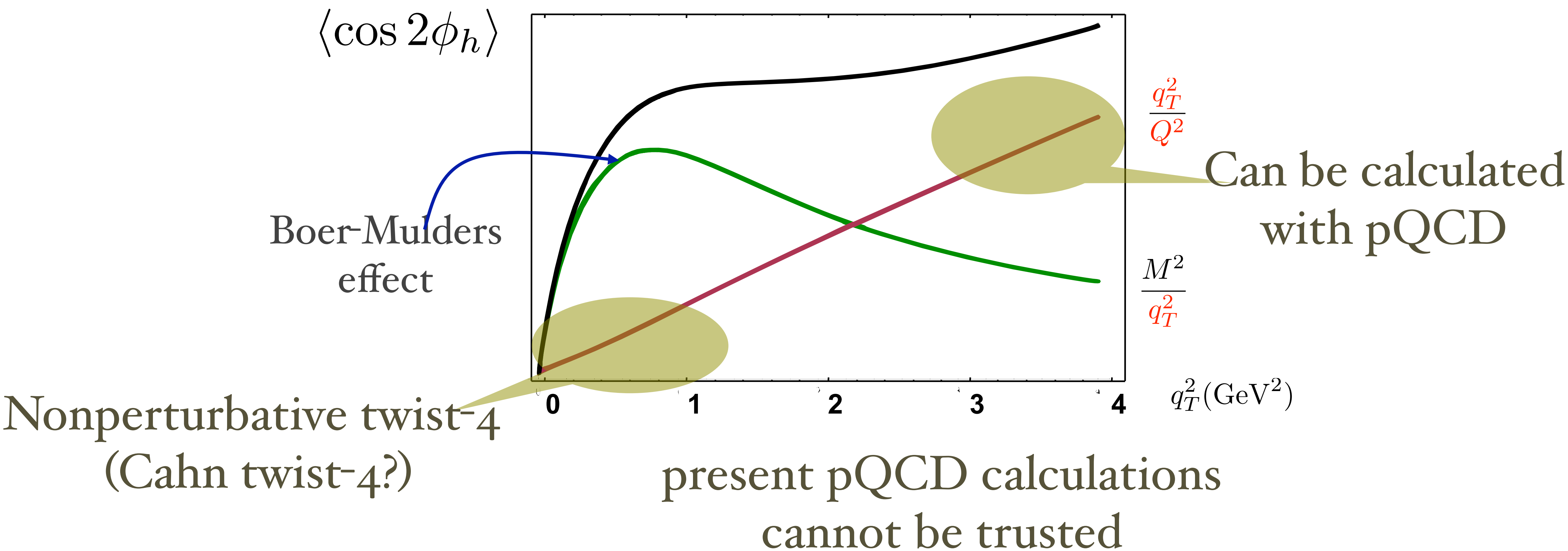
Cos 2Φ asymmetry

see also Barone, Prokudin, Ma 0804.3024



Cos 2Φ asymmetry

see also Barone, Prokudin, Ma 0804.3024



Cos 2$\boldsymbol{\Phi}$ asymmetry

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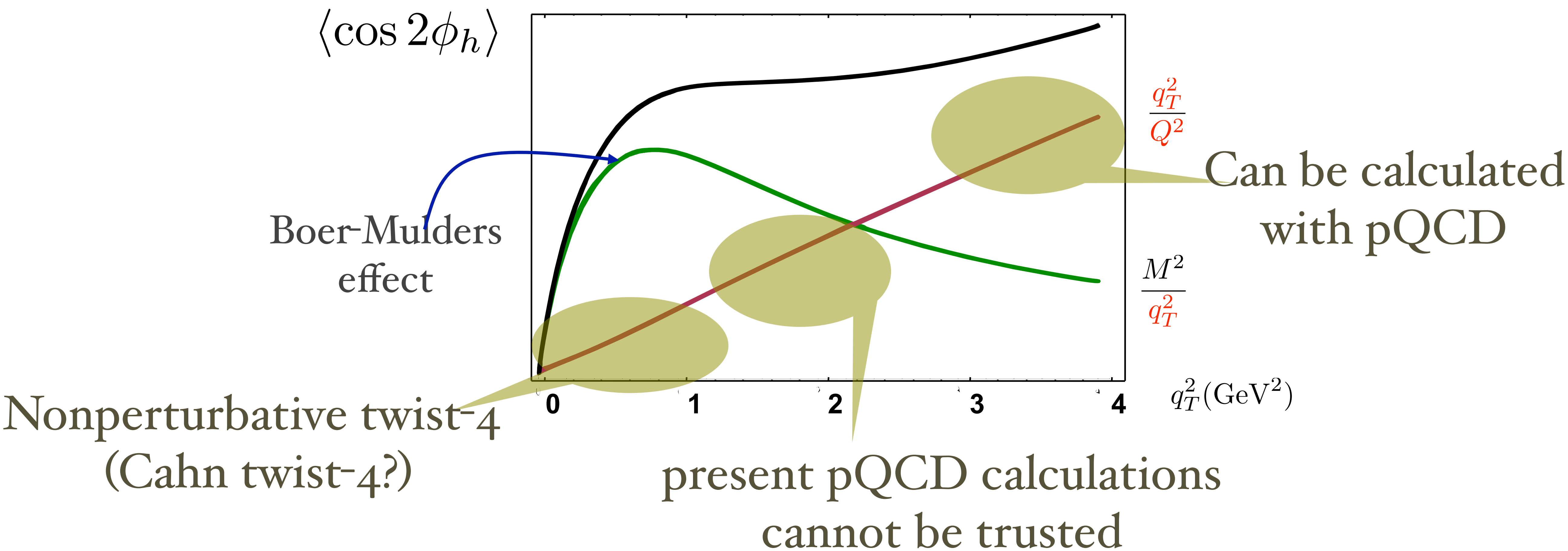


All structure functions

AB, D. Boer, M. Diehl, P.J. Mulders, JHEP 08 (08)

	low-q_T calculation			high-q_T calculation			
observable	twist	order	power	twist	order	power	powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	?
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	?
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$				

conjectures!

Evolution equations

Collinear evolution of transversity

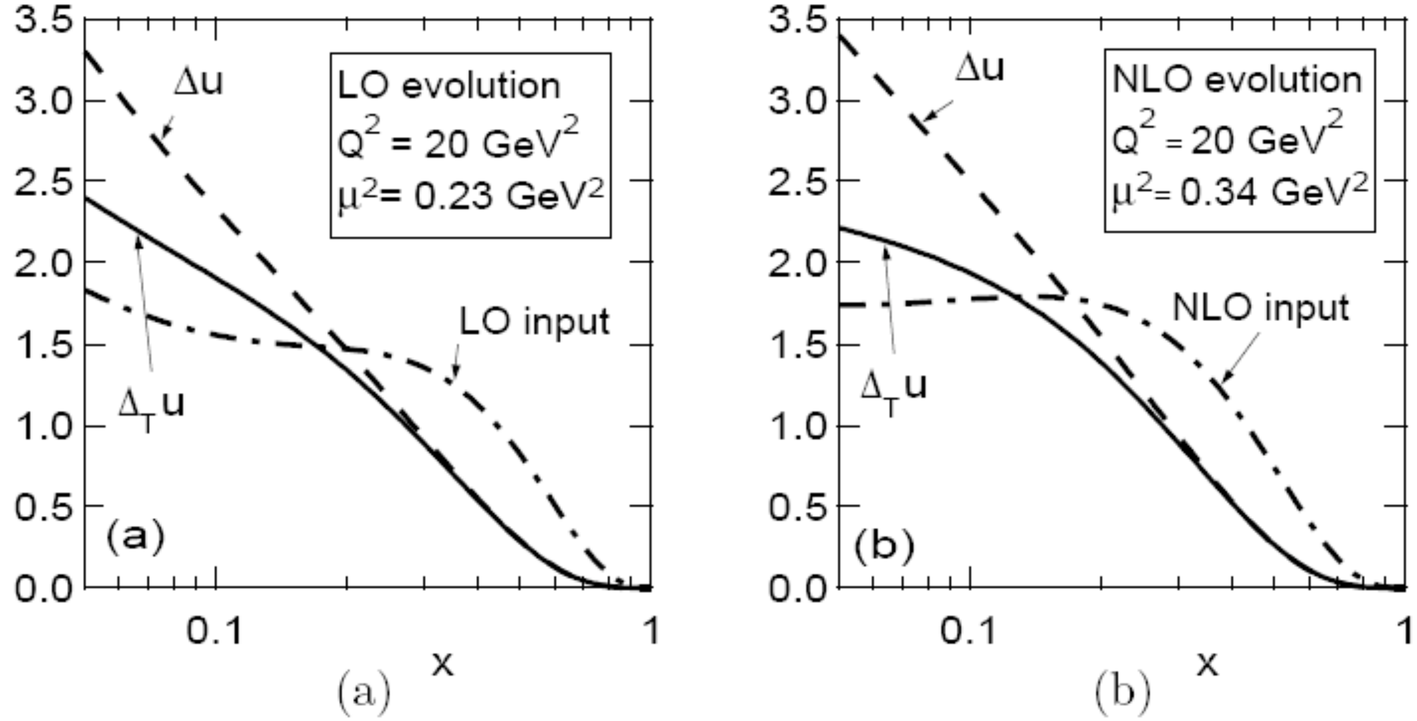


Fig. 19. Comparison of the Q^2-evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO, from [72].

Barone, Drago, Ratcliffe, PR 359 (2002)

Hayashigaki, Kanazawa, Koike, PRD56 (97)

TMDs evolution

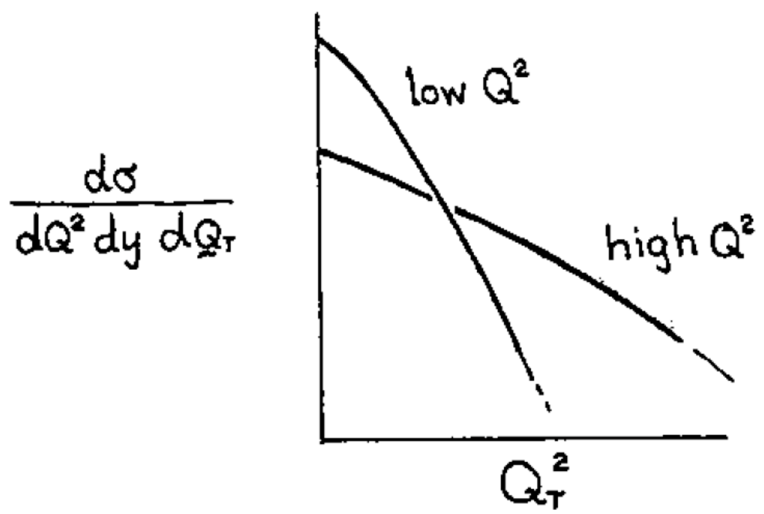


Fig 3 Broadening of the Q_T distribution

Collins, Soper, Sterman, talk at Fermilab Workshop on Drell-Yan Process, Batavia, Ill., Oct 7-8, 1982

TMDs evolution

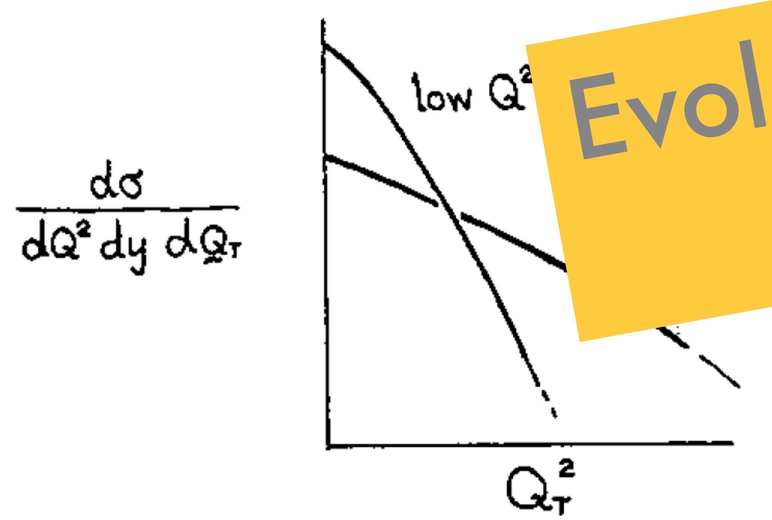
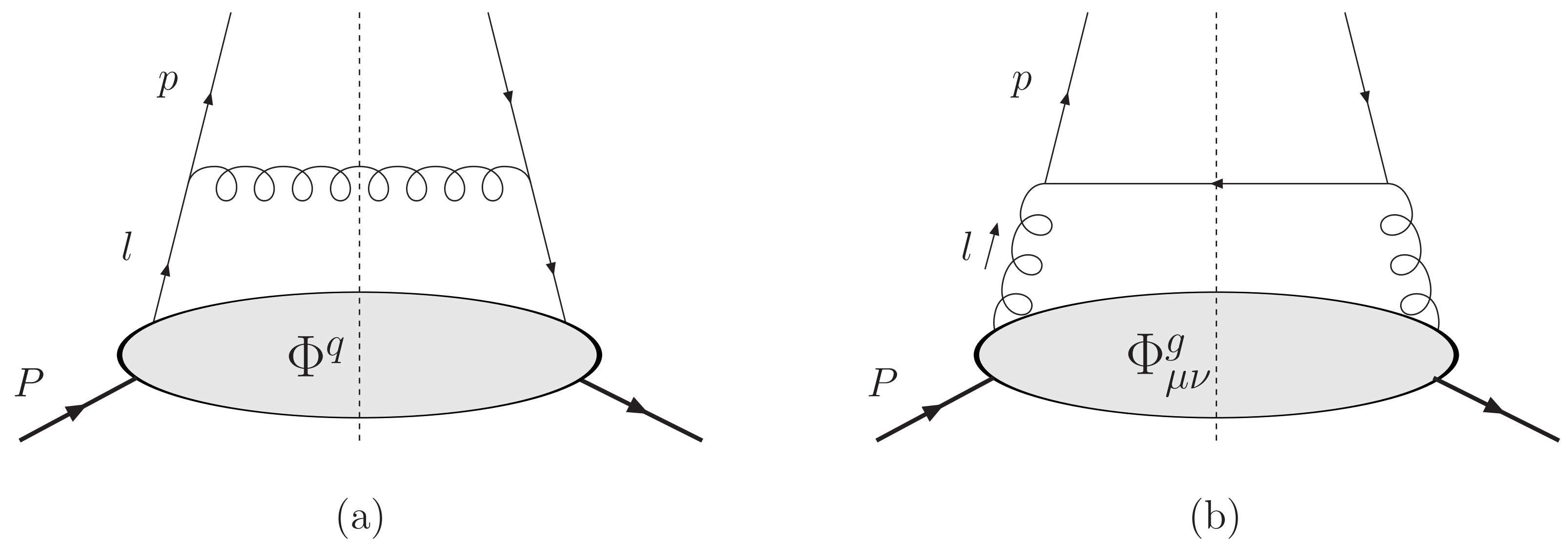


Fig 3 Broadening of the Q_T distribution

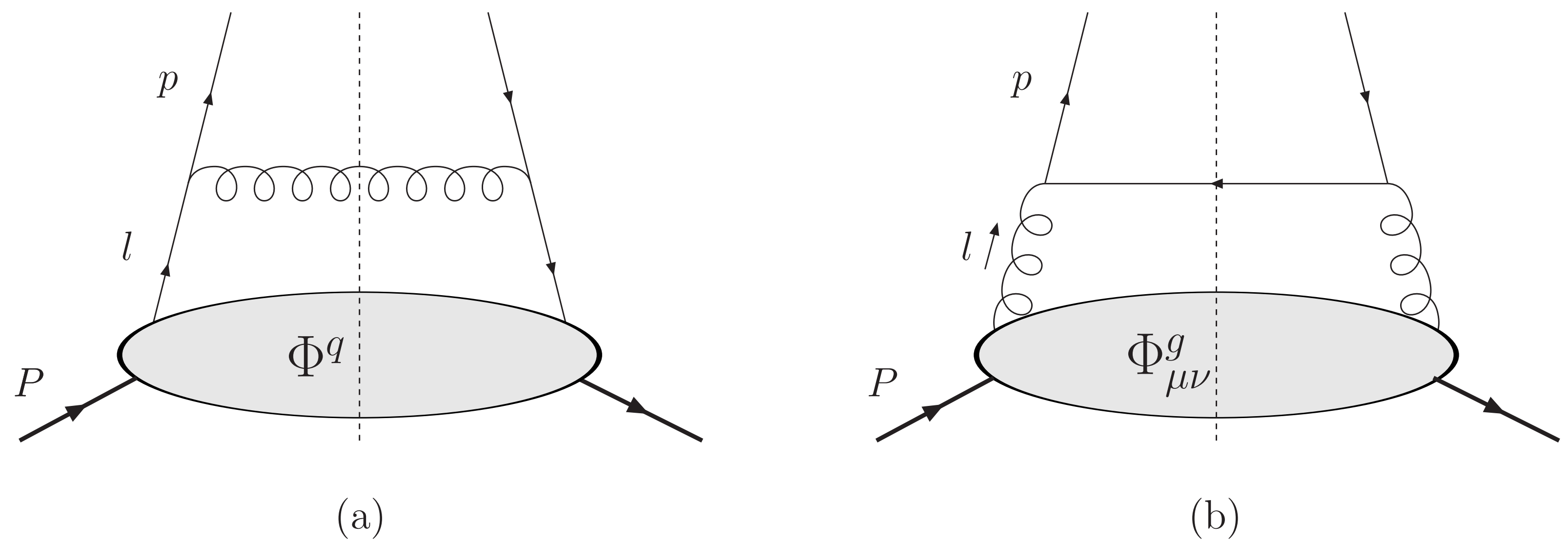
Evolution equations for TMDs are NOT standard DGLAP

Collins, Soper, Sterman, talk at Fermilab Workshop on Drell-Yan Process, Batavia, Ill., Oct 7-8, 1982

Perturbative corrections to TMDs

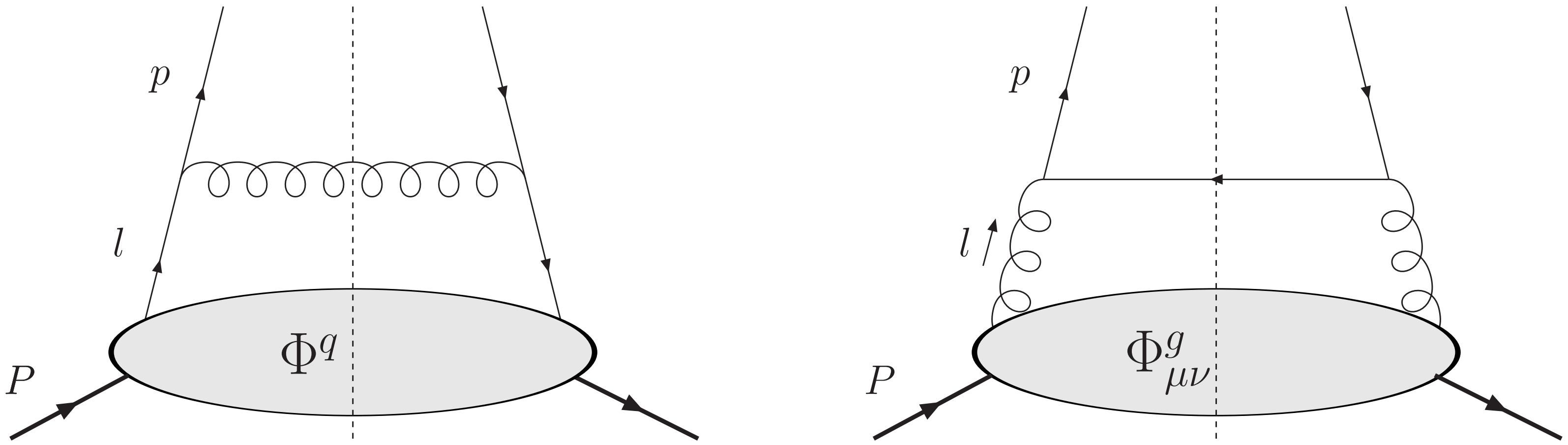


Perturbative corrections to TMDs



$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

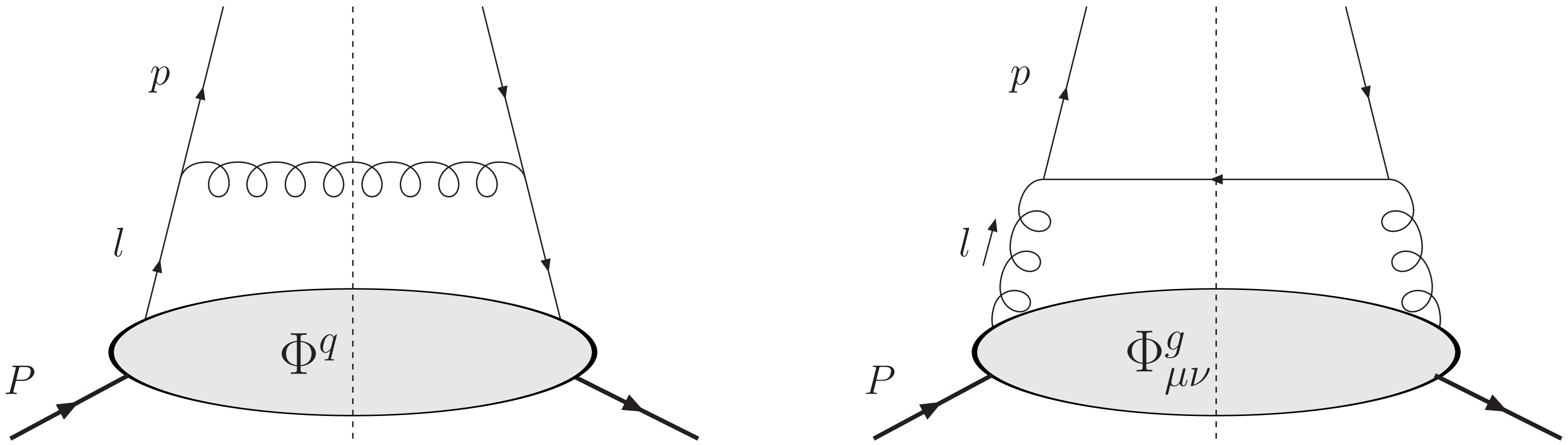
Perturbative corrections to TMDs



(a) (b)

