Part 3: Phenomenolgy

Unpolarized functions

Unpolarized cross section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{h\perp}^2} = \frac{4\pi^2\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)}\left(F_{UU,T}(x,z,P_{h\perp}^2,Q^2) + \varepsilon F_{UU,L}(x,z,P_{h\perp}^2,Q^2)\right),$$

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}, \qquad \gamma = \frac{2Mx}{Q}$$

$$\frac{y^2}{2\,(1-\varepsilon)} = \frac{1}{1+\gamma^2}\left(1-y+\tfrac{1}{2}y^2+\tfrac{1}{4}\gamma^2y^2\right) \approx \left(1-y+\tfrac{1}{2}y^2\right),$$

$$\frac{y^2}{2\,(1-\varepsilon)}\,\varepsilon = \frac{1}{1+\gamma^2}\left(1-y-\tfrac{1}{4}\gamma^2y^2\right) \approx (1-y)$$

Convolution

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big]$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\, e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

$$f \otimes D = x_B \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, f^a(x_B, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \sum_a e_a^2 f_1^a \otimes D_1^a, \qquad F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2}\right)$$

Integrated

$$x_B \int d^2 P_{h\perp} \int d^2 \boldsymbol{p}_T \, d^2 \boldsymbol{k}_T \, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big) \, f_1^a(x_B, p_T^2) \, D_1^a(z, k_T^2)$$

$$= x_B \int d^2 \boldsymbol{p}_T \, f_1^a(x_B, p_T^2) \int z^2 d^2 \boldsymbol{k}_T D_1^a(z, k_T^2)$$

$$= f_1^a(x_B) \, D_1^a(z)$$

$$F_{UU,T} = \sum_a e_a^2 \, f_1^a(x_B) D_1^a(z), \qquad F_{UU,L} = \mathcal{O}(\alpha_s)$$

Fragmentation functions

For the "favored" functions

$$D_1^{u\to\pi^+} = D_1^{\bar{d}\to\pi^+} = D_1^{d\to\pi^-} = D_1^{\bar{u}\to\pi^-}, \equiv D_1^{\mathrm{f}}$$

$$D_1^{u\to K^+} = D_1^{\bar{u}\to K^-}, \equiv D_1^{\mathrm{fd}}$$

$$D_1^{\bar{s}\to K^+} = D_1^{s\to K^-} \equiv D_1^{\mathrm{f}'}$$

for the "unfavored" functions

$$D_1^{\bar{u}\to\pi^+} = D_1^{d\to\pi^+} = D_1^{\bar{d}\to\pi^-} = D_1^{u\to\pi^-} \equiv D_1^{\mathrm{d}},$$

$$D_1^{s\to\pi^+} = D_1^{\bar{s}\to\pi^+} = D_1^{s\to\pi^-} = D_1^{\bar{s}\to\pi^-} \equiv D_1^{\mathrm{df}},$$

$$D_1^{\bar{u}\to K^+} = D_1^{\bar{d}\to K^+} = D_1^{d\to K^+} = D_1^{\bar{d}\to K^-} = D_1^{d\to K^-} = D_1^{u\to K^-} \equiv D_1^{\mathrm{dd}},$$

$$D_1^{s\to K^+} = D_1^{\bar{s}\to K^-} \equiv D_1^{\mathrm{d}'}.$$

Various combinations

$$F_{UU,T}^{p/\pi^+}(x,z,P_{h\perp}^2) = \left(4\,f_1^u + f_1^{\bar{d}}\right) \otimes D_1^{\mathrm{f}} + \left(4\,f_1^{\bar{u}} + f_1^{d}\right) \otimes D_1^{\mathrm{d}} + \left(f_1^s + f_1^{\bar{s}}\right) \otimes D_1^{\mathrm{df}},$$

$$F_{UU,T}^{p/\pi^-}(x,z,P_{h\perp}^2) = \left(4\,f_1^{\bar{u}} + f_1^{d}\right) \otimes D_1^{\mathrm{f}} + \left(4\,f_1^{u} + f_1^{\bar{d}}\right) \otimes D_1^{\mathrm{d}} + \left(f_1^s + f_1^{\bar{s}}\right) \otimes D_1^{\mathrm{df}},$$

$$F_{UU,T}^{n/\pi^+}(x,z,P_{h\perp}^2) = \left(4\,f_1^d + f_1^{\bar{u}}\right) \otimes D_1^{\mathrm{f}} + \left(4\,f_1^{\bar{d}} + f_1^{u}\right) \otimes D_1^{\mathrm{d}} + \left(f_1^s + f_1^{\bar{s}}\right) \otimes D_1^{\mathrm{df}}$$

$$F_{UU,T}^{n/\pi^-}(x,z,P_{h\perp}^2) = \left(4\,f_1^{\bar{d}} + f_1^{u}\right) \otimes D_1^{\mathrm{f}} + \left(4\,f_1^{d} + f_1^{\bar{u}}\right) \otimes D_1^{\mathrm{d}} + \left(f_1^s + f_1^{\bar{s}}\right) \otimes D_1^{\mathrm{df}},$$

$$F_{UU,T}^{p/K^+}(x,z,P_{h\perp}^2) = 4\,f_1^u \otimes D_1^{\mathrm{fd}} + \left(4\,f_1^{\bar{u}} + f_1^d + f_1^{\bar{d}}\right) \otimes D_1^{\mathrm{dd}} + f_1^{\bar{s}} \otimes D_1^{\mathrm{f}'} + f_1^s \otimes D_1^{\mathrm{d}'},$$

$$F_{UU,T}^{p/K^-}(x,z,P_{h\perp}^2) = 4\,f_1^{\bar{u}} \otimes D_1^{\mathrm{fd}} + \left(4\,f_1^{u} + f_1^d + f_1^{\bar{d}}\right) \otimes D_1^{\mathrm{dd}} + f_1^{s} \otimes D_1^{\mathrm{f}'} + f_1^{\bar{s}} \otimes D_1^{\mathrm{d}'},$$

$$F_{UU,T}^{n/K^+}(x,z,P_{h\perp}^2) = 4\,f_1^d \otimes D_1^{\mathrm{fd}} + \left(4\,f_1^{\bar{d}} + f_1^u + f_1^{\bar{u}}\right) \otimes D_1^{\mathrm{dd}} + f_1^{\bar{s}} \otimes D_1^{\mathrm{f}'} + f_1^s \otimes D_1^{\mathrm{d}'},$$

$$F_{UU,T}^{n/K^-}(x,z,P_{h\perp}^2) = 4\,f_1^{\bar{d}} \otimes D_1^{\mathrm{fd}} + \left(4\,f_1^{d} + f_1^u + f_1^{\bar{u}}\right) \otimes D_1^{\mathrm{dd}} + f_1^{s} \otimes D_1^{\mathrm{f}'} + f_1^{\bar{s}} \otimes D_1^{\mathrm{d}'}$$

Valence and pions only

$$F_{UU,T}^{p/\pi^+}(x,z,P_{h\perp}^2) = 4\,f_1^u \otimes D_1^{\mathrm{f}} + f_1^d \otimes D_1^{\mathrm{d}},$$
$$F_{UU,T}^{p/\pi^-}(x,z,P_{h\perp}^2) = f_1^d \otimes D_1^{\mathrm{f}} + 4\,f_1^u \otimes D_1^{\mathrm{d}},$$
$$F_{UU,T}^{n/\pi^+}(x,z,P_{h\perp}^2) = 4\,f_1^d \otimes D_1^{\mathrm{f}} + f_1^u \otimes D_1^{\mathrm{d}},$$
$$F_{UU,T}^{n/\pi^-}(x,z,P_{h\perp}^2) = f_1^u \otimes D_1^{\mathrm{f}} + 4\,f_1^d \otimes D_1^{\mathrm{d}}$$