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

Perturbative corrections to TMDs



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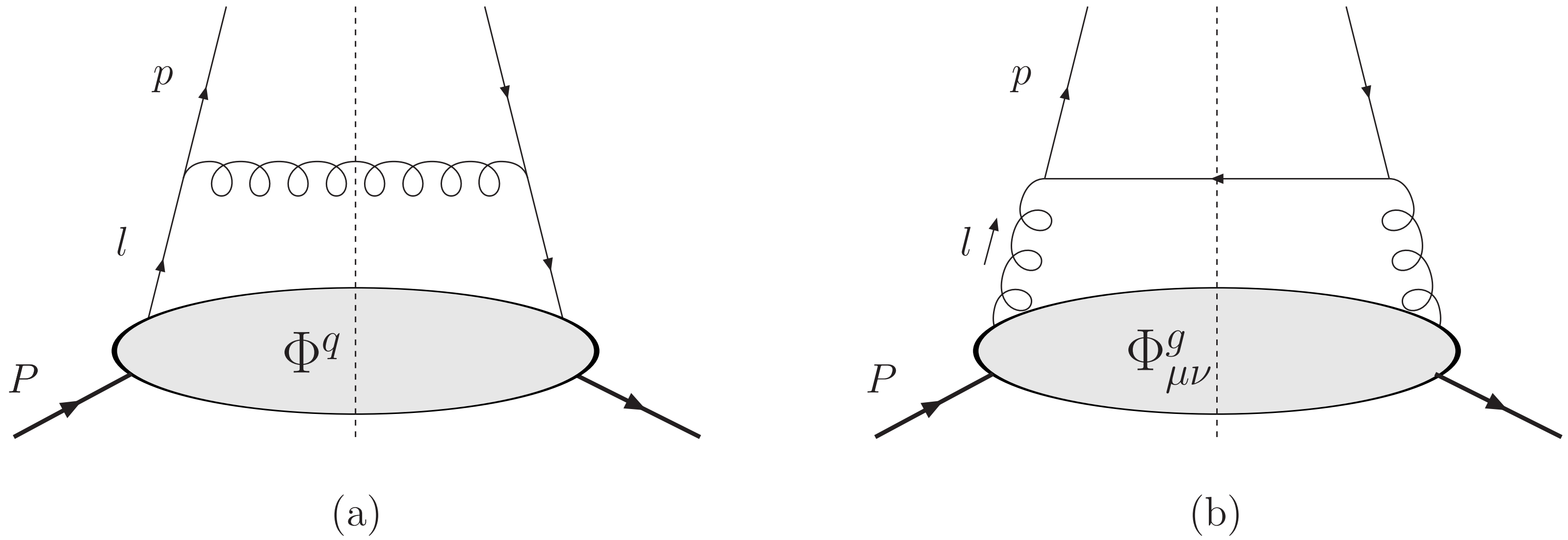
$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\, f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2 \left[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right.$$
$$\left. + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\, D_1^a(z)\right]$$

where $L\!\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$

Perturbative corrections to TMDs



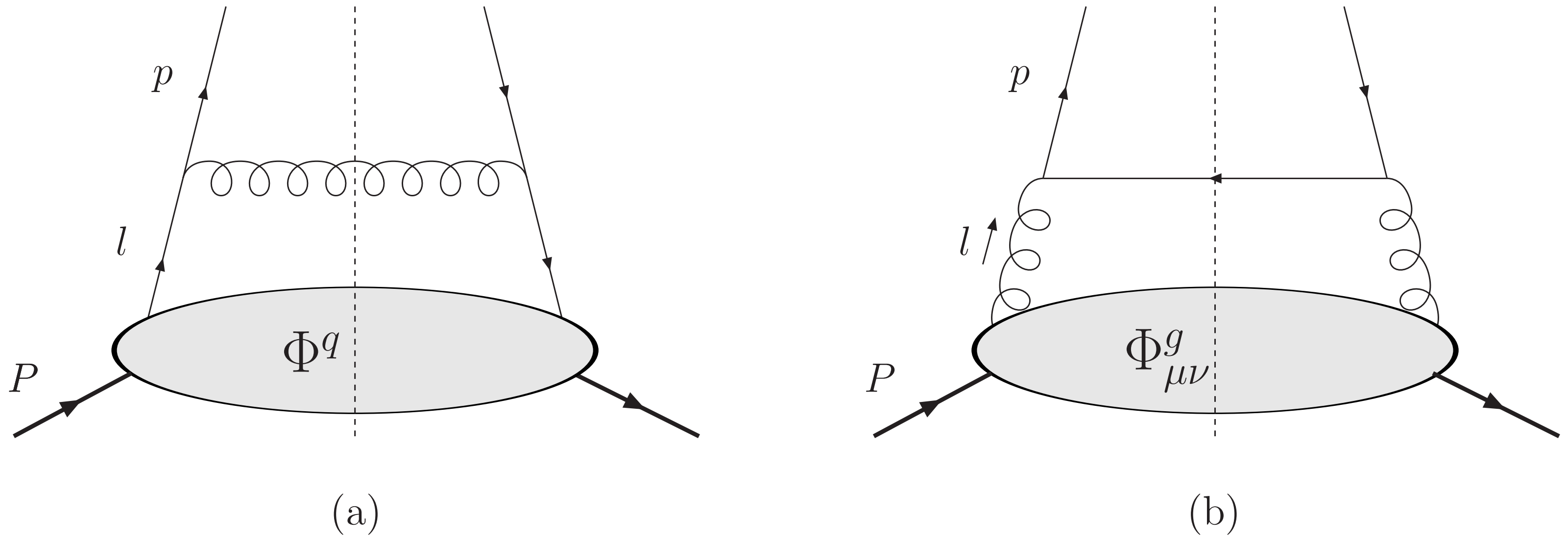
$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2} f_1^q(x) - C_F f_1^q(x) + \big(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\big)(x) \right],$$

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\, \big(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right.$$
$$\left. + \big(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\big)(x)\, D_1^a(z) \right]$$

DGLAP splitting functions

where $L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln \frac{Q^2}{q_T^2} - 3C_F$

Perturbative corrections to TMDs



$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2} f_1^q(x) - C_F f_1^q(x) + \left(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\right)(x) \right],$$

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) \left(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\right)(z) + \left(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\right)(x)\, D_1^a(z) \right]$$

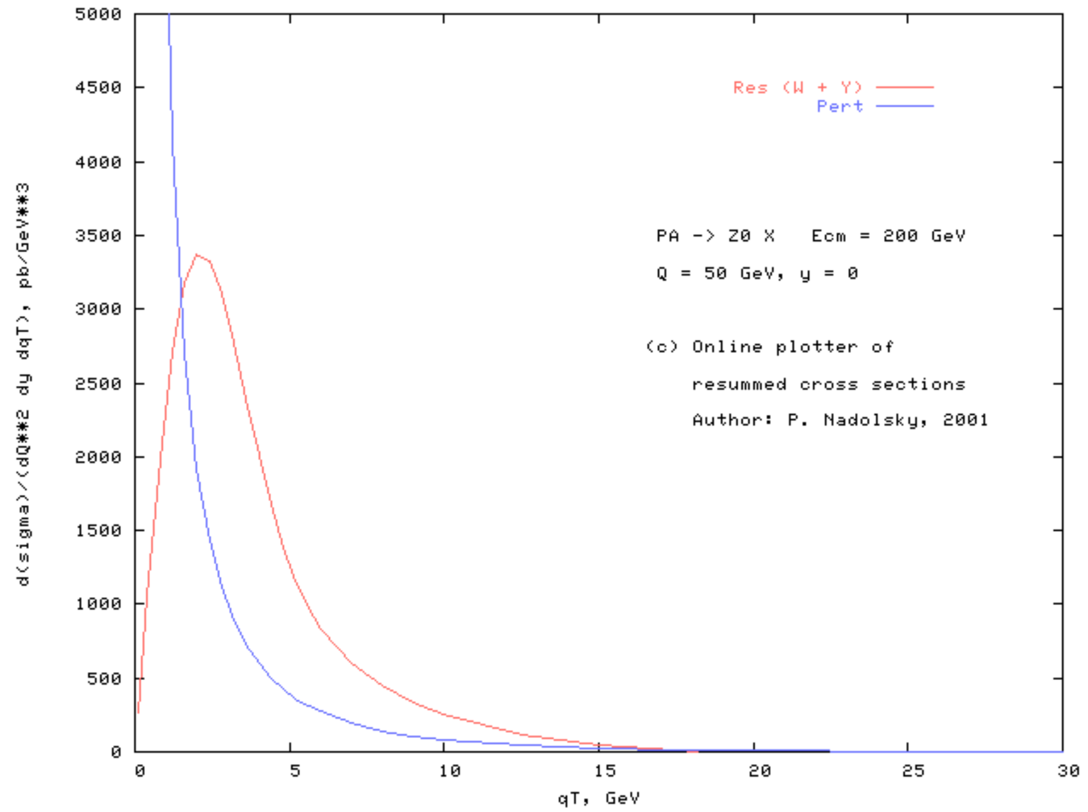
Large log, needs resummation

DGLAP splitting functions

where $L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln \frac{Q^2}{q_T^2} - 3C_F$

Resummation results

$$F_{UU,T}(x,z,b,Q^2) = x \sum_a e_a^2 \left[(f_1^i \otimes \mathcal{C}_{ia})\,(\mathcal{C}_{aj} \otimes D_1^j)\, e^{-S} e^{-S_{NP}} \right]$$



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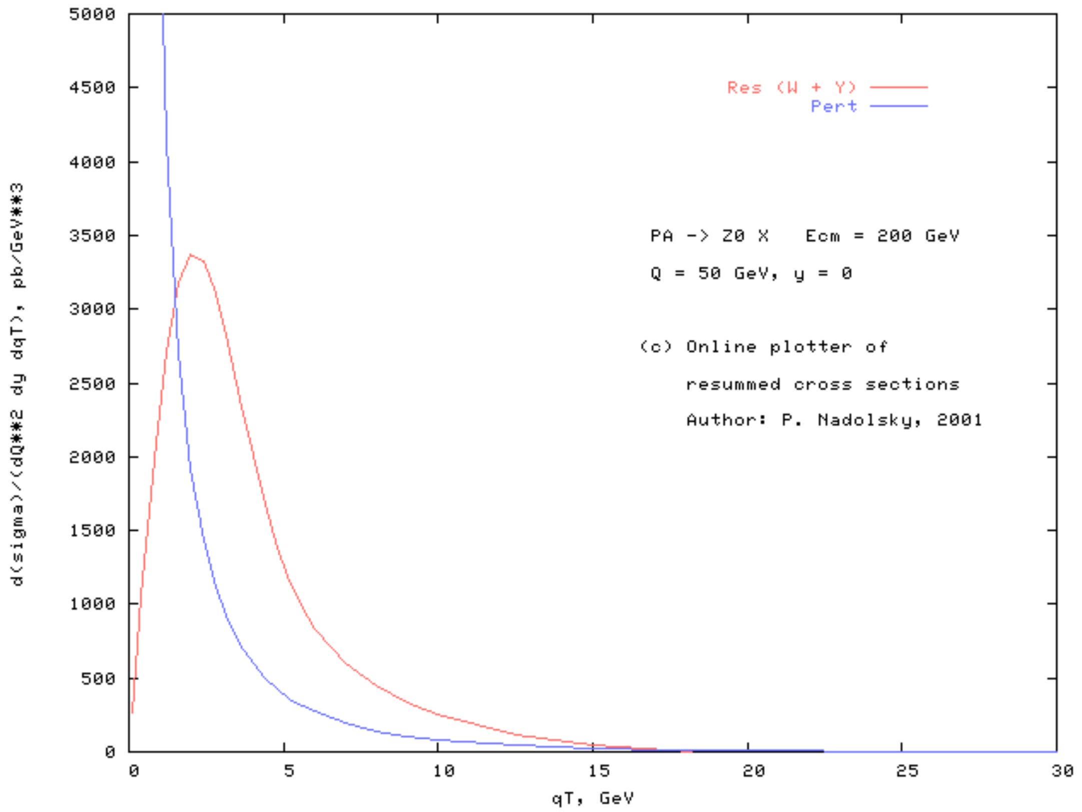
collinear PDF and FF

calculable with pQCD

nonperturbative part of TMDs



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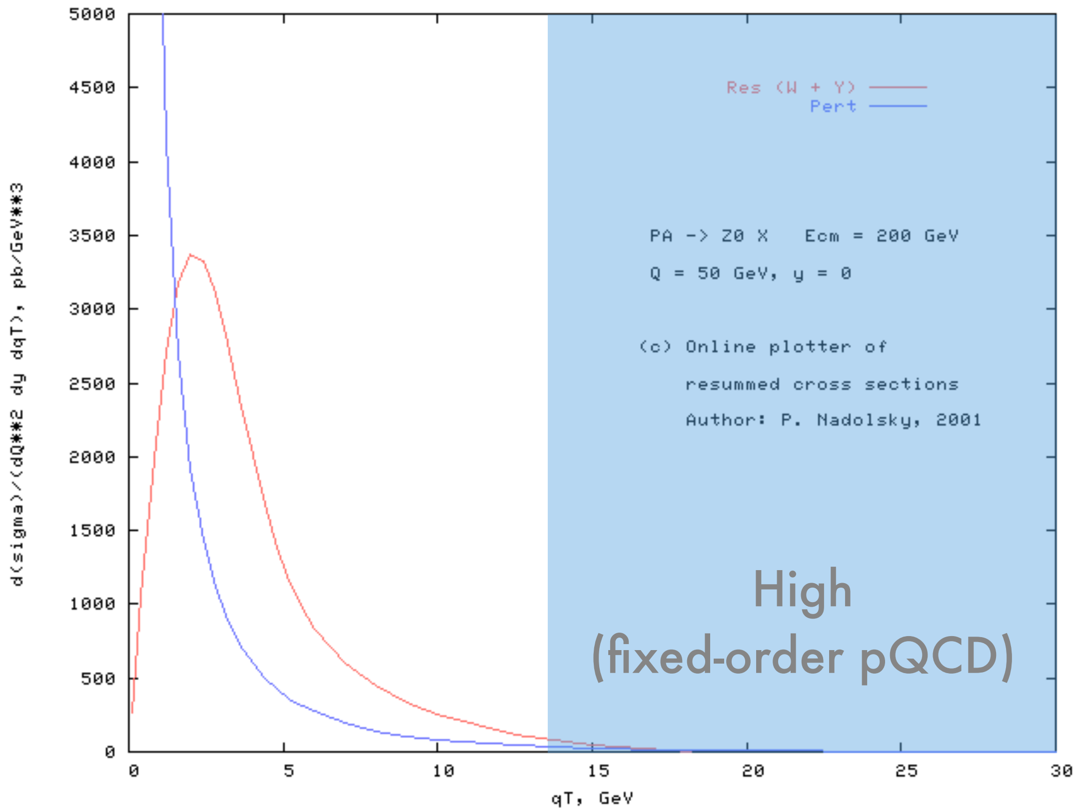

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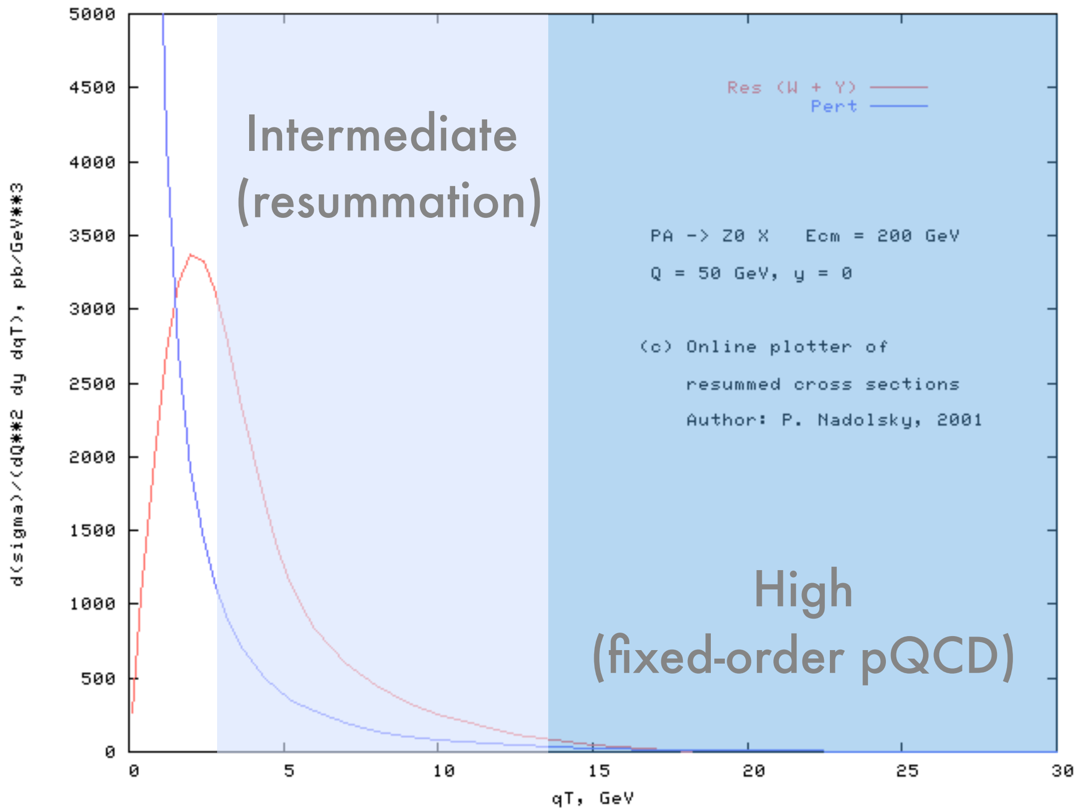
collinear PDF and FF   calculable with pQCD   nonperturbative part of TMDs



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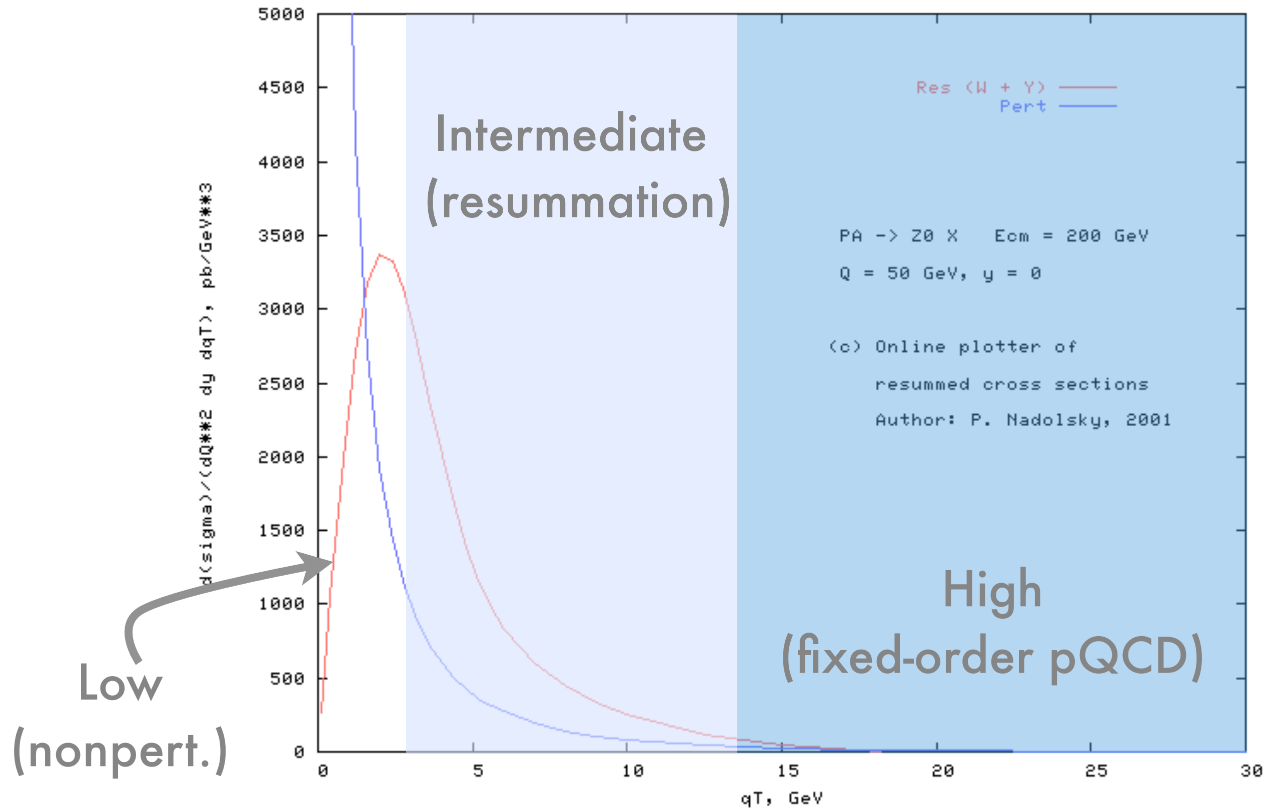