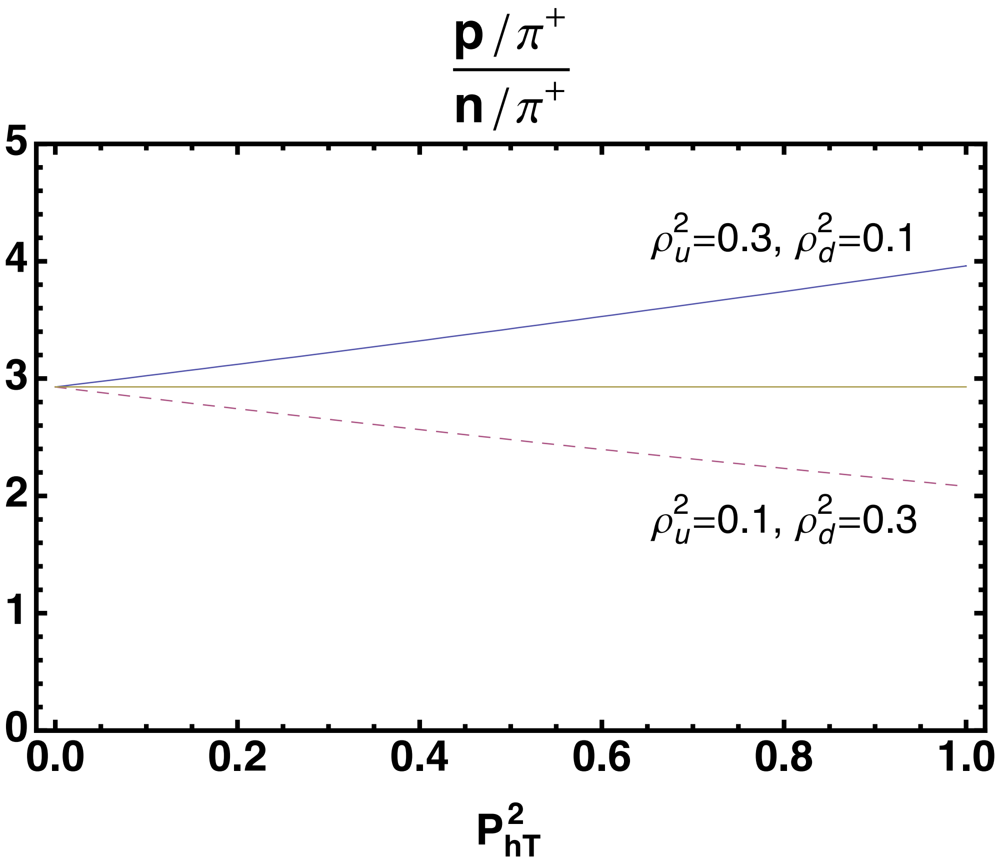
Gaussian ansatz

$$f_1^a(x, p_T^2) = \frac{f_1^a(x)}{\pi\rho_a^2} e^{-\boldsymbol{p}_T^2/\rho_a^2}, \qquad D_1^a(z, k_T^2) = \frac{D_1^a(z)}{\pi\sigma_a^2} e^{-z^2\boldsymbol{k}_T^2/\sigma_a^2}$$

$$f_1^a \otimes D_1^a = \frac{1}{\pi(z^2\rho_a^2 + \sigma_a^2)}\, e^{-\boldsymbol{P}_{h\perp}^2/(z^2\rho_a^2+\sigma_a^2)}$$

Interesting ratio

$$\sigma_{\rm f}^2 = \sigma_{\rm d}^2 = 0.3\ \mathrm{GeV}^2$$
$$f_1^u / f_1^d \approx 0.25$$
$$D_1^{\rm f} / D_1^{\rm f} \approx 0.40$$



Experimental access

Drell-Yan

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 \, f_1^q(x, p_T^2) \otimes f_1^{\bar{q}}(\bar{x}, \bar{p}_T^2)$$

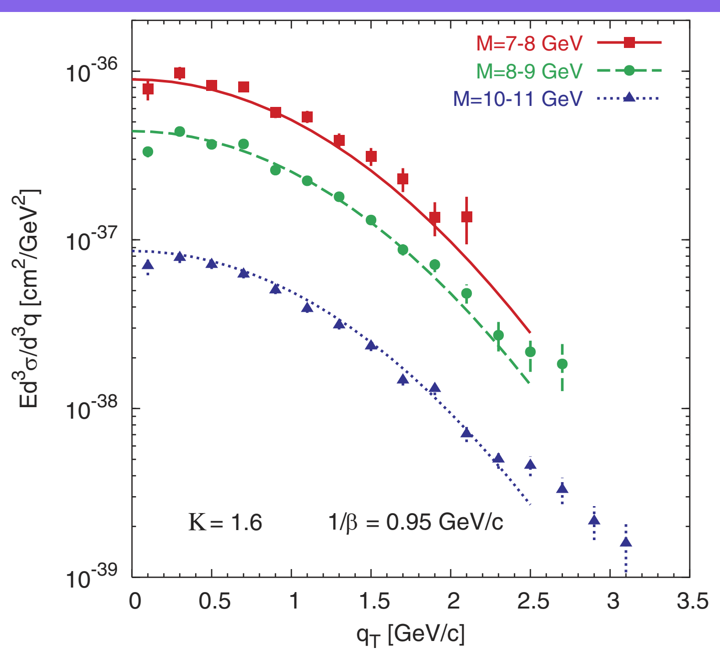
Semi-inclusive DIS

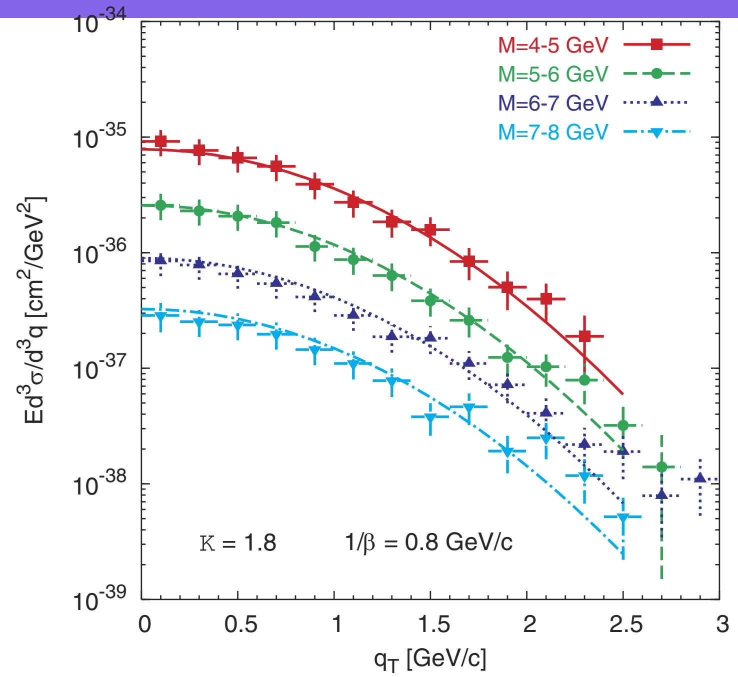
$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 \, f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)$$

electron-positron annihilation

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 \, D_1^q(z, k_T^2) \otimes D_1^{\bar{q}}(\bar{z}, \bar{k}_T^2)$$

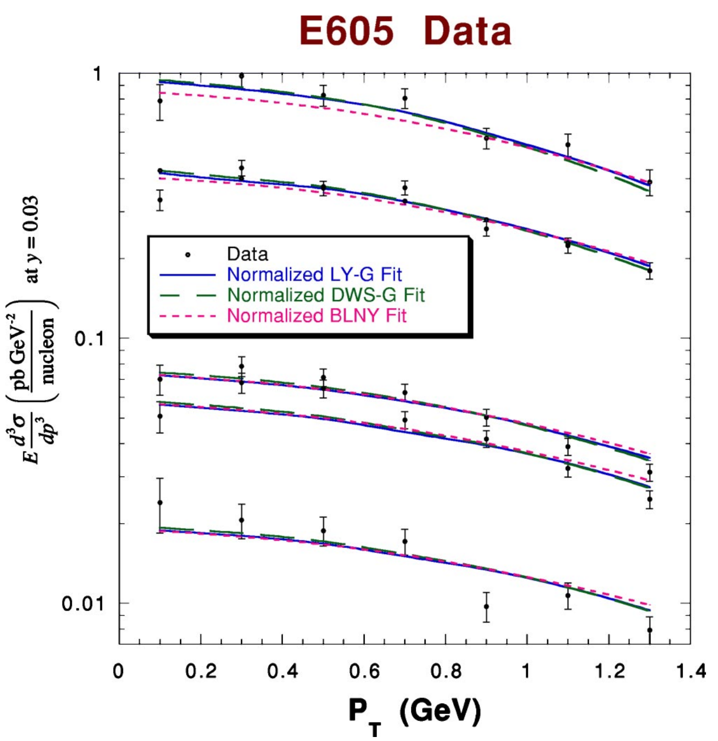
Available studies

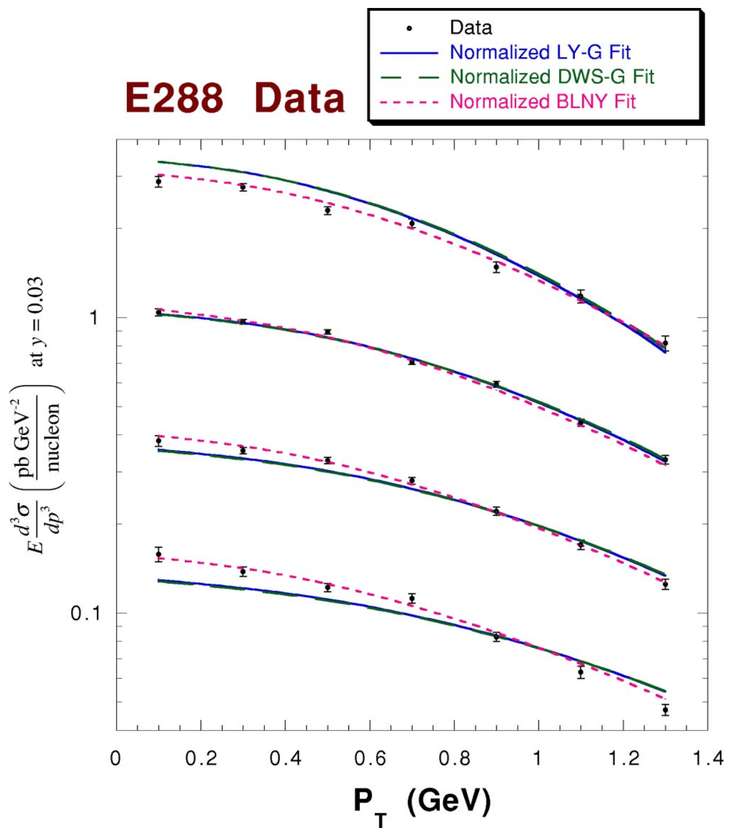




Gaussians

D'Alesio, Murgia, PRD70 (04)





Gaussians
+ kT resummation

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

Nonperturbative part

- In b space

$$\exp\left[-g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 b^2 + g_1 g_3 b^2 \ln(100 x_A x_B)\right]$$

$$g_1 = 0.21 \pm 0.01 \text{ GeV}^2,$$

$$g_2 = 0.68 \pm 0.02 \text{ GeV}^2,$$

$$g_3 = -0.60^{+0.05}_{-0.04} \text{ GeV}^2.$$

$$Q_0 = 1.6 \text{ GeV}$$

111 data points (Drell-Yan)

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

Transversity

Asymmetry

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \sum_a e_a^2\, h_1 \otimes \left(-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} H_1^{\perp}\right)$$