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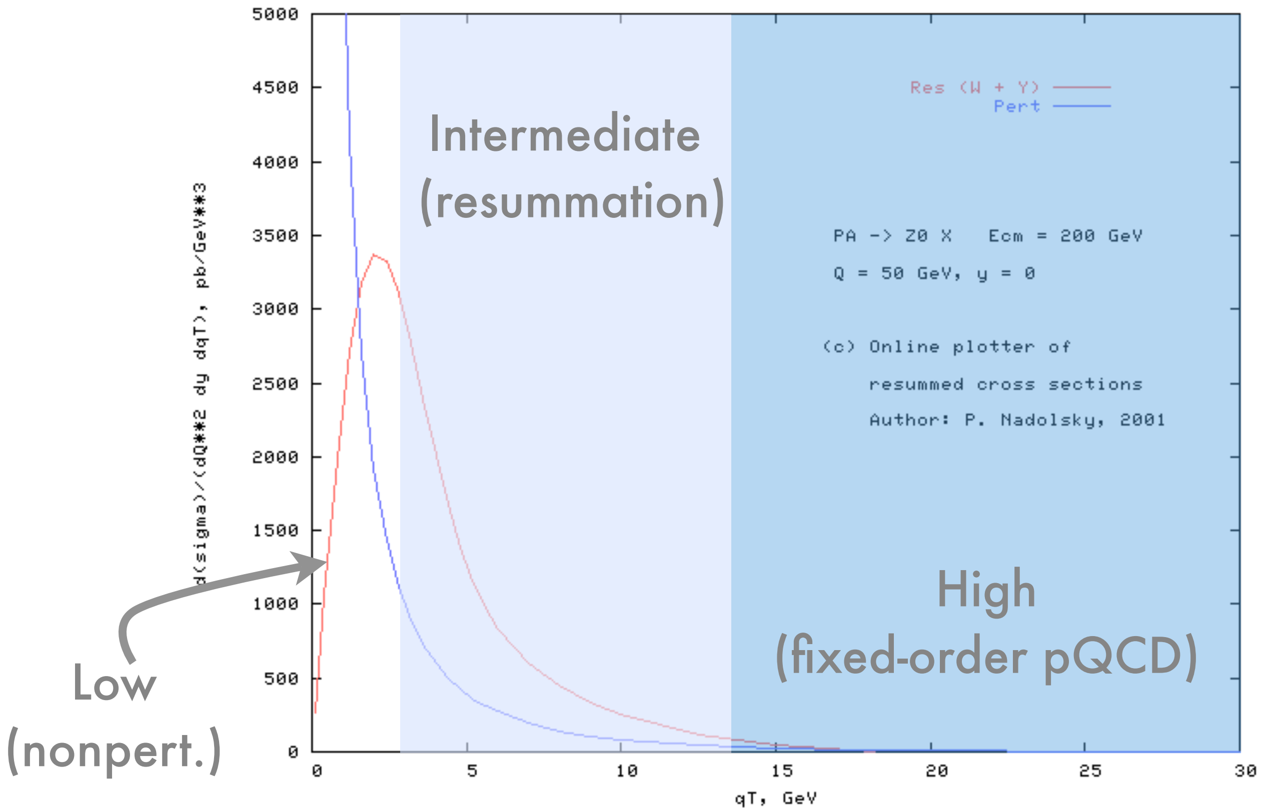
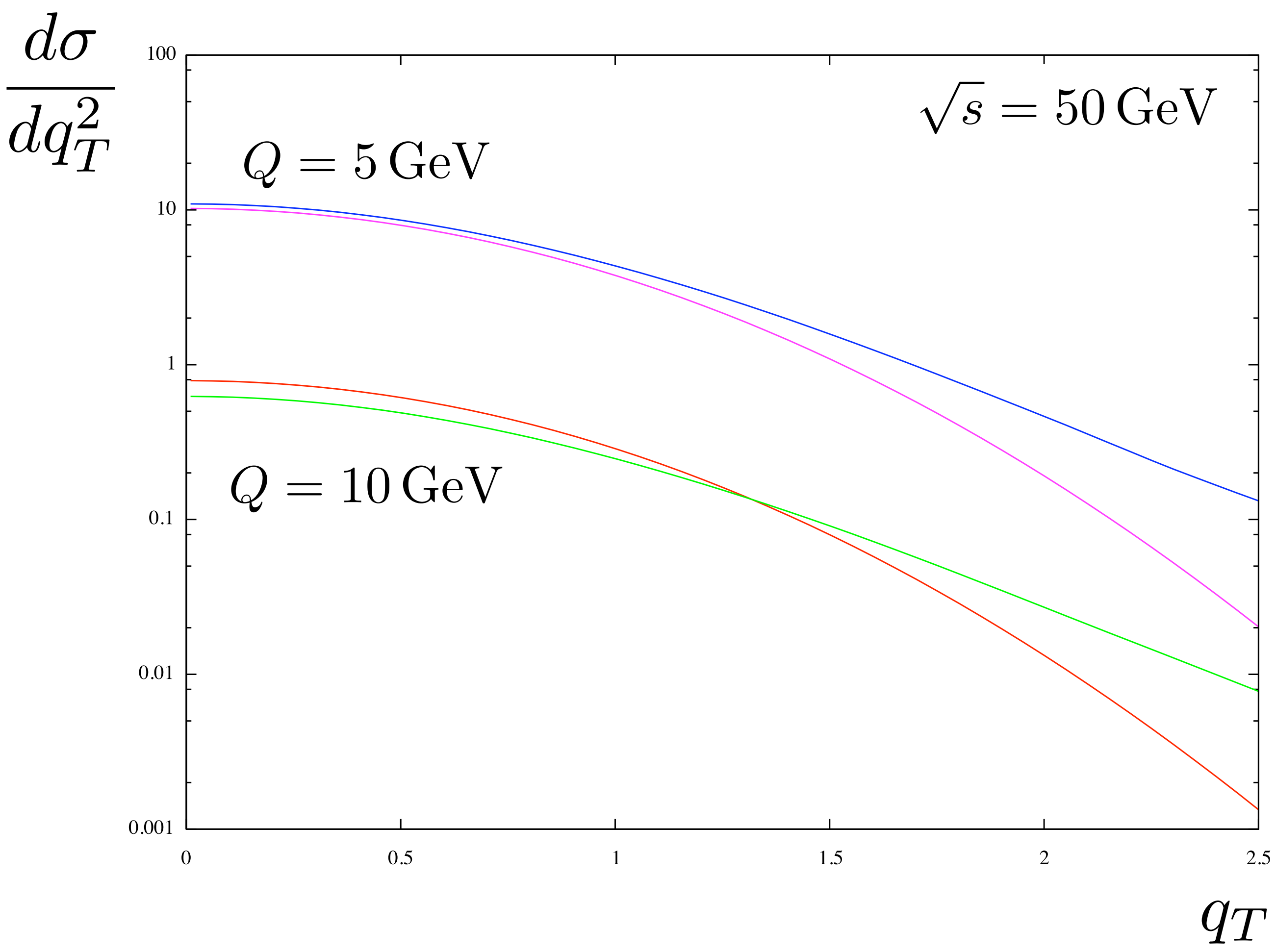
collinear PDF and FF   calculable with pQCD   nonperturbative part of TMDs

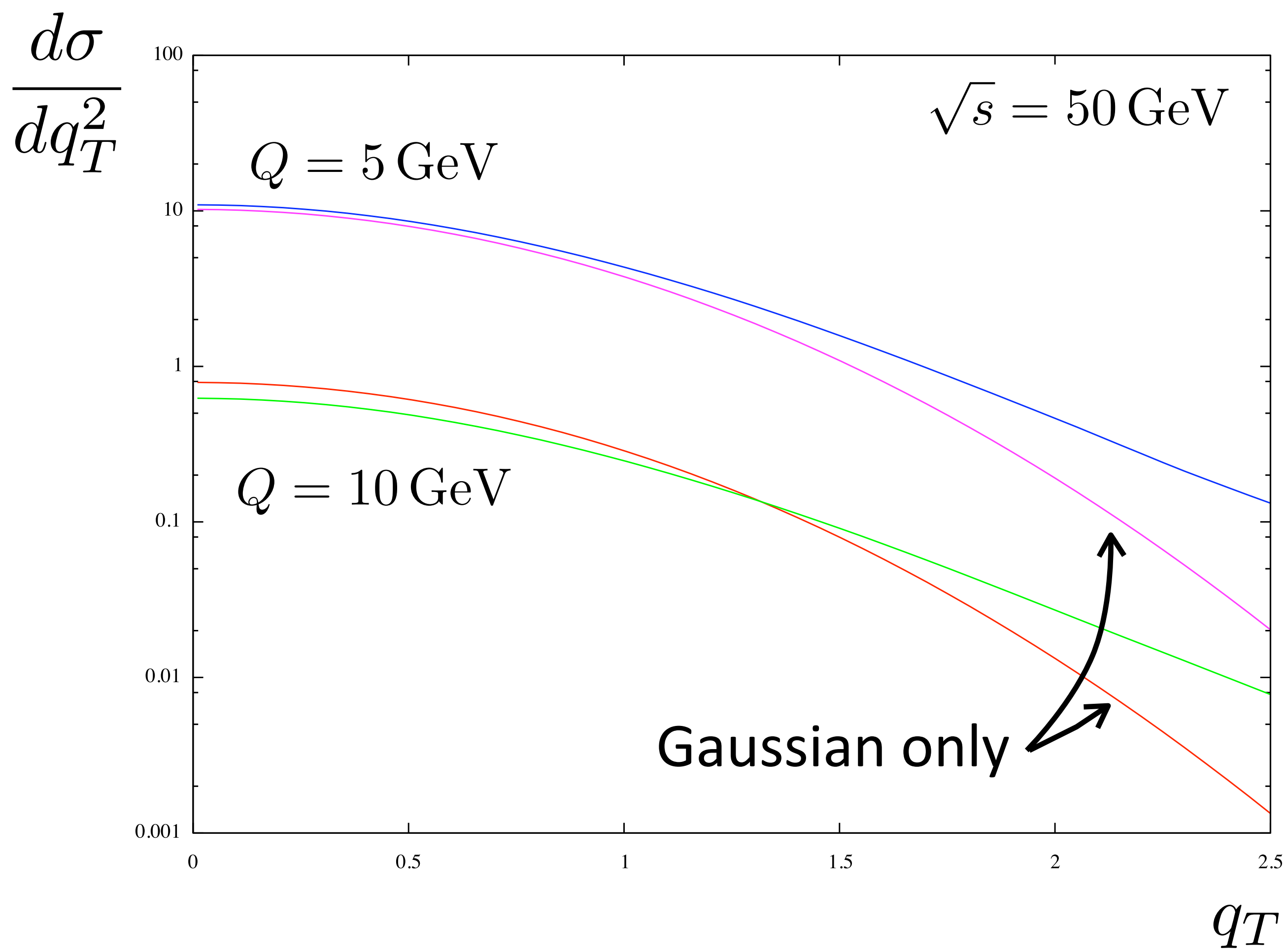


$$F_{UU,T}(x,z,q_T^2,Q^2) = x\sum_a e_a^2 \frac{d}{dq_T^2}\left[(f_1^i \otimes \mathcal{C}_{ia})\,(\mathcal{C}_{aj} \otimes D_1^j)\, e^{-S}\left(1 - e^{-S_{NP}}\right) \right]$$

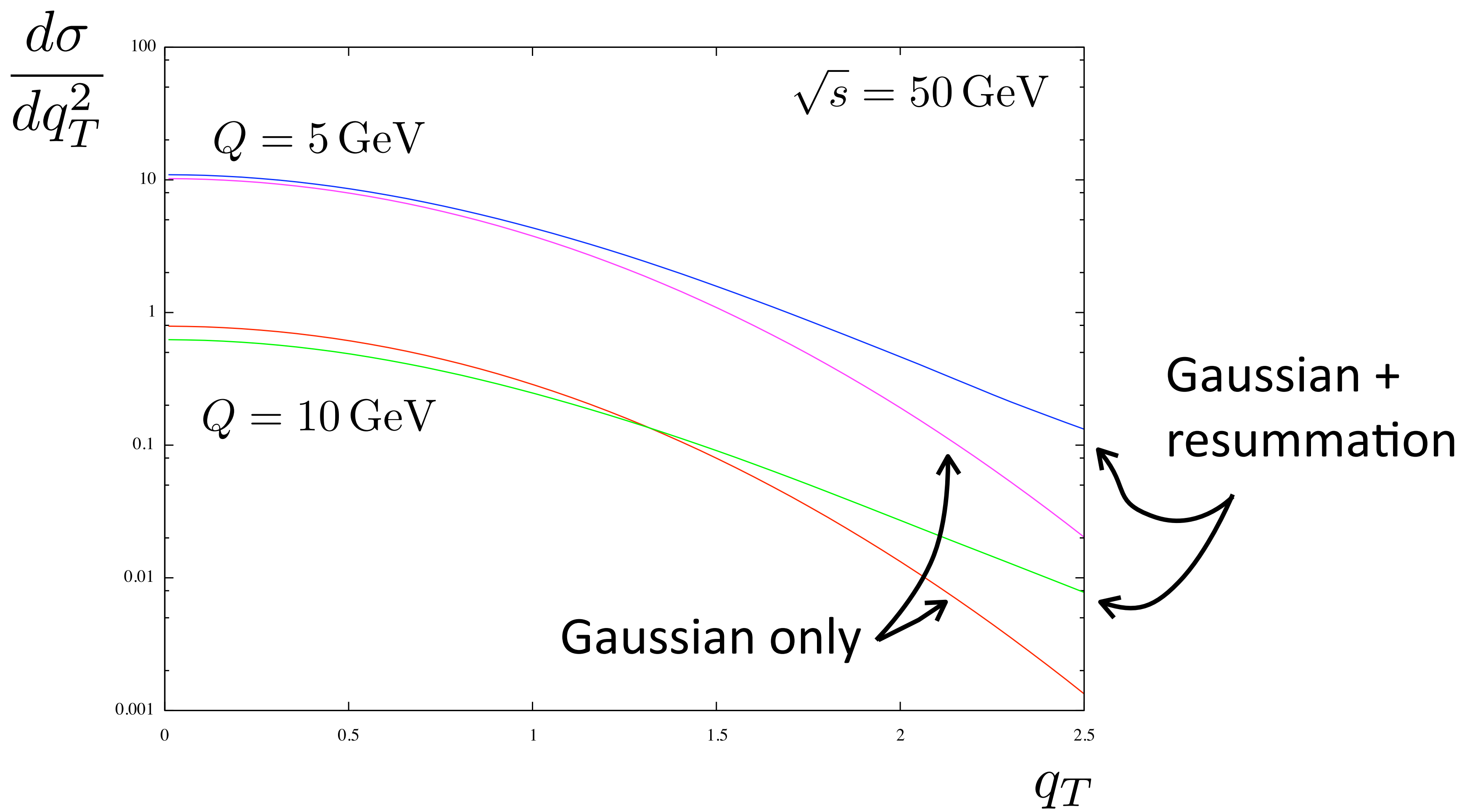
Example of resummation effects



Example of resummation effects



Example of resummation effects



Leading-log formula

Ellis, Veseli, NPB 511 (98)

$$F_{UU,T}(x,z,q_T^2,Q^2) = x \sum_a e_a^2 \, \frac{d}{dq_T^2} \Big[f_1^a(x;[q_T^2]) \, D_1^a(z;[q_T^2]) \, e^{-S} \big(1 - e^{-S_{NP}}\big) \Big]$$

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Ellis, Veseli, NPB 511 (98)

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$$S(q_T^2,Q^2) = -\int_{q_T^2}^{Q^2}\frac{d\mu^2}{\mu^2}\frac{\alpha_S(\mu^2)}{2\pi}\,2C_F\log\frac{Q^2}{\mu^2}$$

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Ellis, Veseli, NPB 511 (98)

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$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0\log(\mu^2/\Lambda^2)}$$

Nonperturbative part

Kulesza, Stirling, JHEP 12 (03)

$$S_{NP} = -\frac{q_T^2}{\langle q_T^2 \rangle}$$

$$\frac{1}{\langle q_T^2 \rangle} = 0.20 + 0.95 \, \log\left(\frac{Q}{3.2}\right) + 1.56 \, \log\left(\frac{\sqrt{s}}{19.4}\right)$$

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Kulesza, Stirling, JHEP 12 (03)

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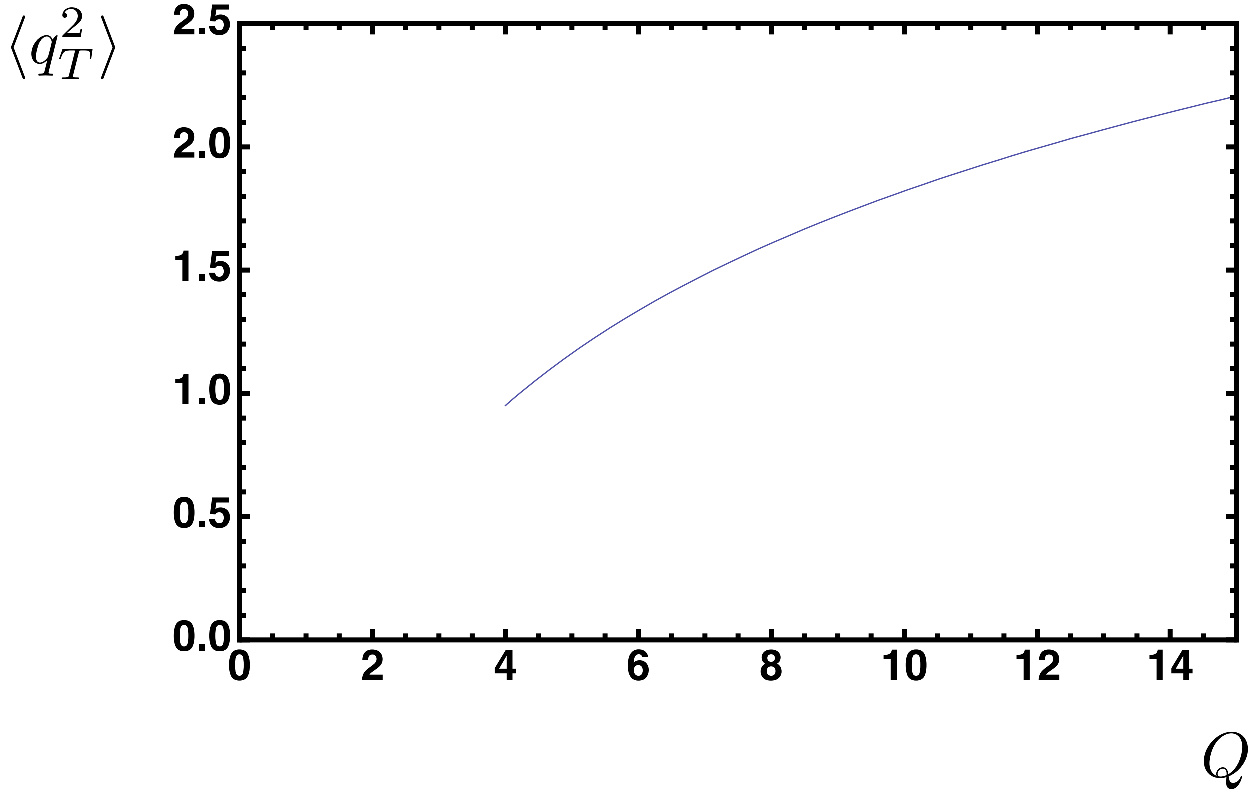
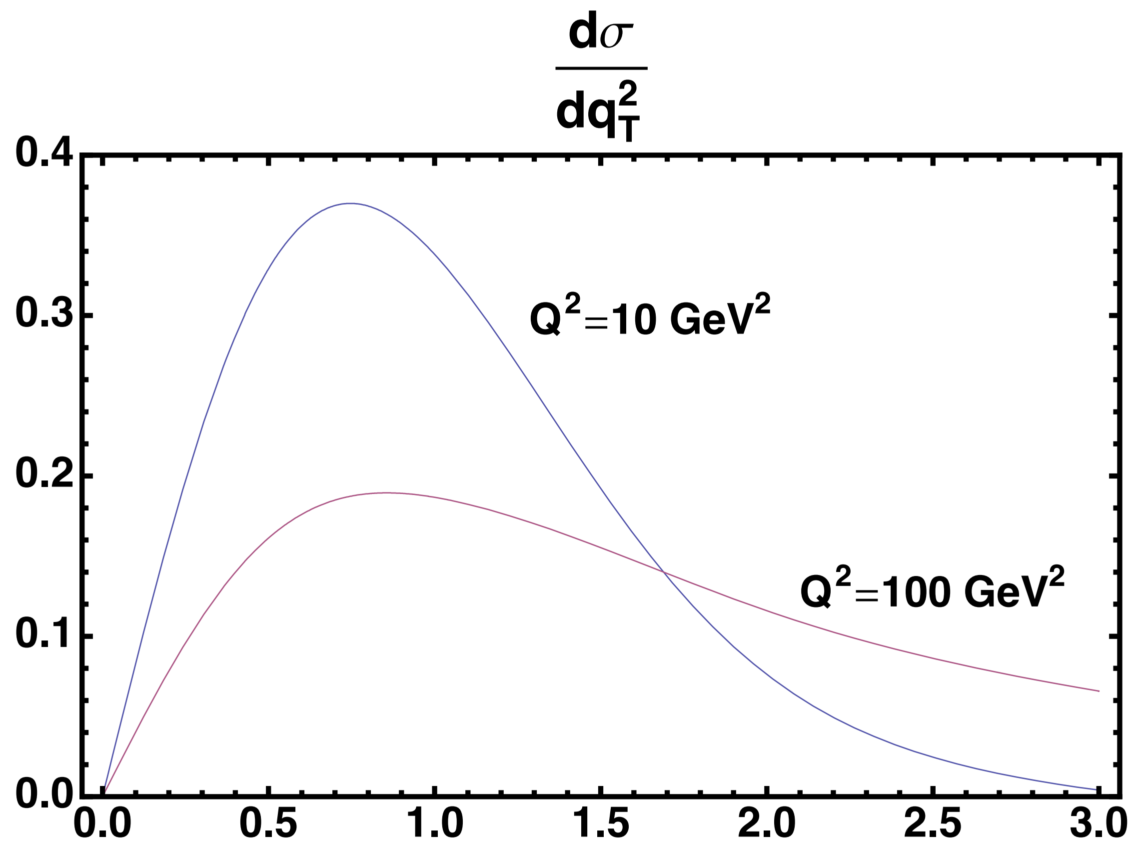


Leading-log evolution



Evolution of Sivers function

$$f_1^{\mathrm{NS}}(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\left(\frac{L(\eta^{-1})}{2} - C_F \right) f_1^{\mathrm{NS}}(x) + \left(P_{qq} \otimes f_1^{\mathrm{NS}} \right) \right]$$

Evolution of Sivers function

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$$\frac{\boldsymbol{p}_T^2}{2M^2} f_{1T}^{\perp \mathrm{NS}}(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{M}{\boldsymbol{p}_T^2} \left[\left(\frac{L(\eta^{-1})}{2} - C_F \right) f_{1T}^{\perp (1) \mathrm{NS}}(x) + \ldots \right]$$

Evolution of Sivers function

$$f_1^{\mathrm{NS}}(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\left(\frac{L(\eta^{-1})}{2} - C_F\right) f_1^{\mathrm{NS}}(x) + \big(P_{qq}\otimes f_1^{\mathrm{NS}}\big)\right]$$

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$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + \ldots\right]$$

Evolution of Sivers function

$$f_1^{\mathrm{NS}}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\left(\frac{L(\eta^{-1})}{2} - C_F\right) f_1^{\mathrm{NS}}(x) + \left(P_{qq}\otimes f_1^{\mathrm{NS}}\right)\right]$$

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$$\frac{q_T}{M}\, F_{UT,T}^{\sin(\phi_h-\phi_s)} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[-f_{1T}^{\perp(1)a}(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + \ldots\right]$$

Evolution of Sivers function

$$f_1^{\mathrm{NS}}(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\left(\frac{L(\eta^{-1})}{2} - C_F \right) f_1^{\mathrm{NS}}(x) + \left(P_{qq} \otimes f_1^{\mathrm{NS}} \right) \right]$$

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Collins asymmetry, *b* space analysis

D. Boer, NPB 806 (08)

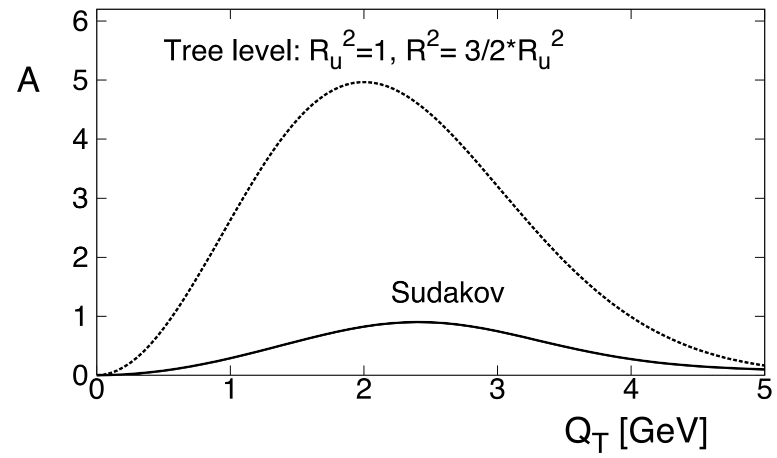

FIG. 6: The asymmetry factor $\mathcal{A}(Q_T)$ at $Q = 10\,\mathrm{GeV}$ (solid curve) and the tree level quantity $\mathcal{A}^{(0)}(Q_T)$ using $R_u^2 = 1\,\mathrm{GeV}^{-2}$ and $R^2/R_u^2 = 3/2$. Both factors are given in units of M^2.

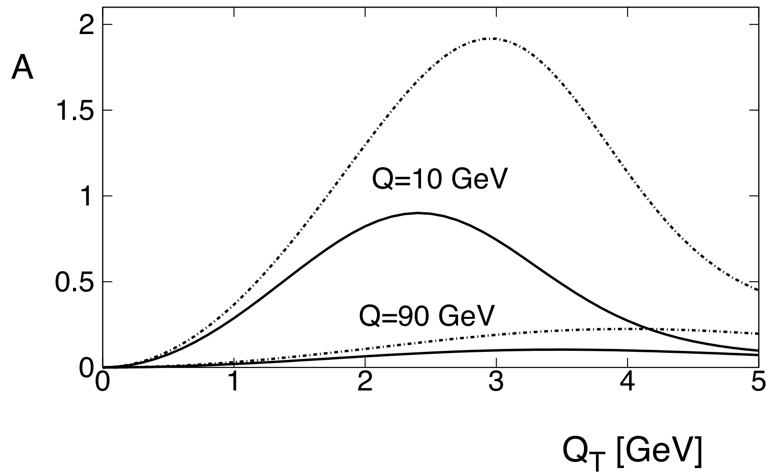

FIG. 5: The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M^2) at $Q = 10\,\mathrm{GeV}$ and $Q = 90\,\mathrm{GeV}$. The solid curves are obtained with the method explained in the text; the dashed-dotted curves are from the earlier analysis of Ref. [64].

Evolution of transverse moment of Sivers function

Vogelsang, Yuan, talk at SPIN08

Kang, Qiu, arXiv:0811.3101 [hep-ph]

$$\frac{\partial f_1^{\rm NS}(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_1^{\rm NS}(\xi,\mu^2) P_{qq}(z)\Big|_{z=x/\xi}$$

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$$\begin{aligned}\frac{\partial \mathcal{T}_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ & P_{qq}(z)\,\mathcal{T}_{q,F}(\xi,\xi,\mu_F) \\ & + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[\mathcal{T}_{q,F}(\xi,x,\mu_F) - \mathcal{T}_{q,F}(\xi,\xi,\mu_F)\right] + z\,\mathcal{T}_{q,F}(\xi,x,\mu_F)\right] \\ & + \frac{C_A}{2}\left[\mathcal{T}_{\Delta q,F}(x,\xi,\mu_F)\right]\Big\},\end{aligned}$$

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$$T_F(x,x) \equiv 2M f_{1T}^{\perp(1)}(x)$$

Evolution of transverse moment of Sivers function

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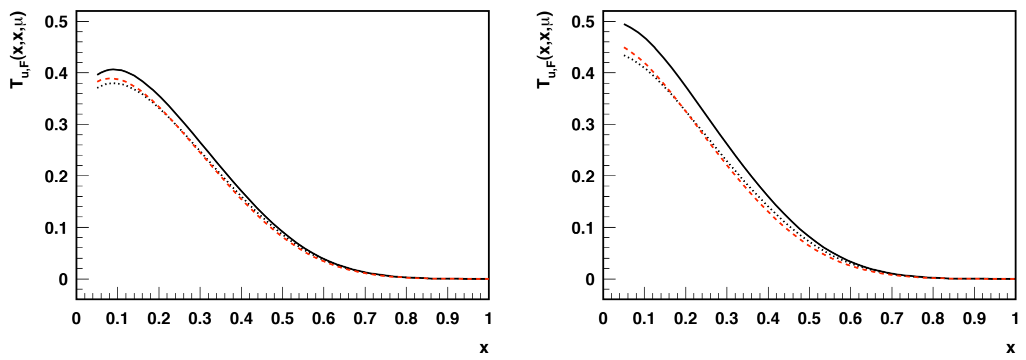
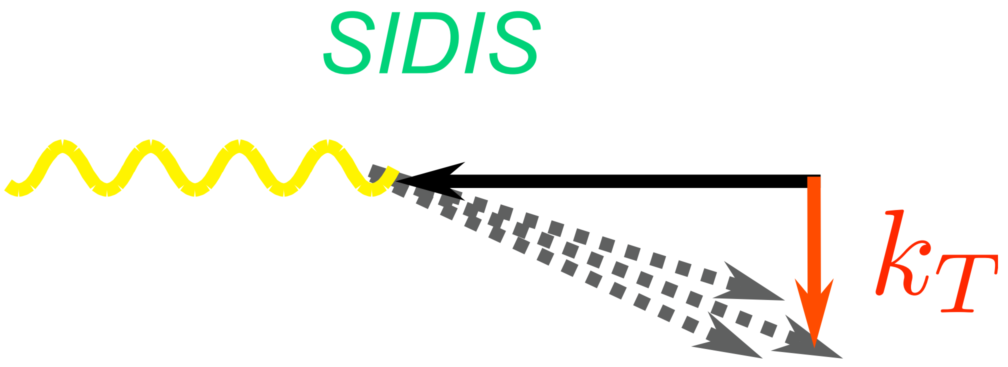


FIG. 12: Twist-3 up-quark-gluon correlation $T_{u,F}(x,x,\mu_F)$ as a function of x at $\mu_F = 4$ GeV (left) and $\mu_F = 10$ GeV (right). The factorization scale dependence is a solution of the flavor non-singlet evolution equation in Eq. (99). Solid and dotted curves correspond to $\sigma = 1/4$ and $1/8$, while the dashed curve is obtained by keeping only the DGLAP evolution kernel $P_{qq}(z)$ in Eq. (99).