Collins asymmetries

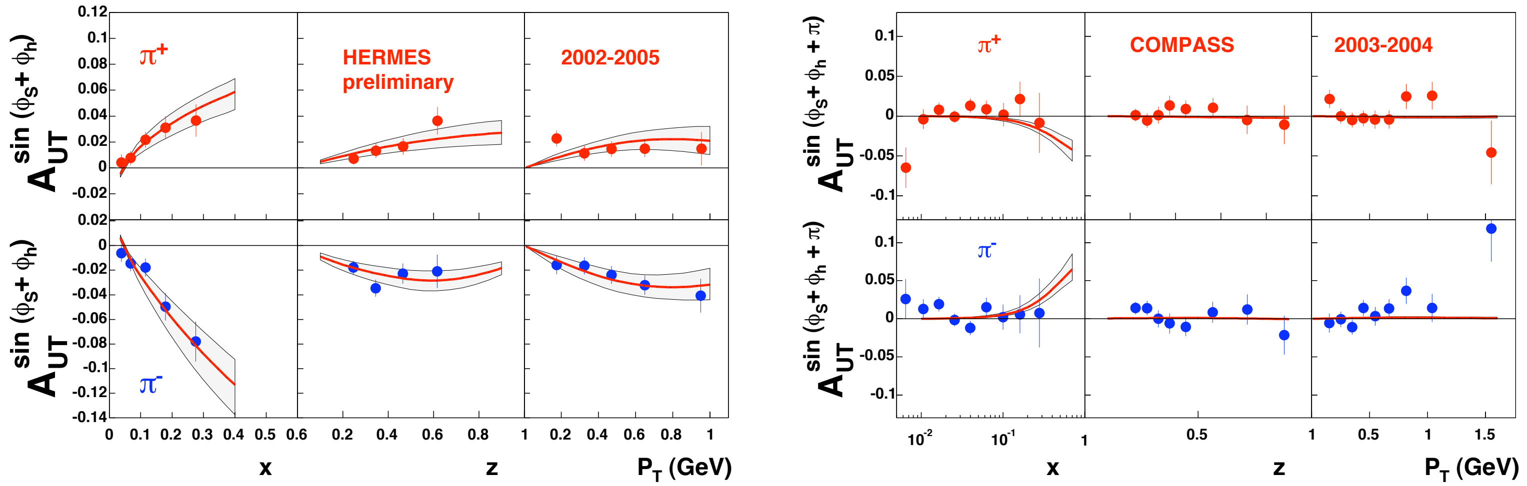


Figure 2: Fits of HERMES [4] and COMPASS [5] data. The shaded area corresponds to the uncertainty in the parameter values, see Ref. [3].

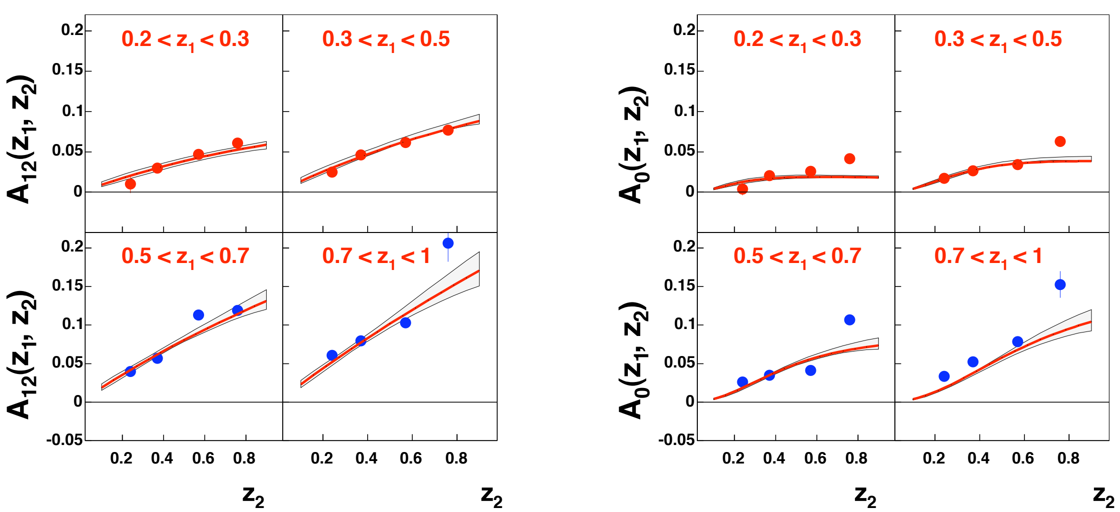


Figure 3: Left panel: fit of the BELLE [6] data on the A_{12} asymmetry ($\cos(\varphi_1+\varphi_2)$ method). Right panel: predictions for the A_0 BELLE asymmetry ($\cos(2\,\varphi_0)$ method).