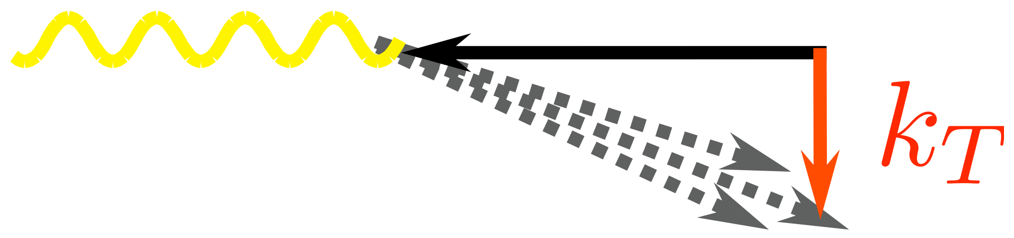
Factorization and universality

Different processes

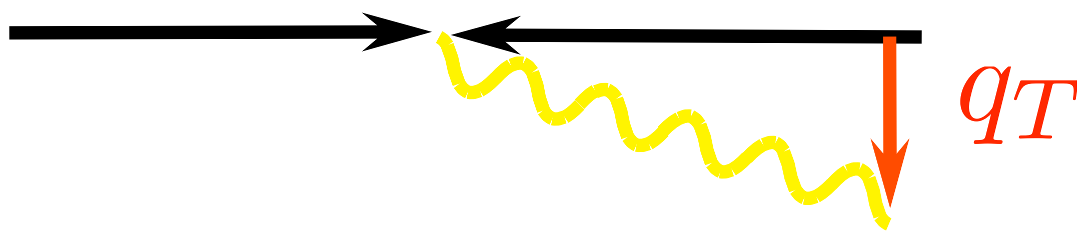


Different processes

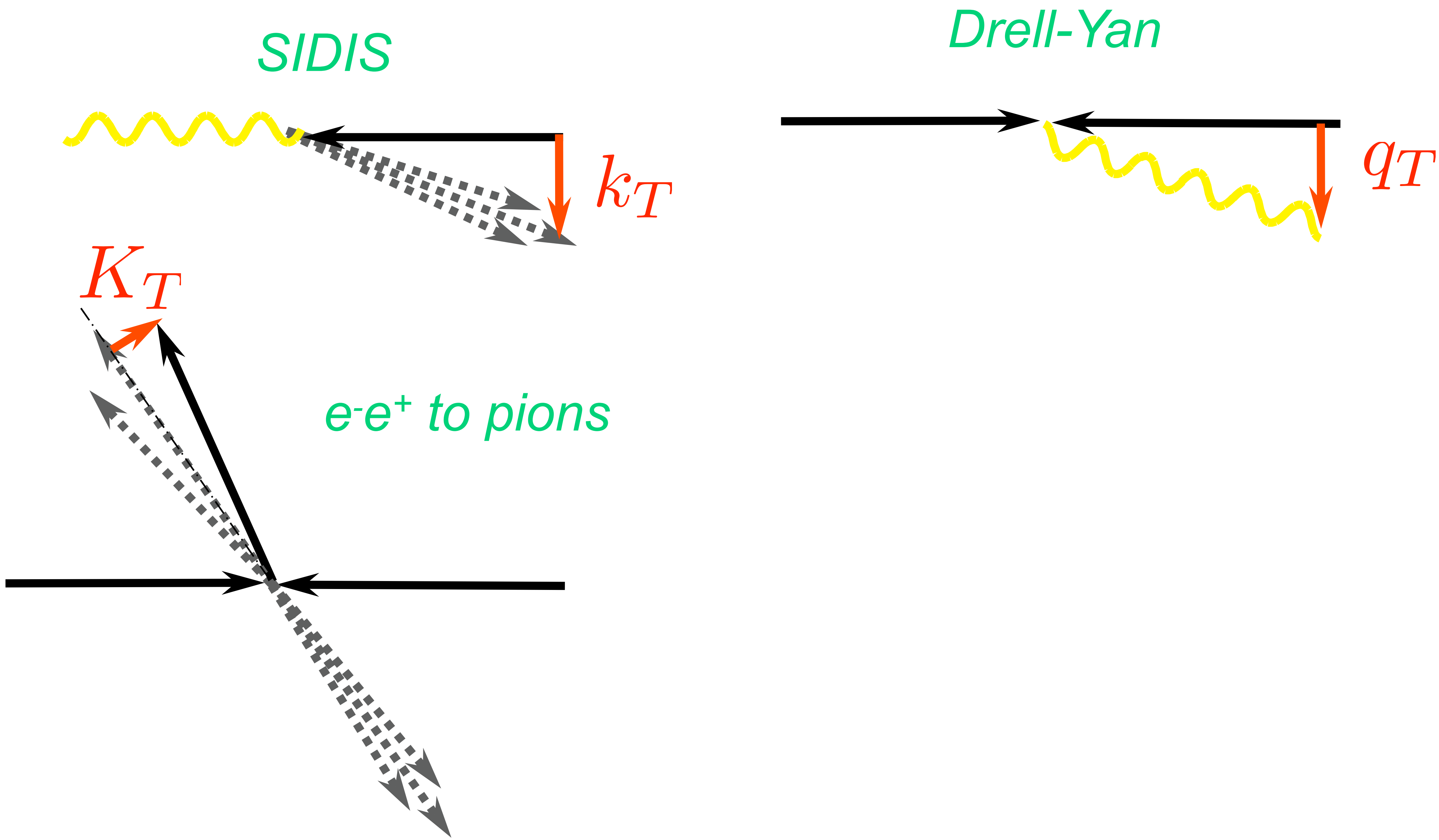
SIDIS



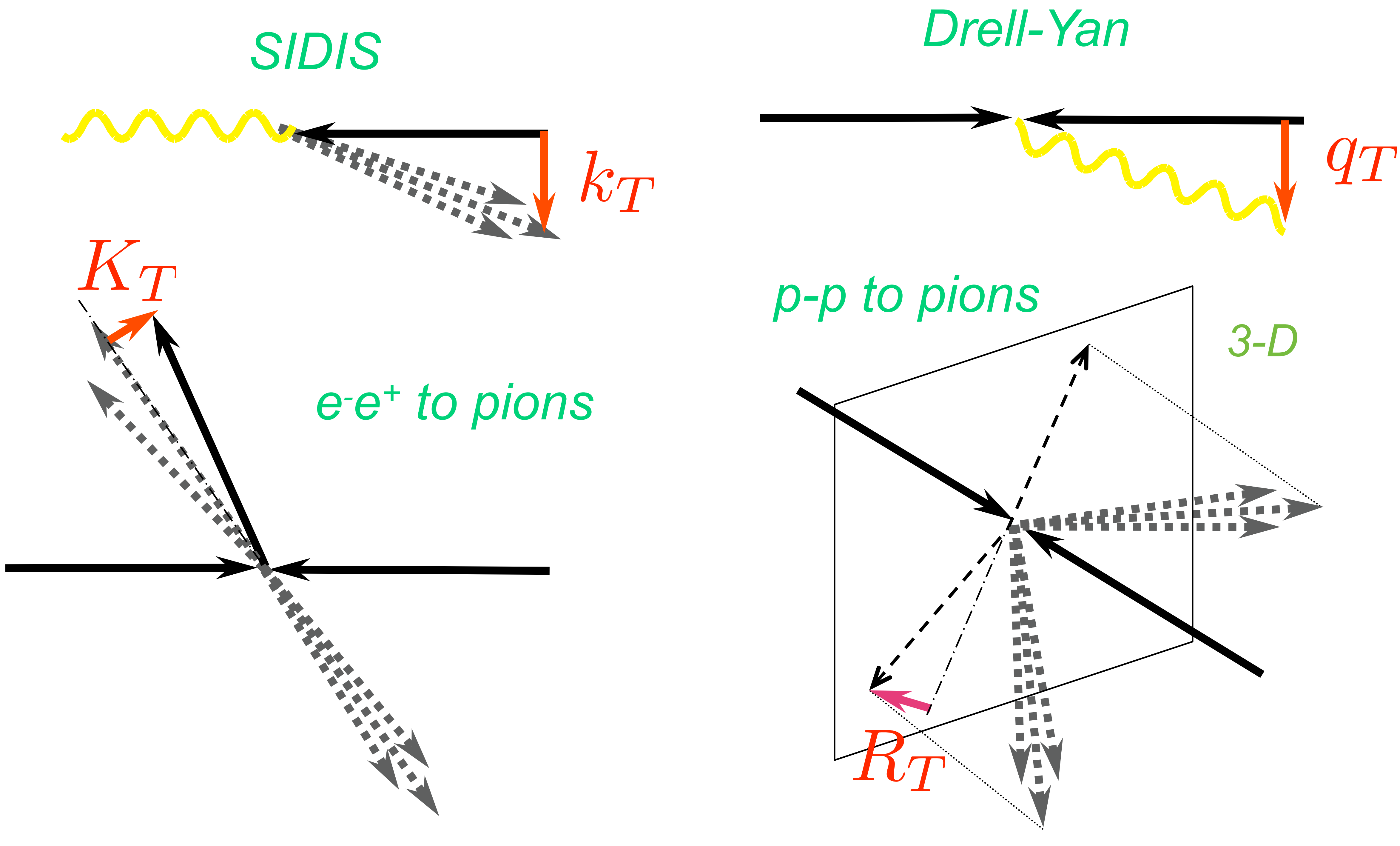
Drell-Yan



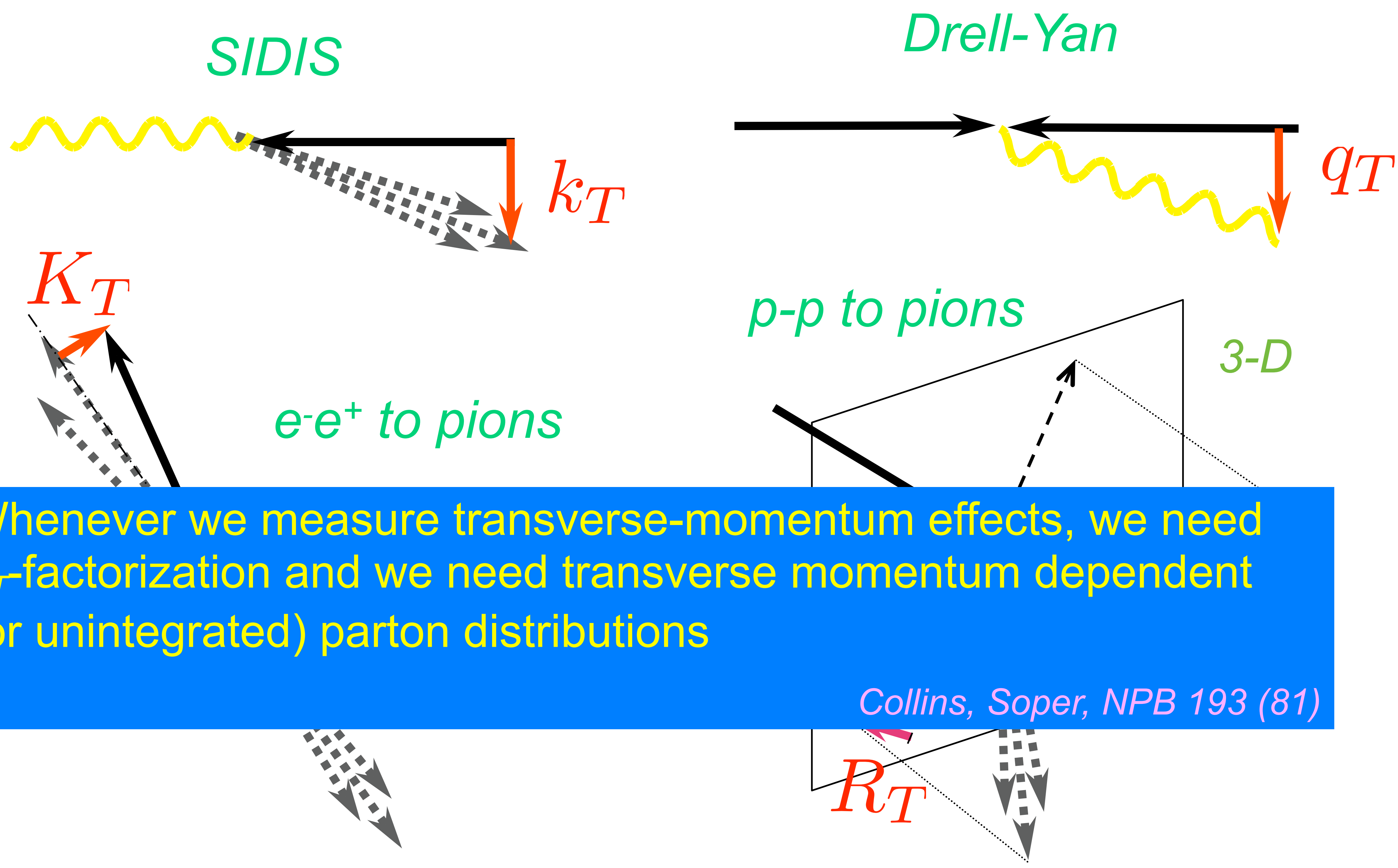
Different processes



Different processes



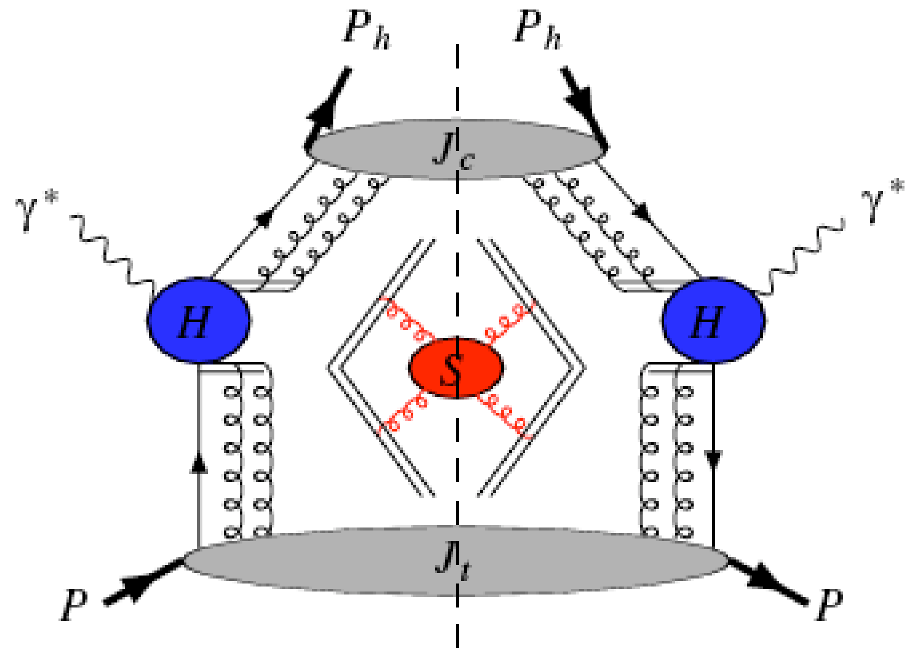
Different processes



Whenever we measure transverse-momentum effects, we need k_T–factorization and we need transverse momentum dependent (or unintegrated) parton distributions

Collins, Soper, NPB 193 (81)

k_T factorization



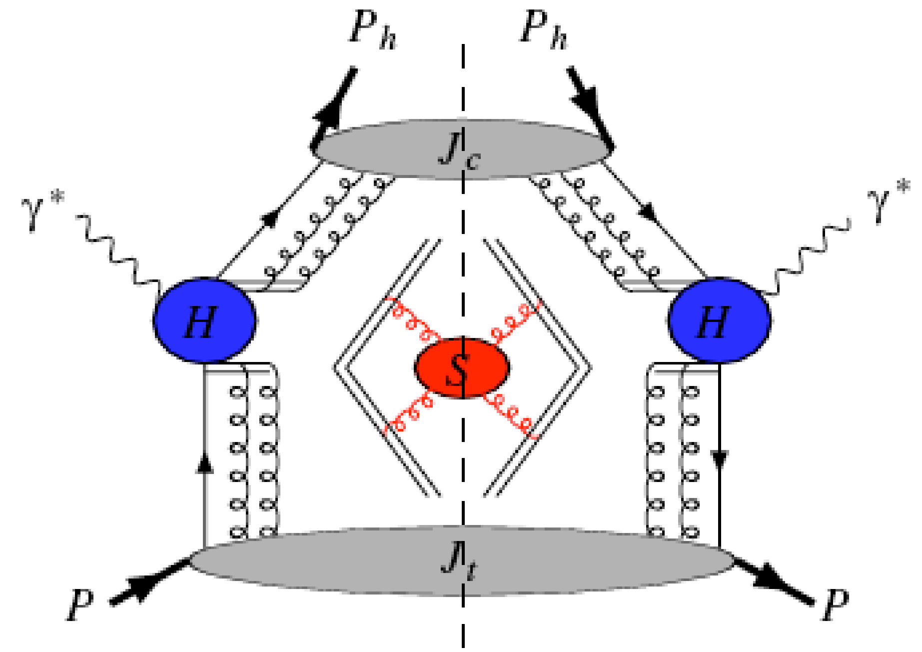
$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}\big[f_1 D_1\big]$$

$$= \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x,p_T^2,\mu^2)\, D_1^a(z,k_T^2,\mu^2)\, U(l_T^2,\mu^2) H(Q^2,\mu^2)$$

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

k_T factorization



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$$x \sum_a e_a^2\, f_1^a(x,p_T^2,\mu^2)\, D_1^a(z,k_T^2,\mu^2)\, U(l_T^2,\mu^2) H(Q^2,\mu^2)$$

TMD PDF — TMD FF — Soft factor — Hard part

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

Consequences

$$d\sigma_{DIS} = H_{DIS} \otimes f$$

$$d\sigma_{DY} = H_{DY} \otimes f$$

$$d\sigma^{\uparrow}_{DIS} - d\sigma^{\downarrow}_{DIS} = K_{DIS} \otimes g$$

$$d\sigma^{\uparrow}_{DIS} - d\sigma^{\downarrow}_{DIS} = -K_{DY} \otimes g$$

Consequences

- The real part of the gauge link remains unchanged

$$d\sigma_{DIS} = H_{DIS} \otimes f \qquad d\sigma_{DY} = H_{DY} \otimes f$$

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Consequences

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- The imaginary part changes sign

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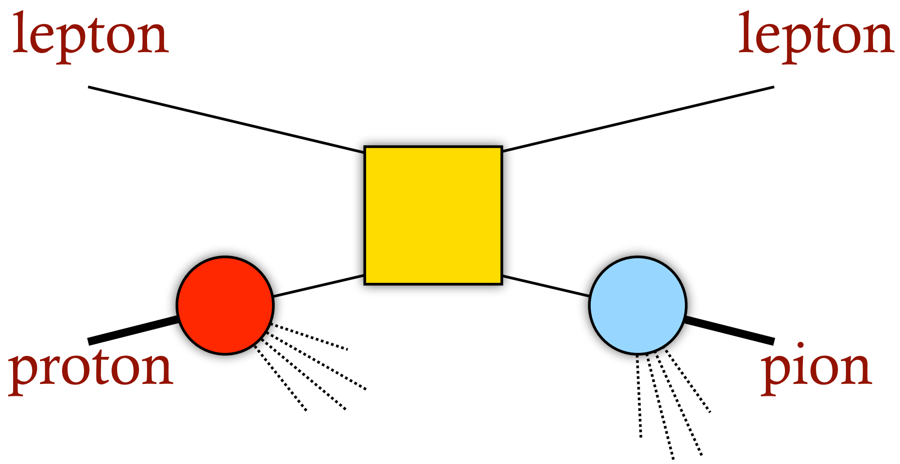
Consequences

- The real part of the gauge link remains unchanged
- The imaginary part changes sign
- Observables sensitive to the imaginary part (e.g. single spin asymmetries) acquire an extra minus sign (generalization of universality)

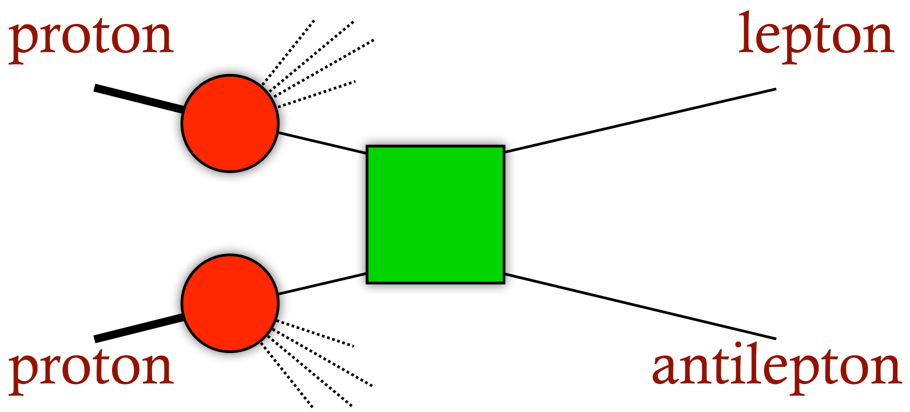
$$d\sigma_{DIS} = H_{DIS} \otimes f \qquad d\sigma_{DY} = H_{DY} \otimes f$$

$$d\sigma^{\uparrow}_{DIS} - d\sigma^{\downarrow}_{DIS} = K_{DIS} \otimes g \qquad d\sigma^{\uparrow}_{DIS} - d\sigma^{\downarrow}_{DIS} = -K_{DY} \otimes g$$