Anselimino et al., arXiv:0807.0173

Transversity and Collins

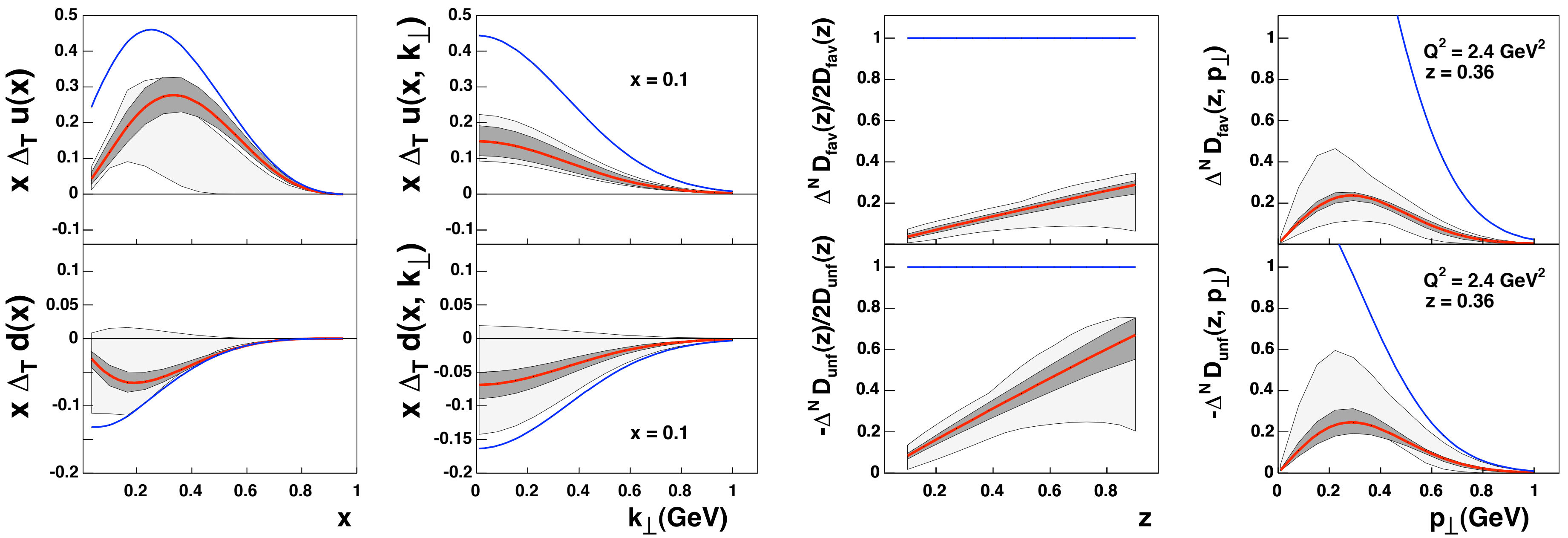
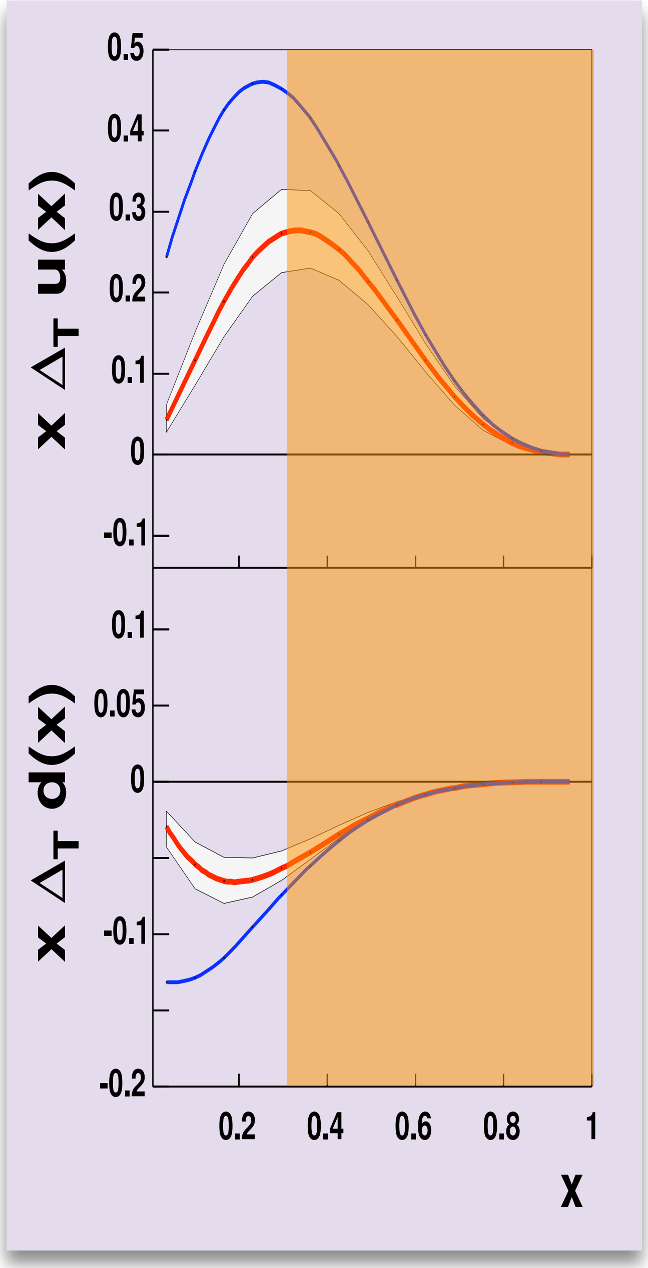