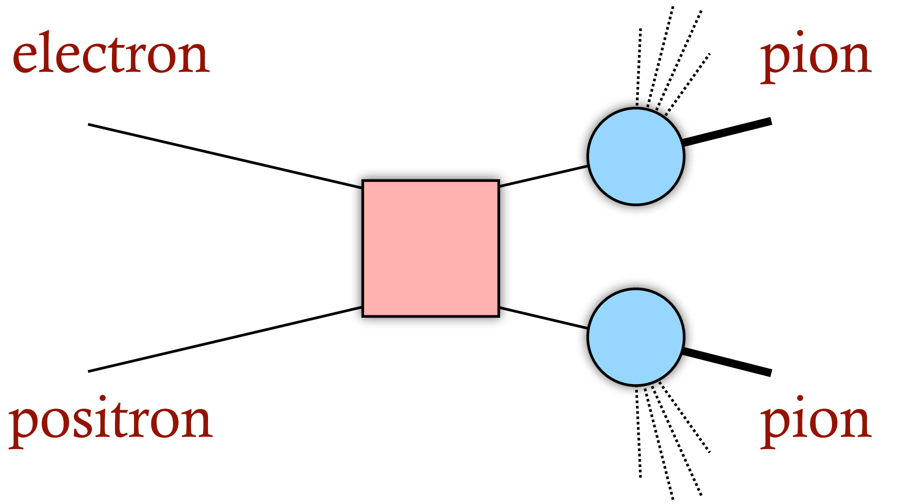
Generalized universality



SIDIS

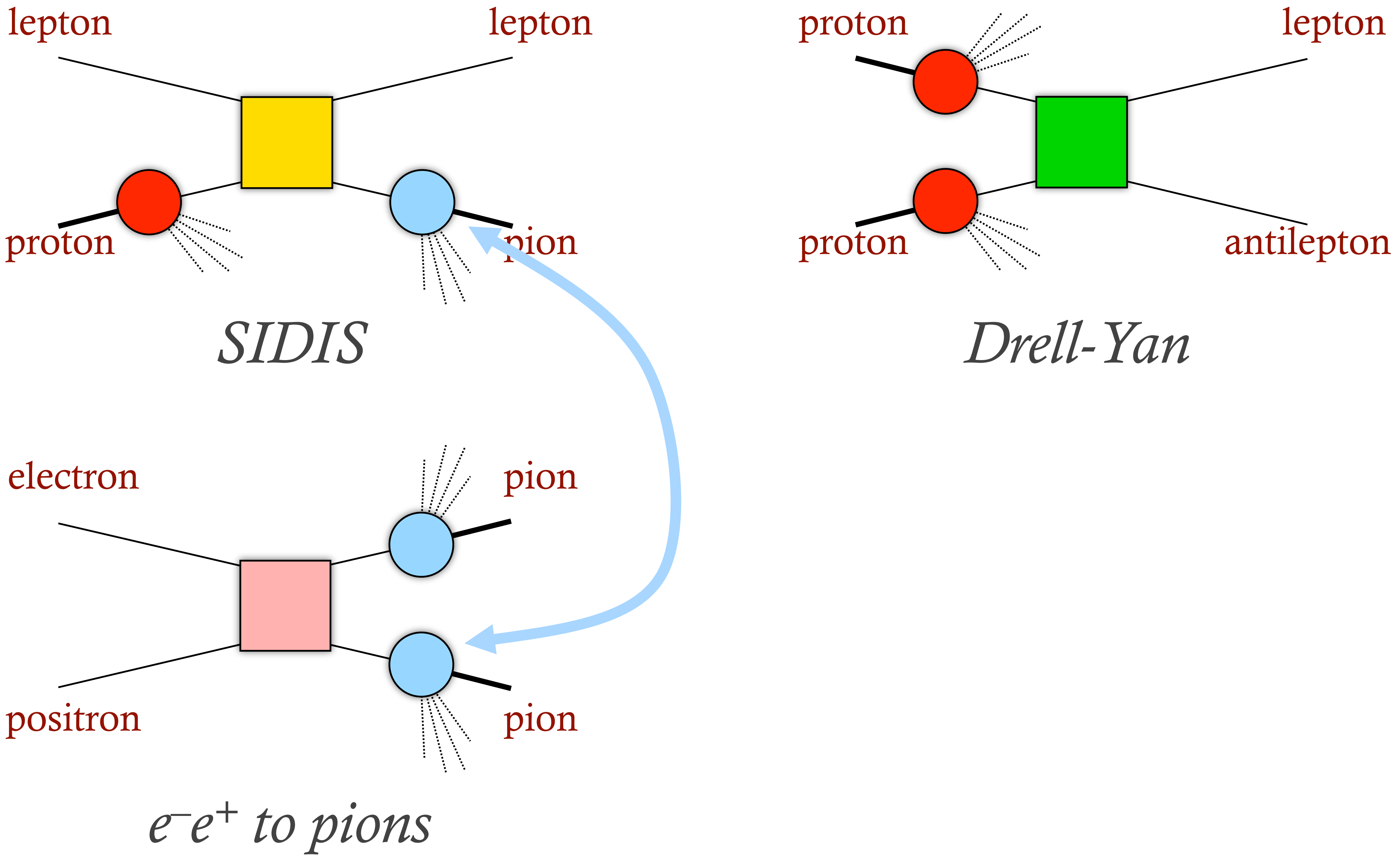


Drell-Yan

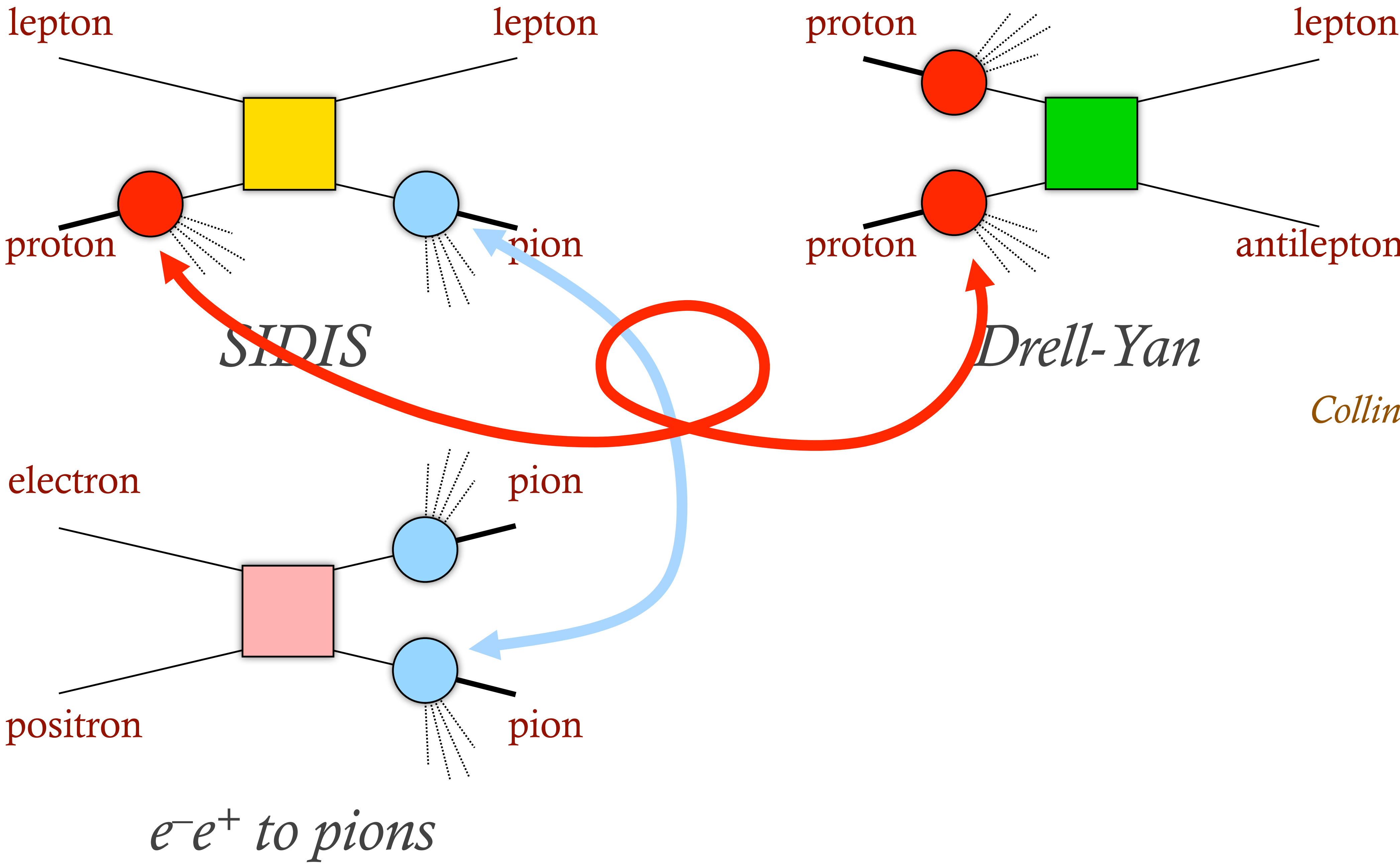


e^-e^+ to pions

Generalized universality

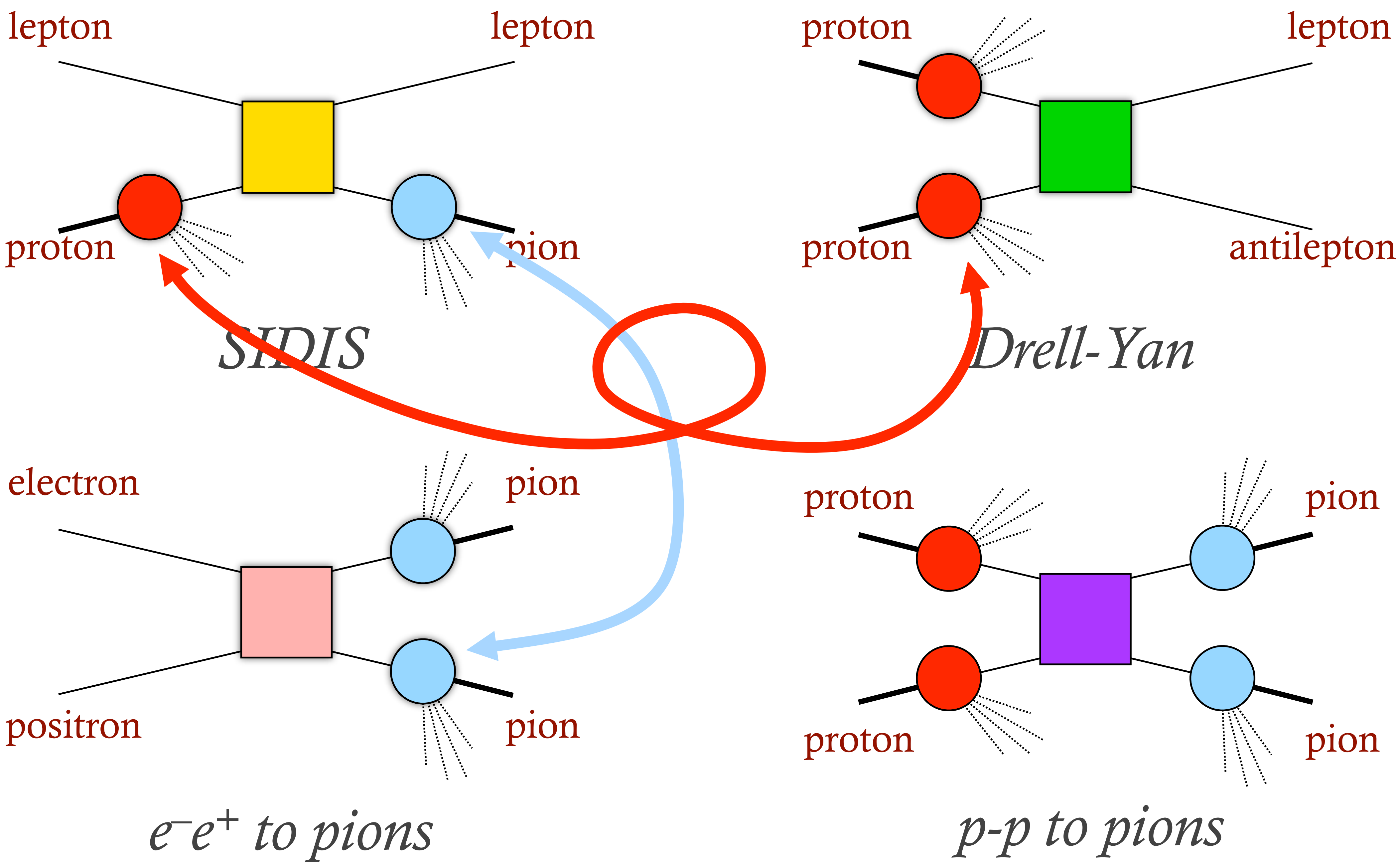


Generalized universality

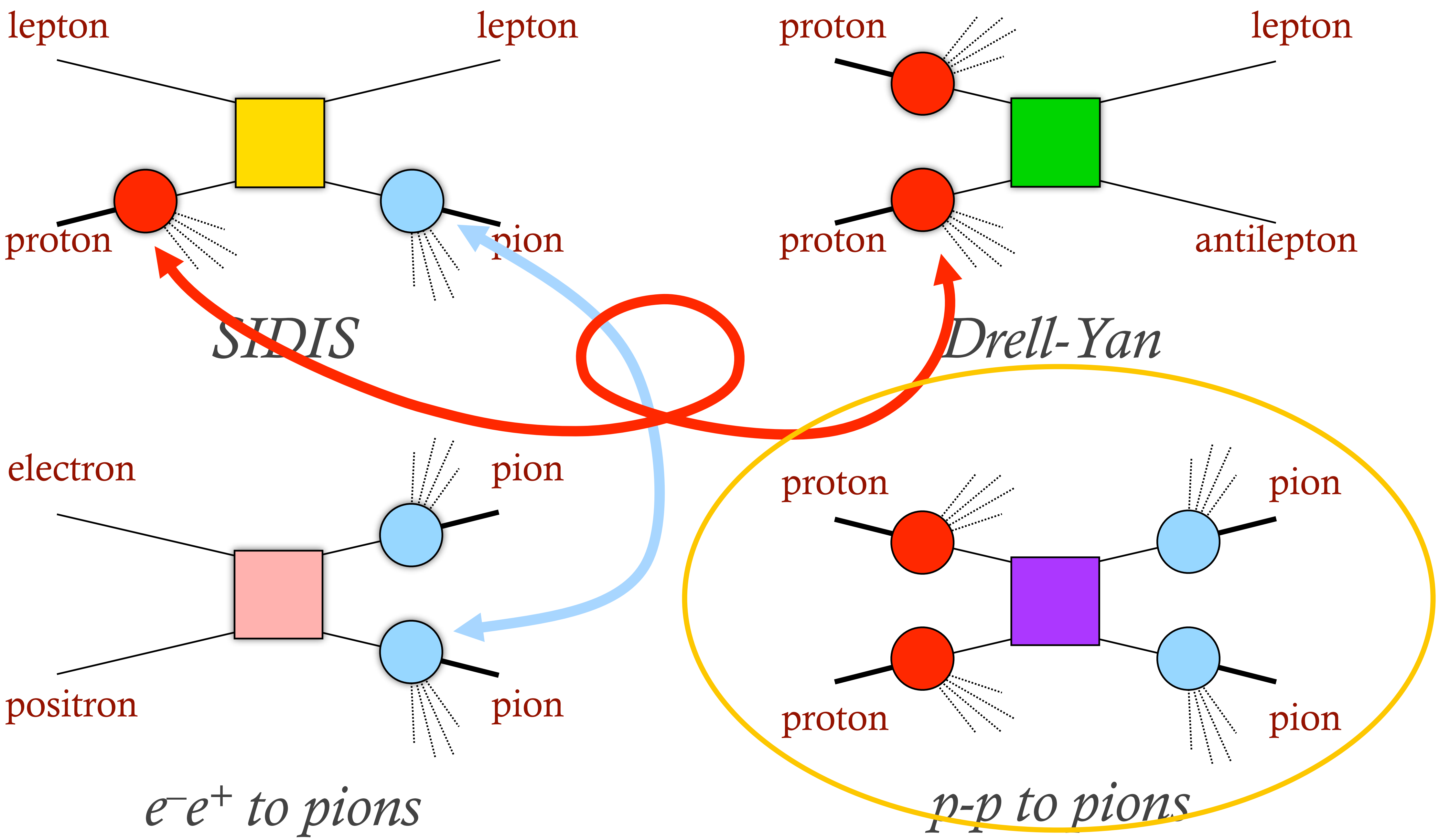


Collins, PLB 536 (02)

Hadrons to hadrons

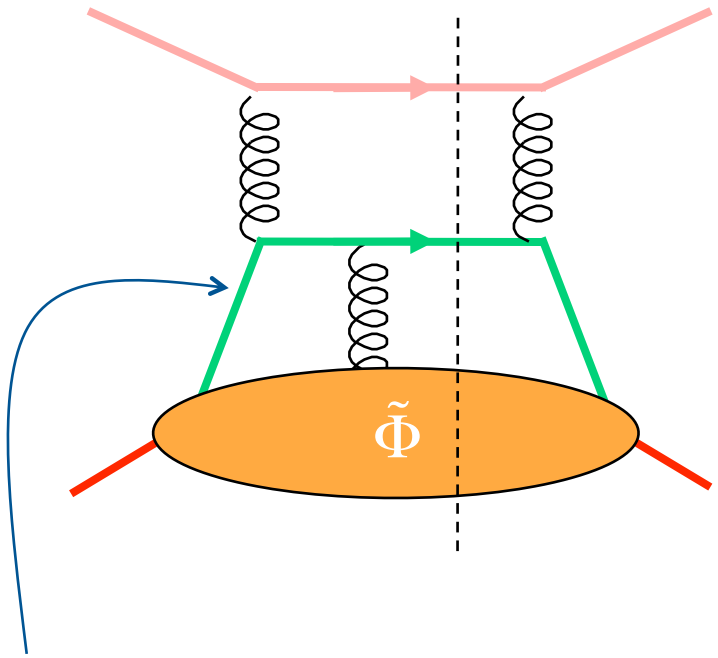


Hadrons to hadrons

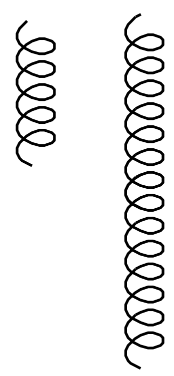


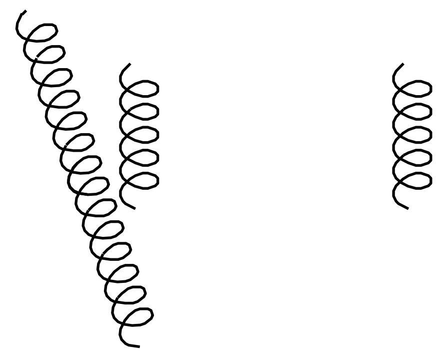
A slightly more complex example

Collins, Qiu, PRD 75 (07)



parton with charge g_1





$$\frac{g_1}{[-l^+ + i\epsilon]}$$

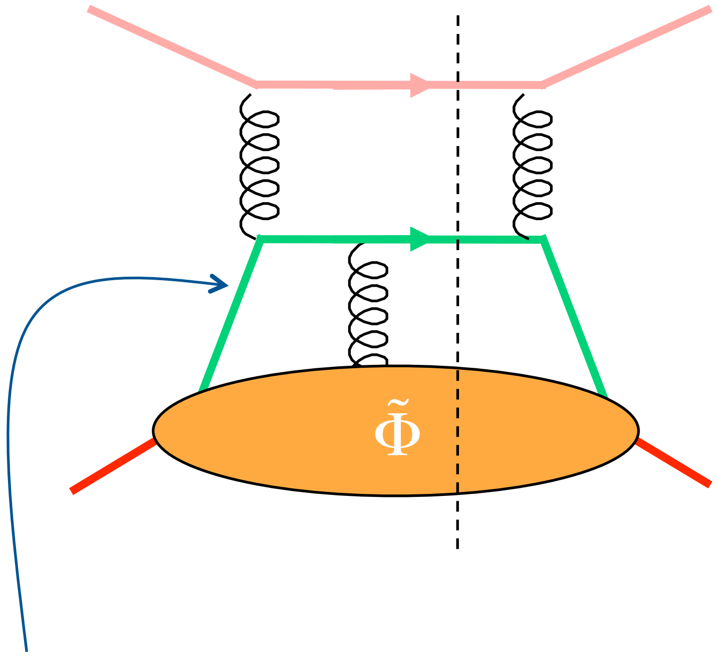
$$\frac{g_2}{[-l^+ + i\epsilon]}$$

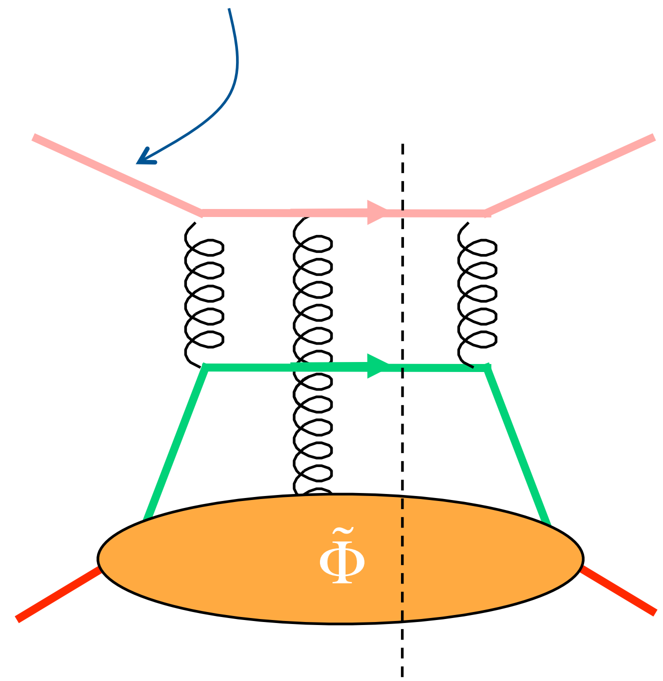
$$-\frac{g_2}{[-l^+ + i\epsilon]}$$

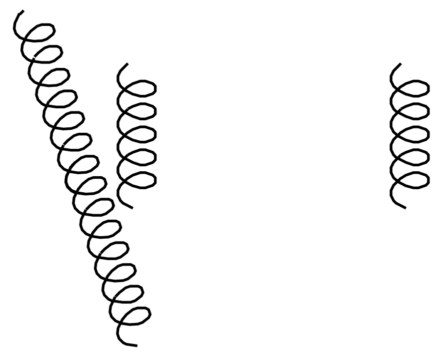
A slightly more complex example

parton with charge g_2

Collins, Qiu, PRD 75 (07)



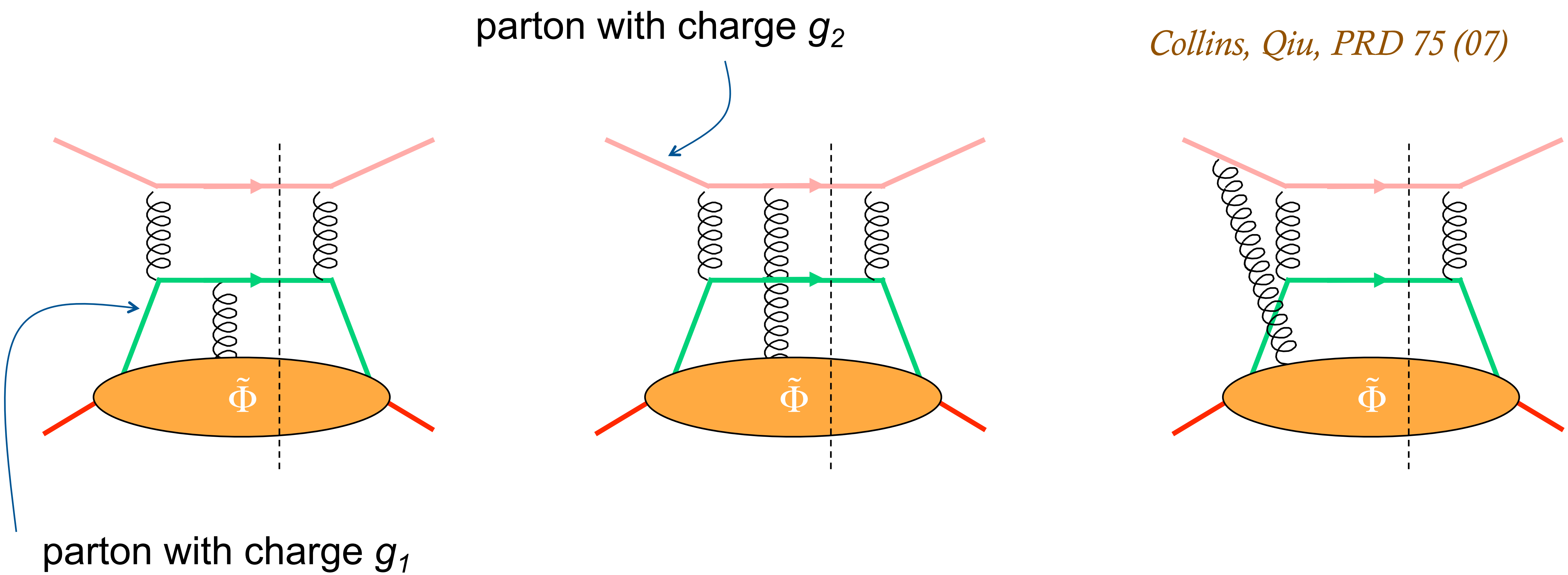




parton with charge g_1

$$\frac{g_1}{[-l^+ + i\epsilon]} \qquad \frac{g_2}{[-l^+ + i\epsilon]} \qquad -\frac{g_2}{[-l^+ + i\epsilon]}$$

A slightly more complex example



$$\frac{g_1}{[-l^+ + i\epsilon]} \qquad \frac{g_2}{[-l^+ + i\epsilon]} \qquad -\frac{g_2}{[-l^+ + i\epsilon]}$$

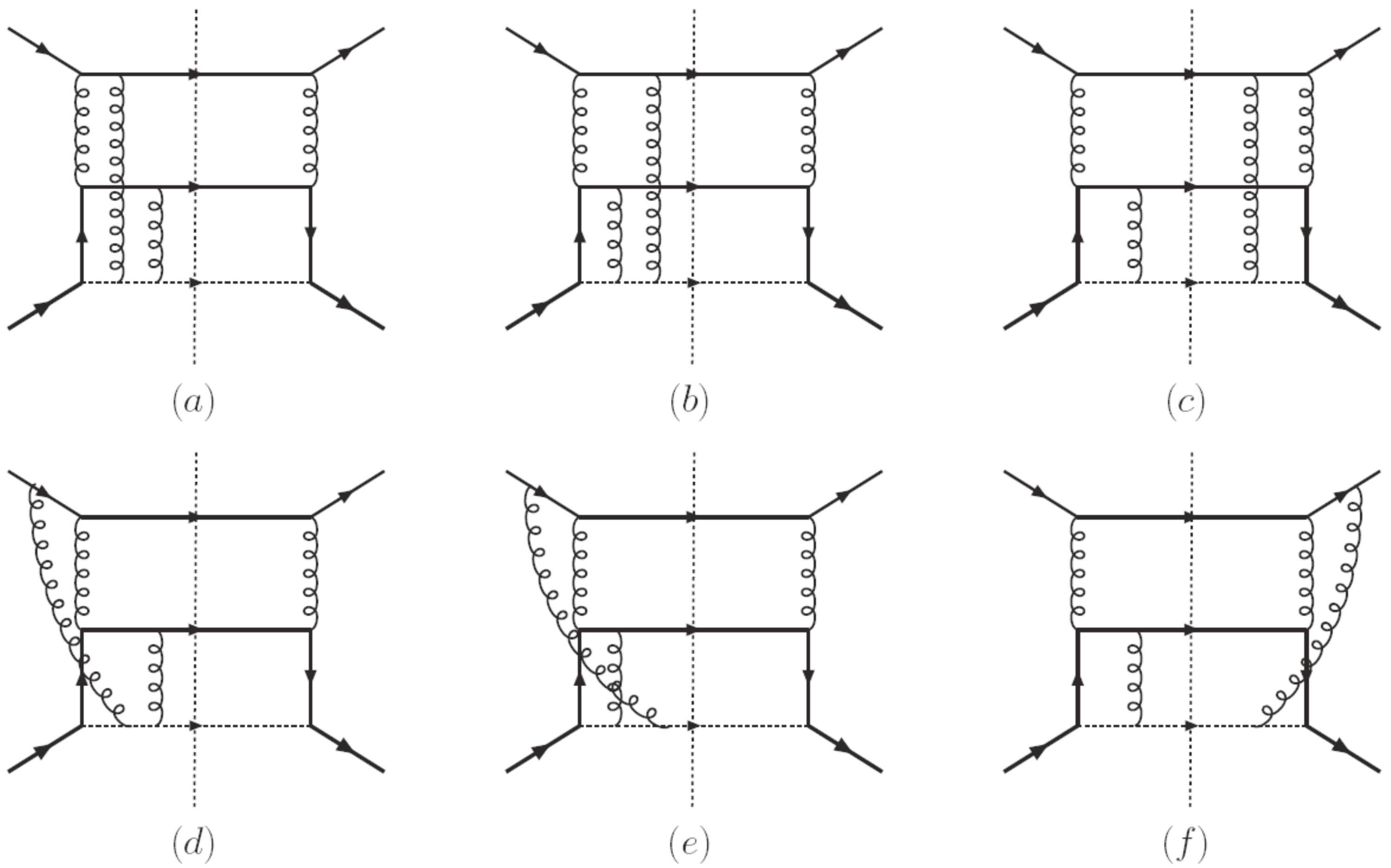
Consequences

$$\frac{g_1}{[-l^+ + i\epsilon]} + \frac{g_2}{[-l^+ + i\epsilon]} - \frac{g_2}{[-l^+ + i\epsilon]} = -i\pi(2g_2 + g_1)\delta(l^+) - PV\frac{g_1}{l^+}$$

- Up to this order, the real part is unchanged, the imaginary part gets more than just a simple sign change and depends on the charge of ANOTHER parton!
- Still possible to get around it: PDFs could still be universal, but the ones sensitive to the imaginary part (those involved in single spin asymmetries) have to be multiplied by $g_1/(2g_2+g_1)$

Two-gluon exchange

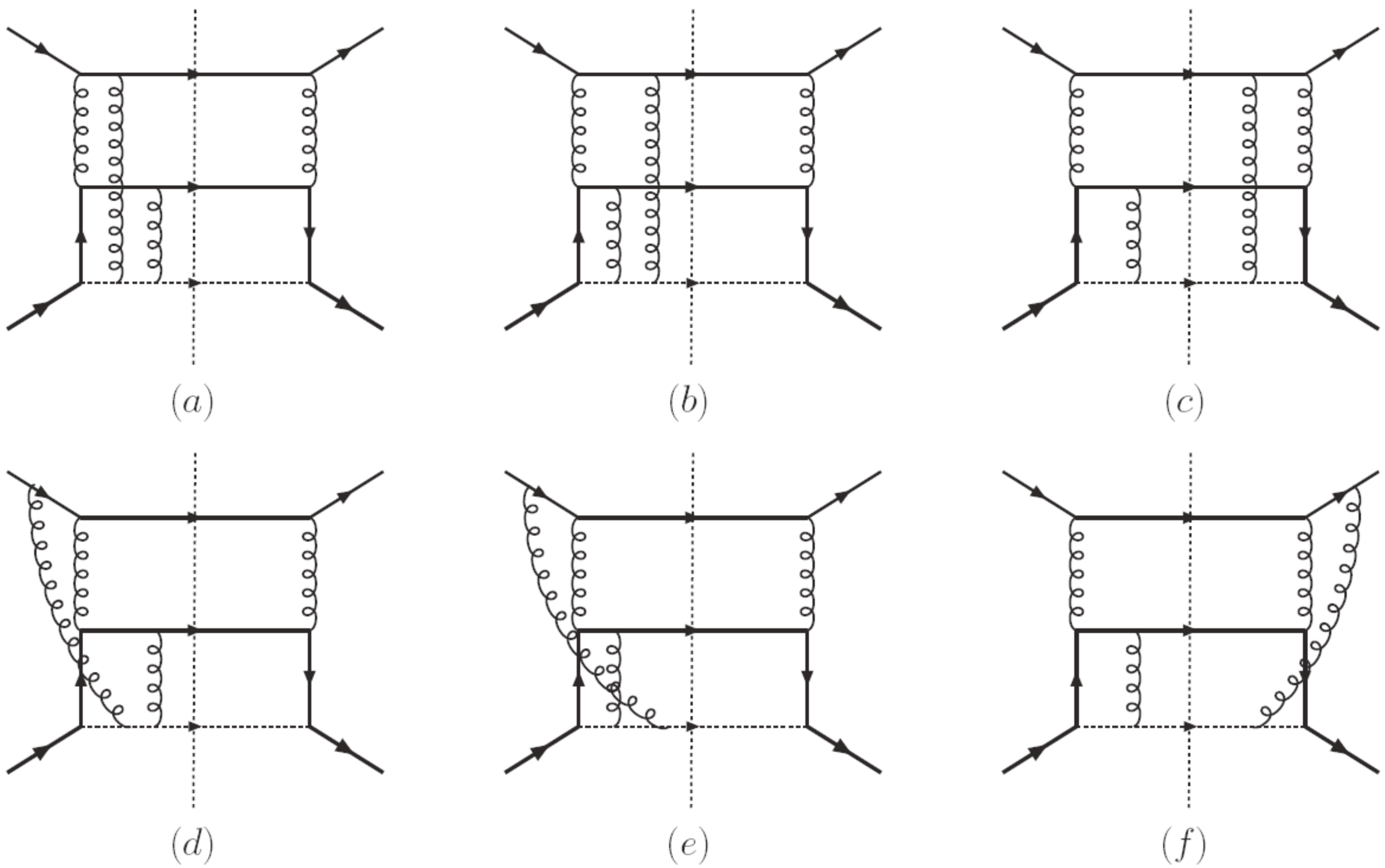
Collins, 0708.4410 [hep-ph] *Vogelsang, Yuan, 0708.4398 [hep-ph]*



+ more

Two-gluon exchange

Collins, 0708.4410 [hep-ph] *Vogelsang, Yuan, 0708.4398 [hep-ph]*



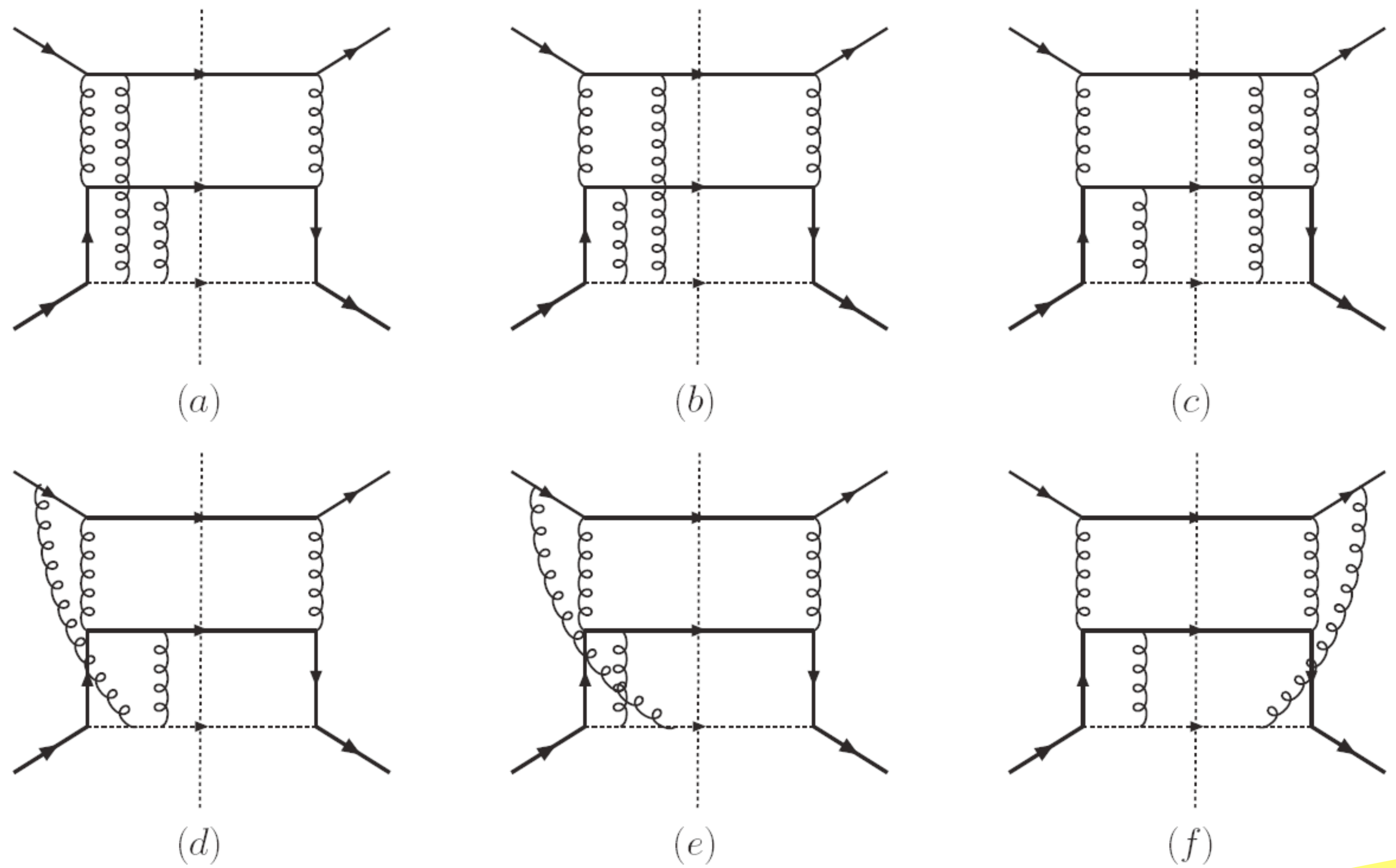
+ more

$$g_1^2 \left[\frac{1}{k_1^+ k_2^+} + (-i\pi)^2 \delta(k_1^+)\delta(k_2^+) \right] + g_1(g_1 + 2g_2)(i\pi) \left[\frac{\delta(k_2^+)}{k_1^+} + \frac{\delta(k_1^+)}{k_2^+} \right]$$

$$+4\left(g_1 g_2 + g_2^2\right)(-i\pi)^2 \delta(k_1^+)\delta(k_2^+) \ .$$

Two-gluon exchange

Collins, 0708.4410 [hep-ph] *Vogelsang, Yuan, 0708.4398 [hep-ph]*



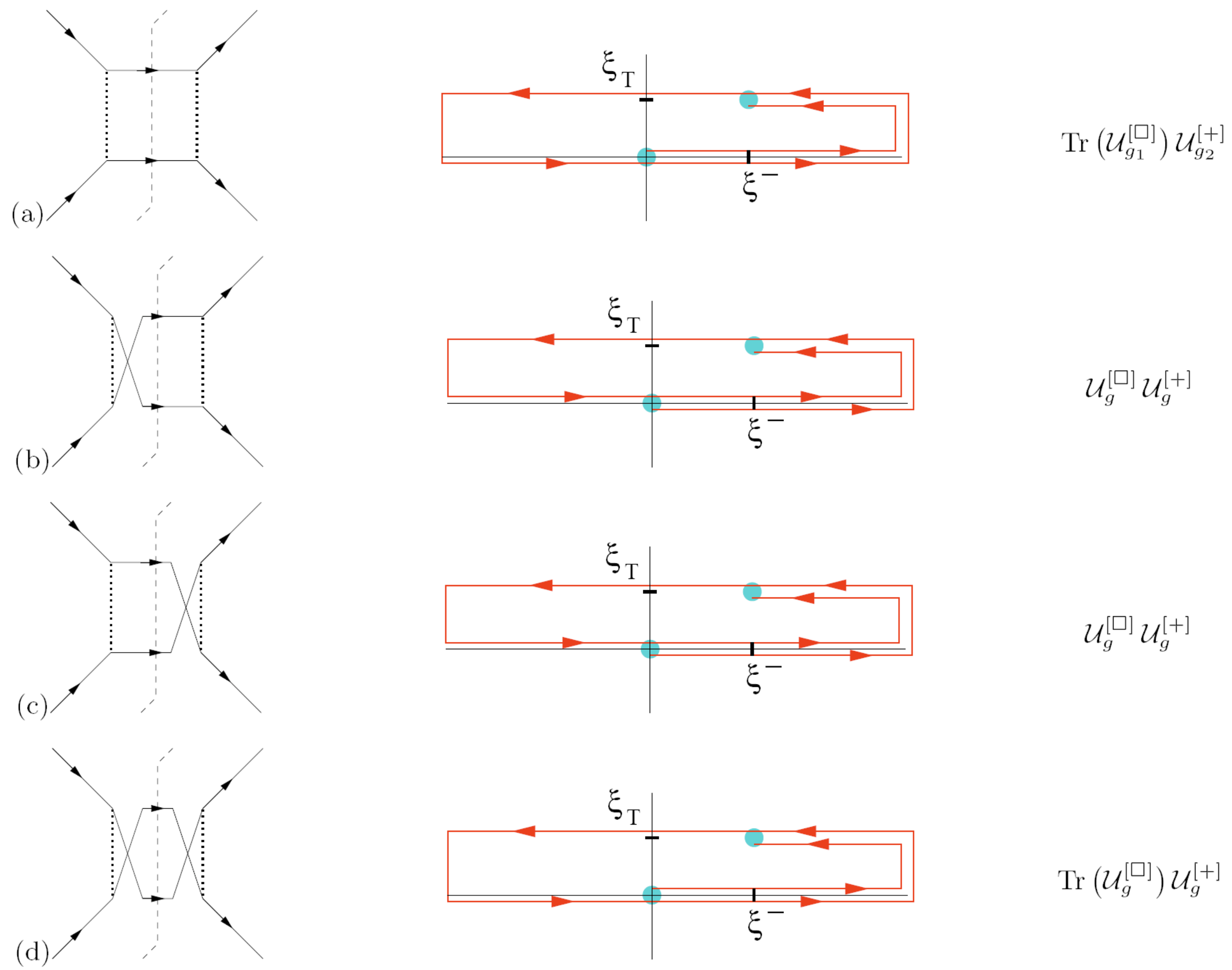
+ more

$$g_1^2 \left[\frac{1}{k_1^+ k_2^+} + (-i\pi)^2 \delta(k_1^+)\delta(k_2^+) \right] + \cdots \frac{\cdots}{k_2^+} \Bigg]$$

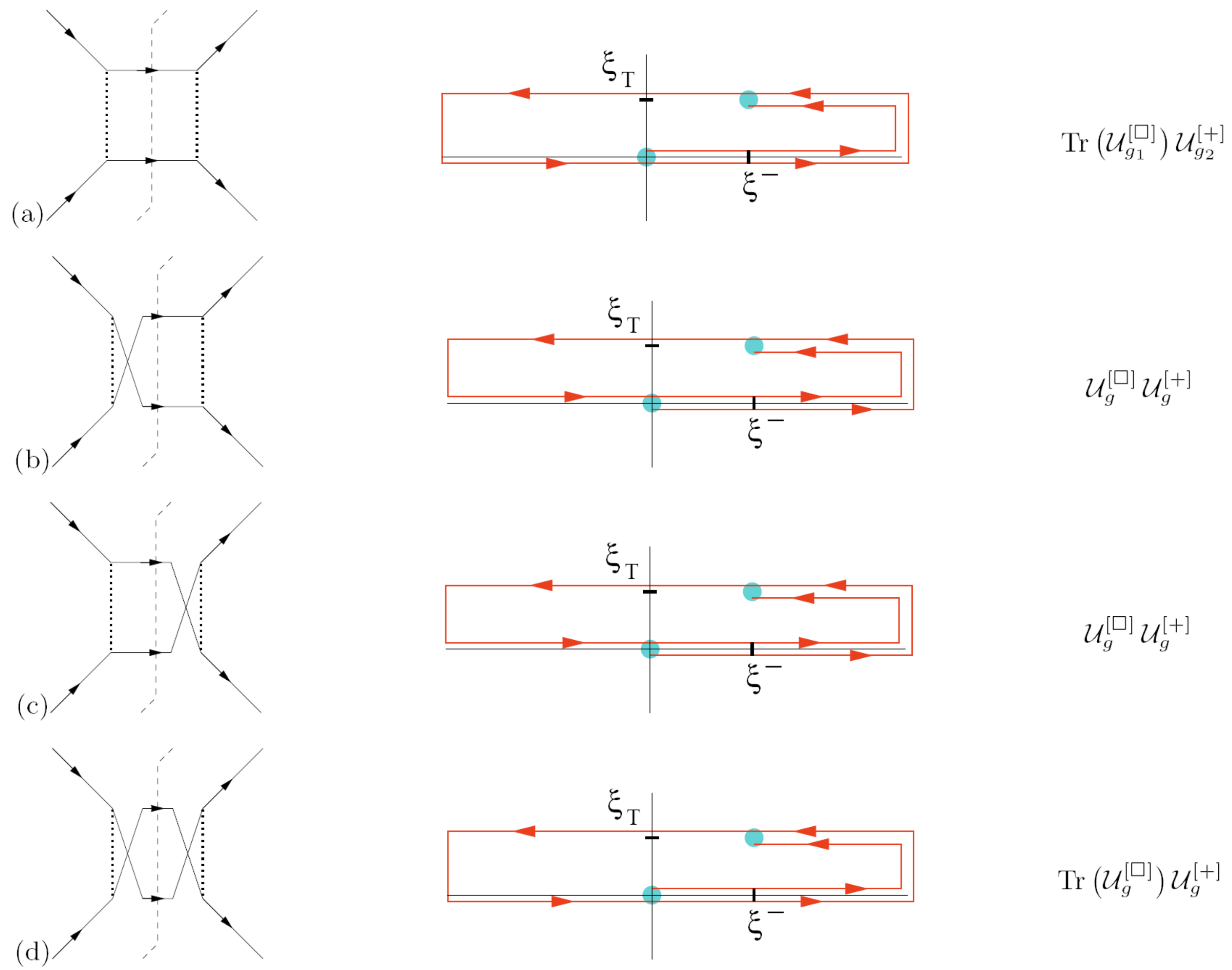
$$+4\left(g_1 g_2 + g_2^2\right)(-i\pi)^2 \delta(k_1^+)\delta(k_2^+) \ .$$

Breaking of universality, and not only in single-spin asymmetries

A forest of gauge links

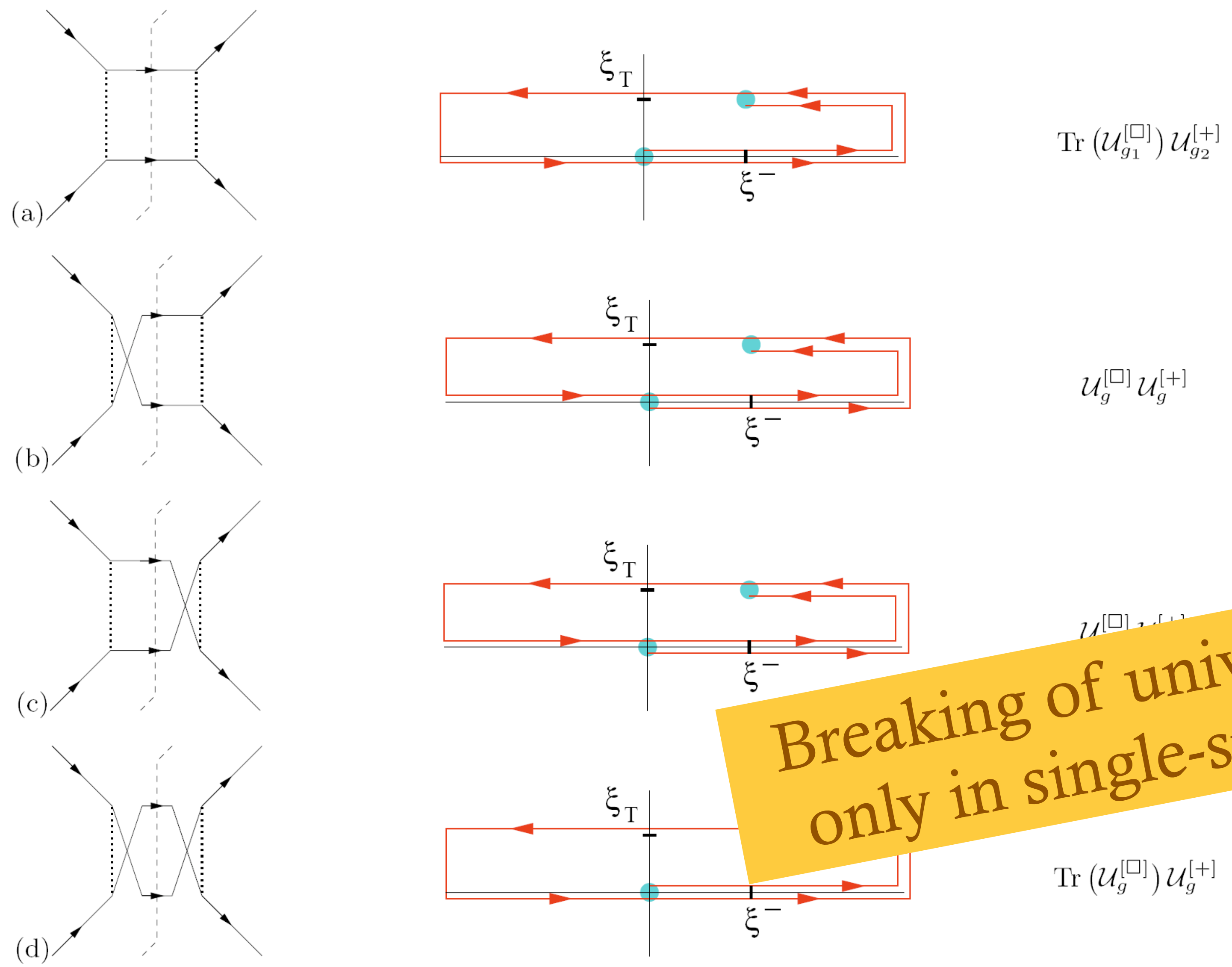


A forest of gauge links



Bomhof, Mulders, Pijlman, PLB 596 (04)
Collins, Qiu, PRD 75 (07)
Vogelsang, Yuan, PRD76 (07)