Figure 1: Left panel: the transversity distribution functions for u and d flavours as determined by our global fit; we also show the Soffer bound (highest or lowest lines) and the (wider) bands of our previous extraction [3]. Right panel: favoured and unfavoured Collins fragmentation functions as determined by our global fit; we also show the positivity bound and the (wider) bands as obtained in Ref. [3].

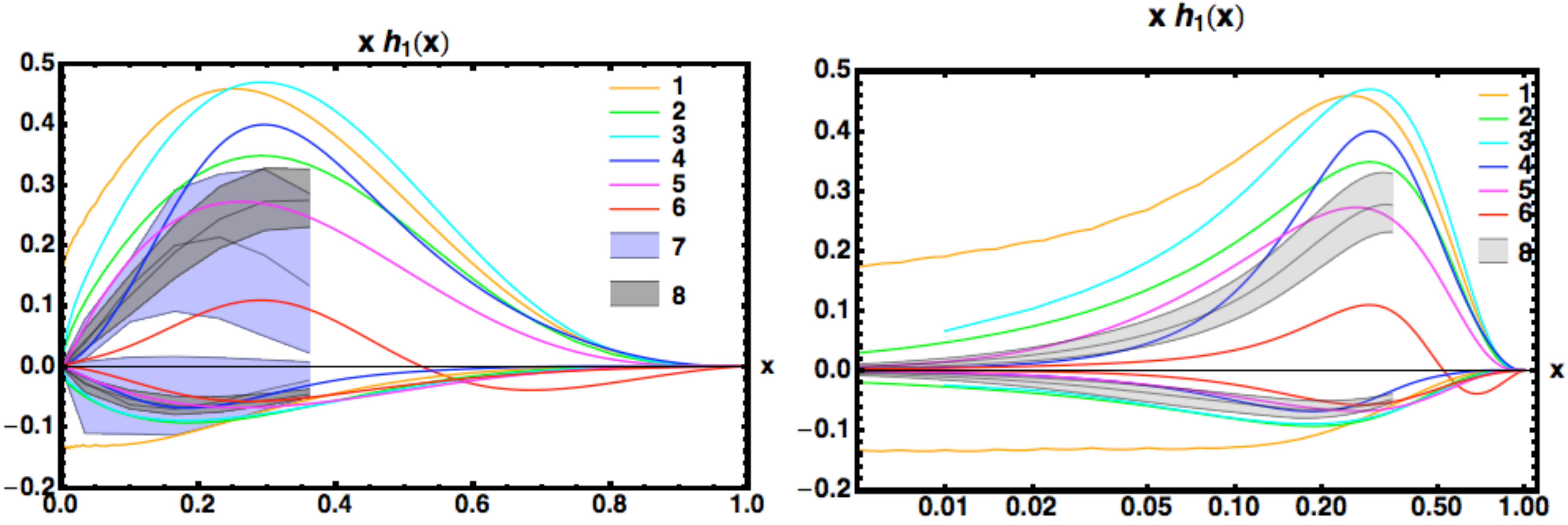
Transversity



- Data from HERMES, COMPASS, BELLE
- 96 data points (some correlations -- cf. 467 points for Δq fits)
- no sys errors taken into account
- $\chi^2 \approx 1.4$
- Statistical uncertainty only ($\Delta\chi^2 \approx 17$)