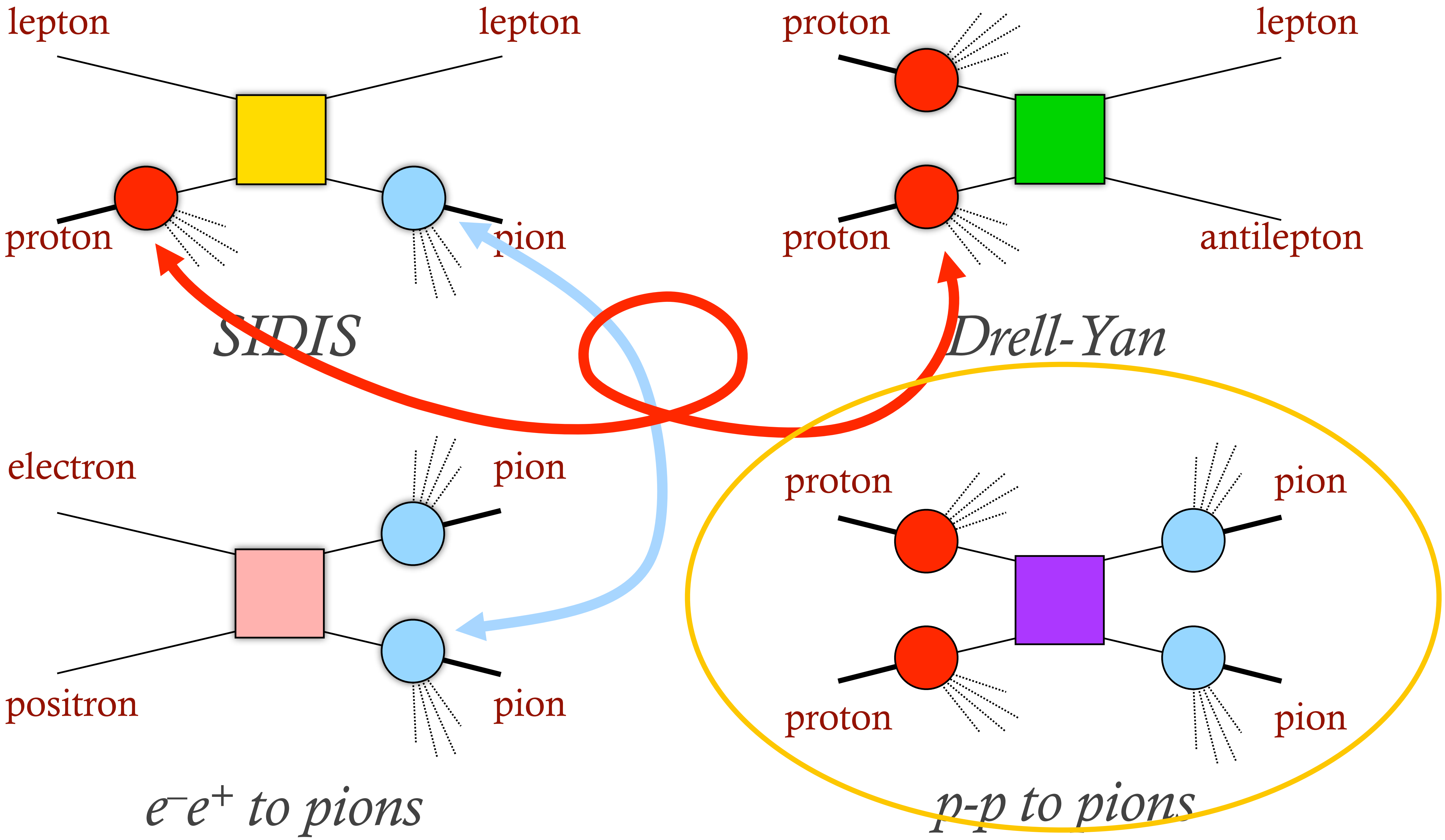
A forest of gauge links



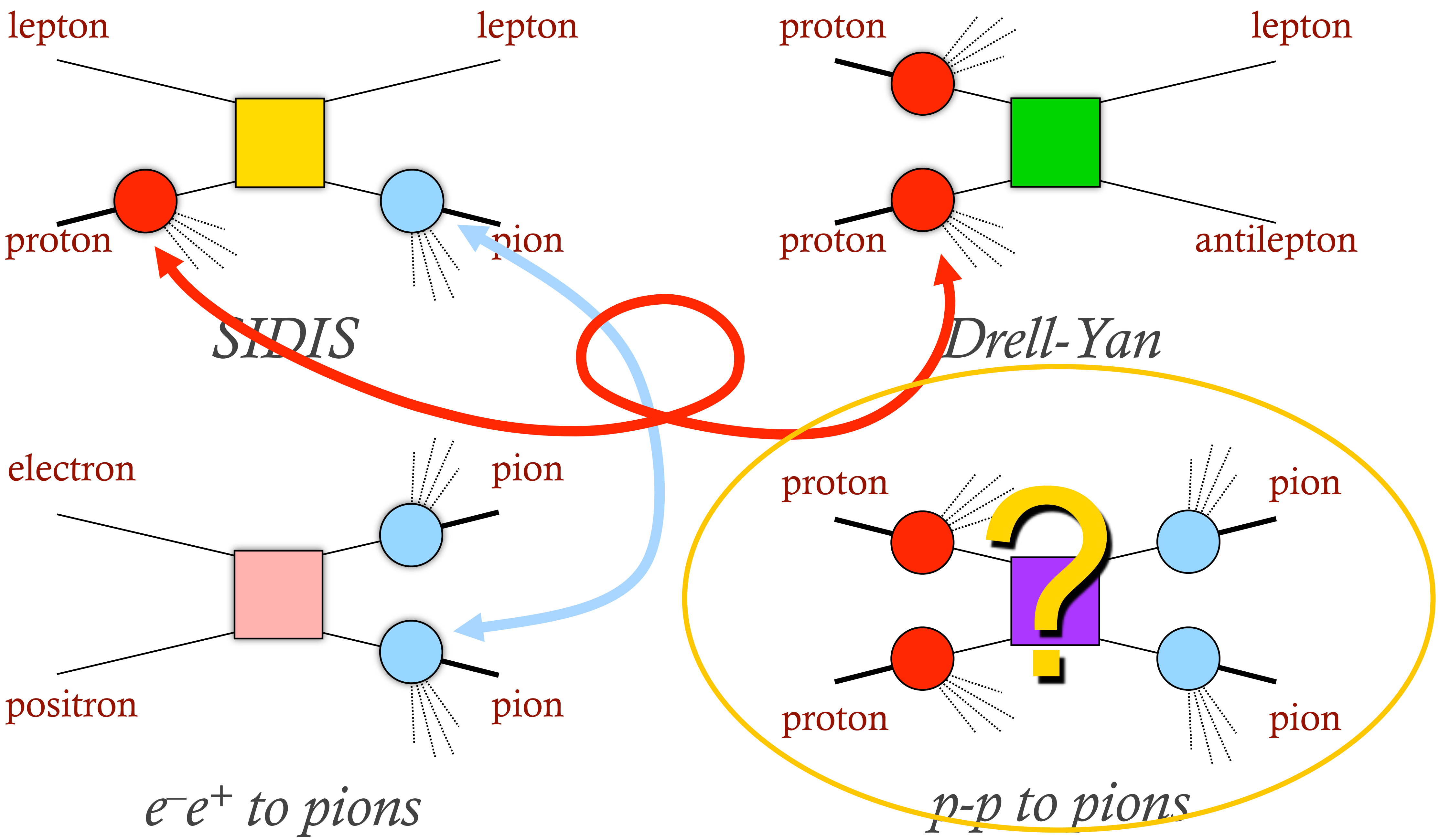
Breaking of universality, and not only in single-spin asymmetries

Bomhof, Mulders, Pijlman, PLB 596 (04)
Collins, Qiu, PRD 75 (07)
Vogelsang, Yuan, PRD76 (07)

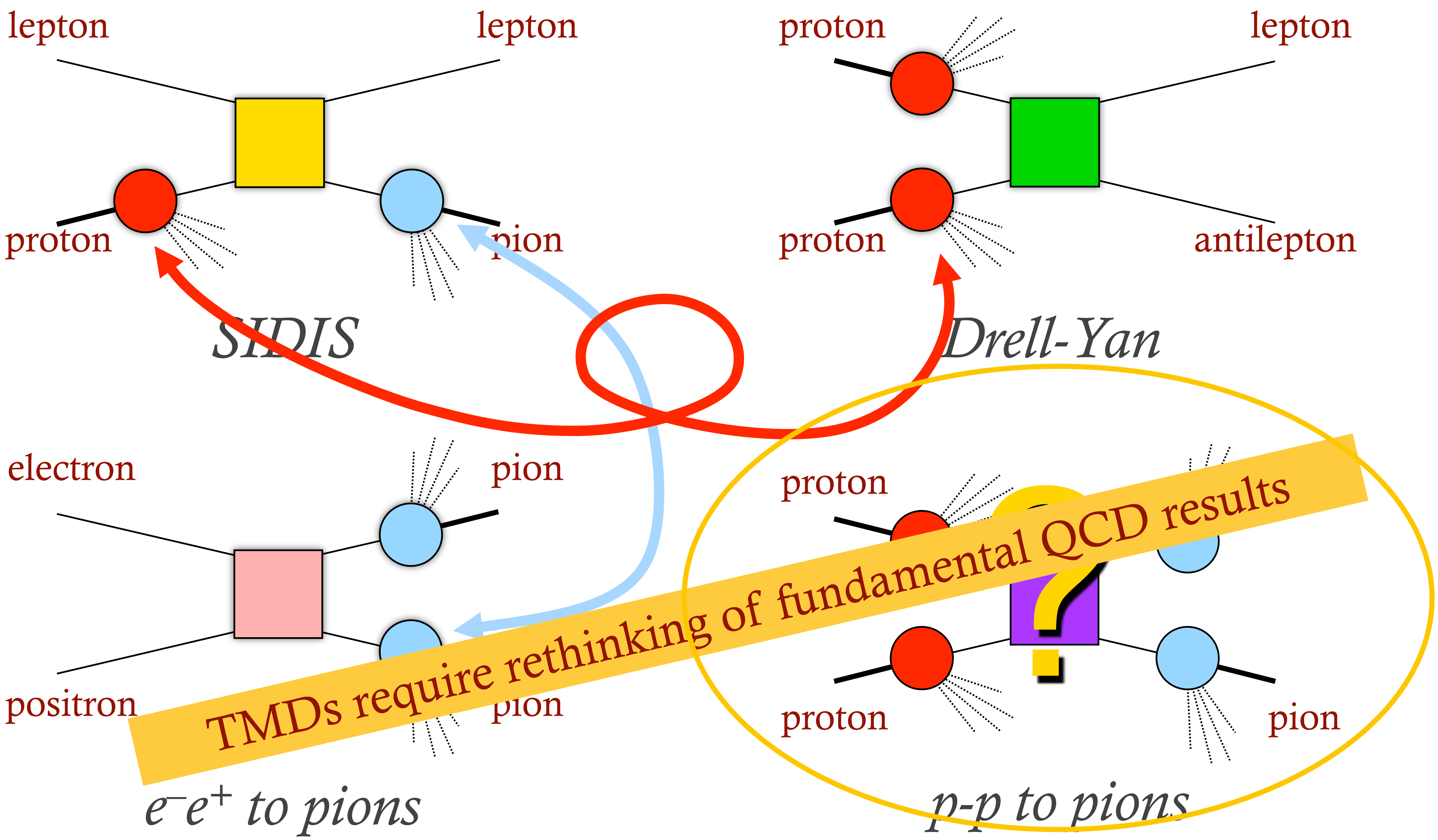
Hadrons to hadrons



Hadrons to hadrons



Hadrons to hadrons



Weighted asymmetries

$$\int \frac{d\sigma_{DIS}}{dq_T} dq_T = H_{DIS} \otimes f$$

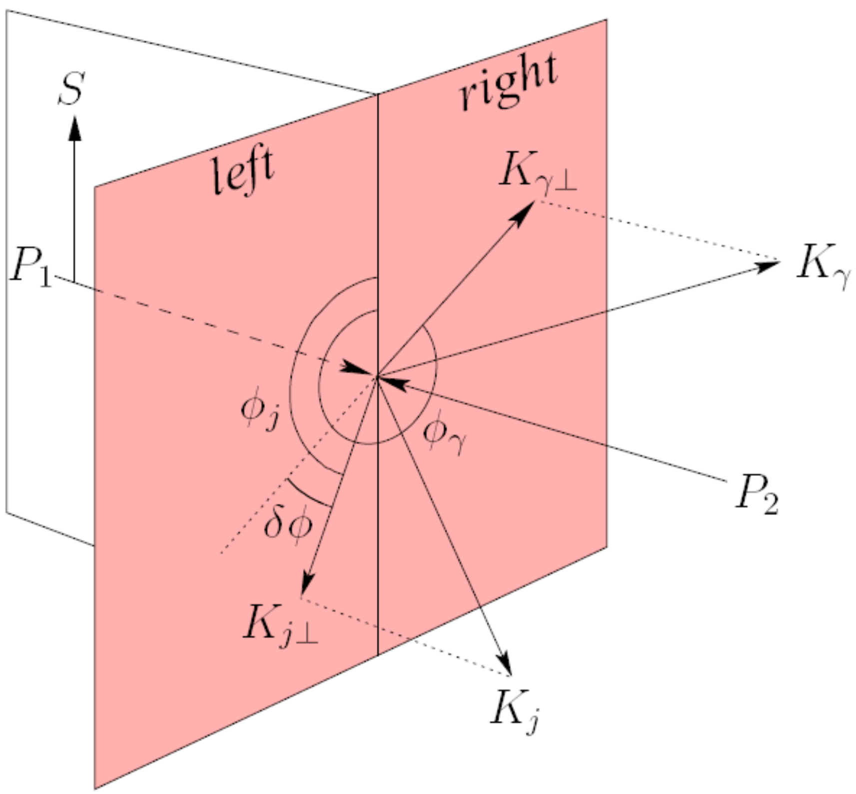
$$\int \frac{d\sigma_{pp}}{dq_T} dq_T = H_{pp} \otimes f$$

$$\int q_T \frac{d\sigma_{DIS}}{dq_T} dq_T = K_{DIS} \otimes g$$

$$\int q_T \frac{d\sigma_{pp}}{dq_T} dq_T = K_{pp} \otimes g' = C K_{pp} \otimes g$$

Weighted asymmetries

A.B., D'Alesio, Bomhof, Mulders, Murgia, PRL99 (07)



$p^\uparrow p$ ® γ jet X at RHIC

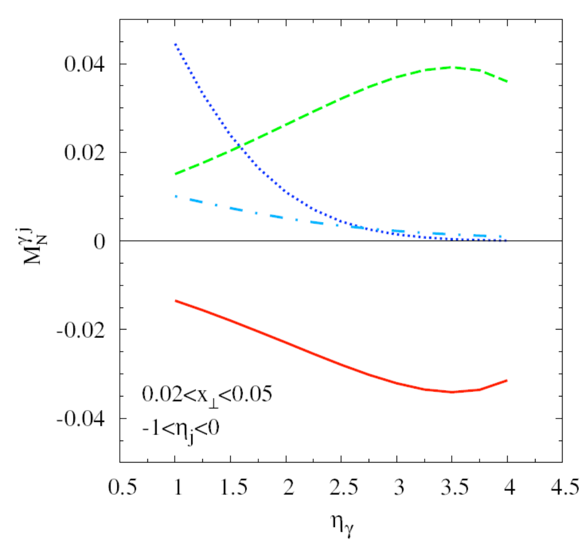
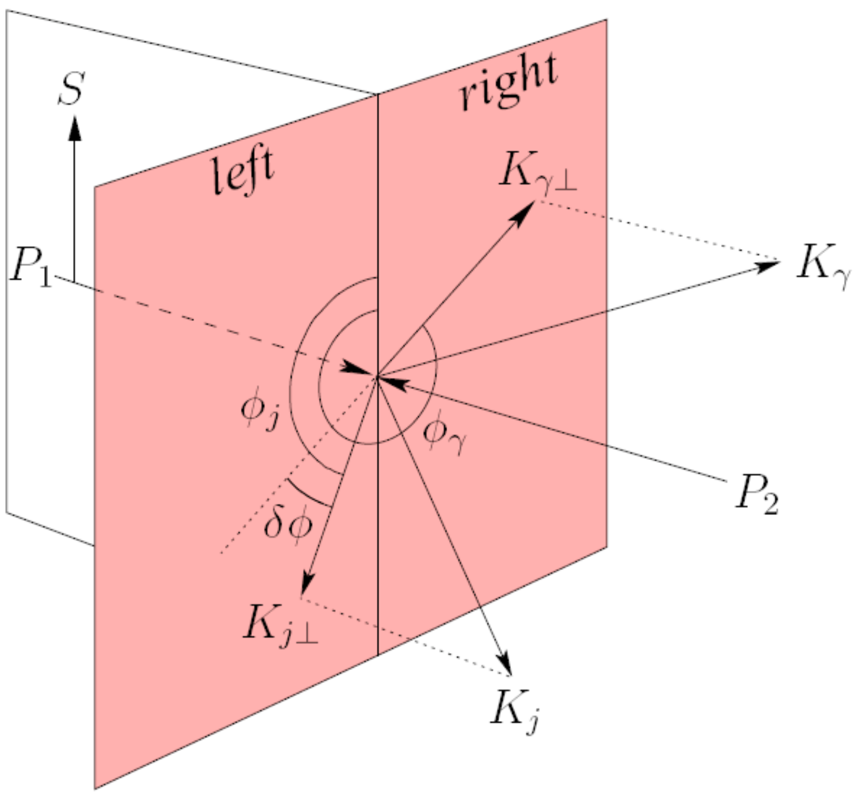


FIG. 5: Prediction for the azimuthal moment $M_N^{\gamma j}$ at $\sqrt{s} =$ 200 GeV, as a function of η_γ, integrated over $-1 \leq \eta_j \leq 0$ and $0.02 \leq x_\perp \leq 0.05$. Solid line: using gluonic-pole cross sections. Dashed line: using standard partonic cross sections. Dotted line: maximum contribution from the gluon Sivers function (absolute value). Dot-dashed line: maximum contribution from the Boer-Mulders function (absolute value).

Weighted asymmetries

A.B., D'Alesio, Bomhof, Mulders, Murgia,PRL99 (07)



"Standard" universality

$p^{\uparrow} p$ ® γ jet X at RHIC

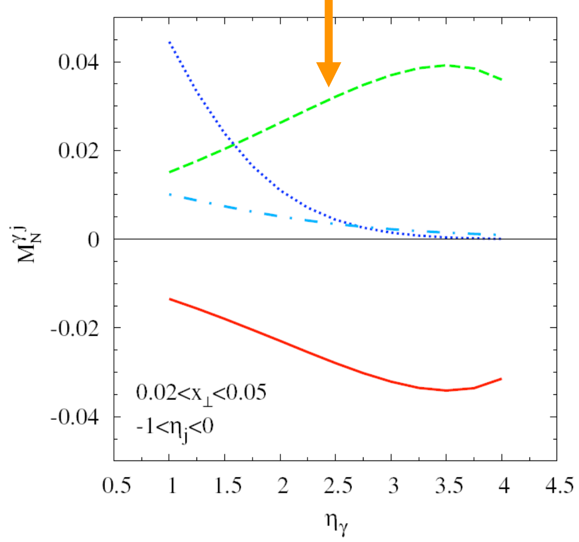
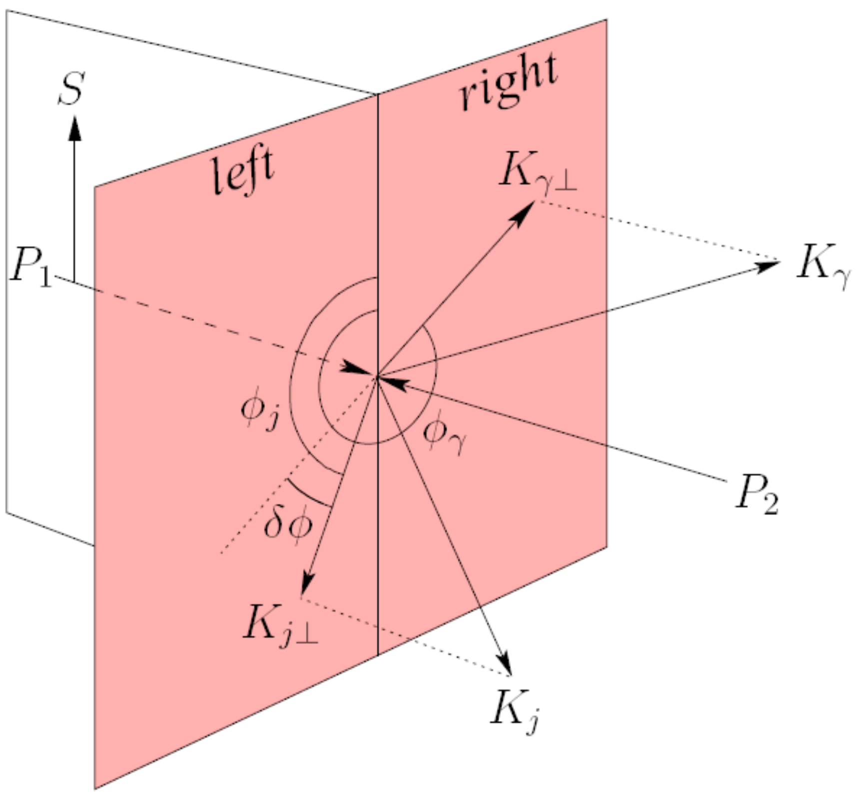


FIG. 5: Prediction for the azimuthal moment $M_N^{\gamma j}$ at $\sqrt{s}$ = 200 GeV, as a function of η_γ, integrated over $-1 \le \eta_j \le 0$ and $0.02 \le x_\perp \le 0.05$. Solid line: using gluonic-pole cross sections. Dashed line: using standard partonic cross sections. Dotted line: maximum contribution from the gluon Sivers function (absolute value). Dot-dashed line: maximum contribution from the Boer-Mulders function (absolute value).

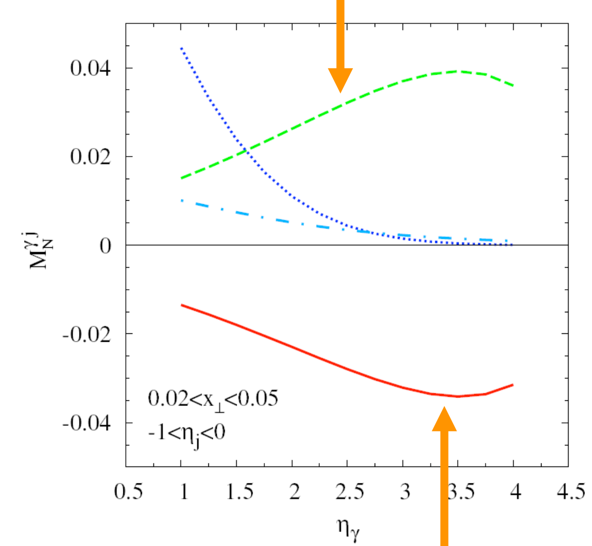
Weighted asymmetries

A.B., D'Alesio, Bomhof, Mulders, Murgia, PRL99 (07)



"Standard" universality

$p^\uparrow p \rightarrow \gamma$ jet X at RHIC



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"Generalized" universality