A. Prokudin, talk at DIS08 (extraction by Anselmino et al.)

Comparison with models



[1] *Soffer et al. PRD 65 (02)*

[2] *Korotkov et al. EPJC 18 (01)*

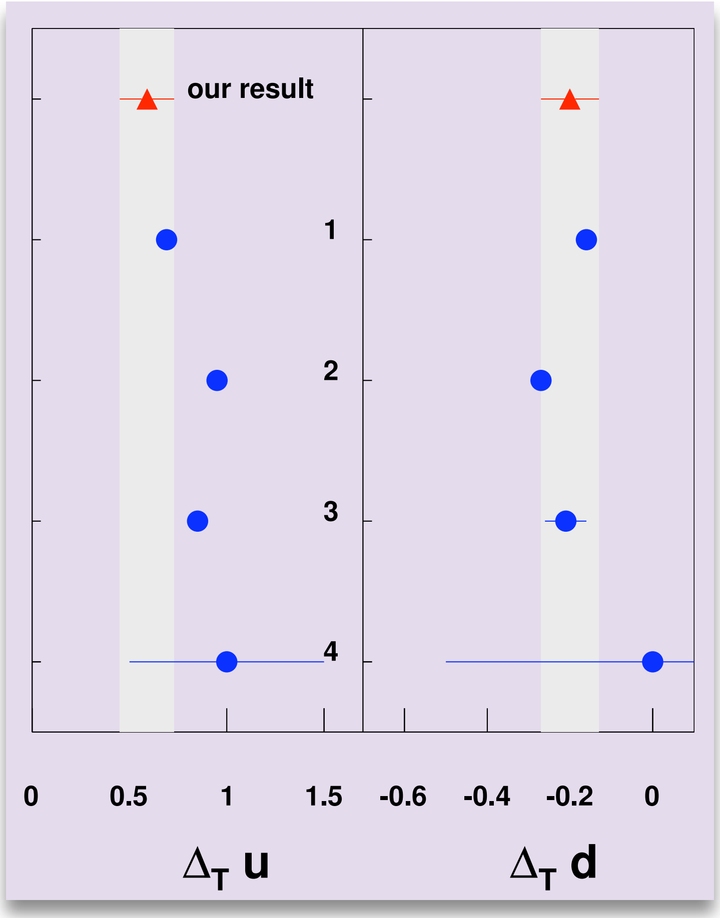
[3] *Schweitzer et al., PRD 64 (01)*

[4] *Wakamatsu, PLB 509 (01)*

[5] *Pasquini et al., PRD 72 (05)*

[6] *Bacchetta, Conti, Radici, PRD 78 (08)*

Tensor charge



[our result] Anselmino et al. DIS 08

[1] Diquark spectator model,
Cloet, Bentz, Thomas, PLB 659 (08)

[2] Chiral quark soliton model,
Wakamatsu, PLB 653 (07)

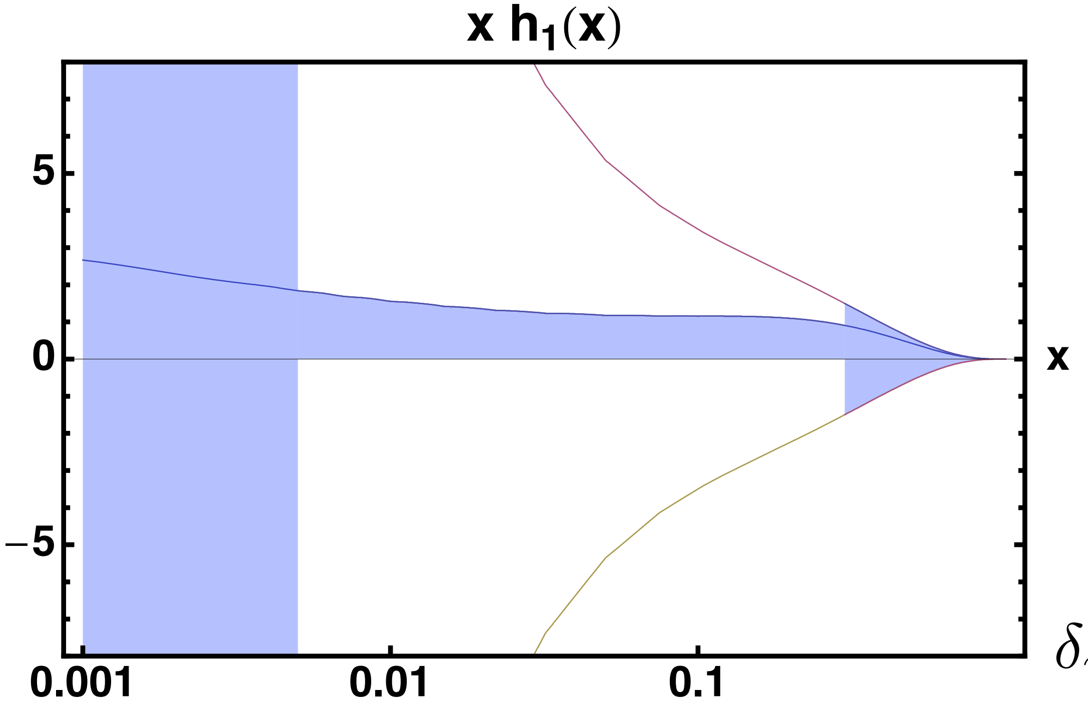
[3] Lattice QCD,
Goekeler et al. PLB 627 (05)

[4] QCD sum rules,
He, Ji, PRD 52 (95)

The first x-moments of the transversity distribution – related to the tensor charge, and defined as $\Delta_T q \equiv \int_0^1 dx \Delta_T q(x)$ – are found to be $\Delta_T u = 0.59^{+0.14}_{-0.13}$, $\Delta_T d = -0.20^{+0.05}_{-0.07}$ at $Q^2 = 0.8$ GeV^2.

Anselimino et al., arXiv:0807.0173

Tensor charge: extremes



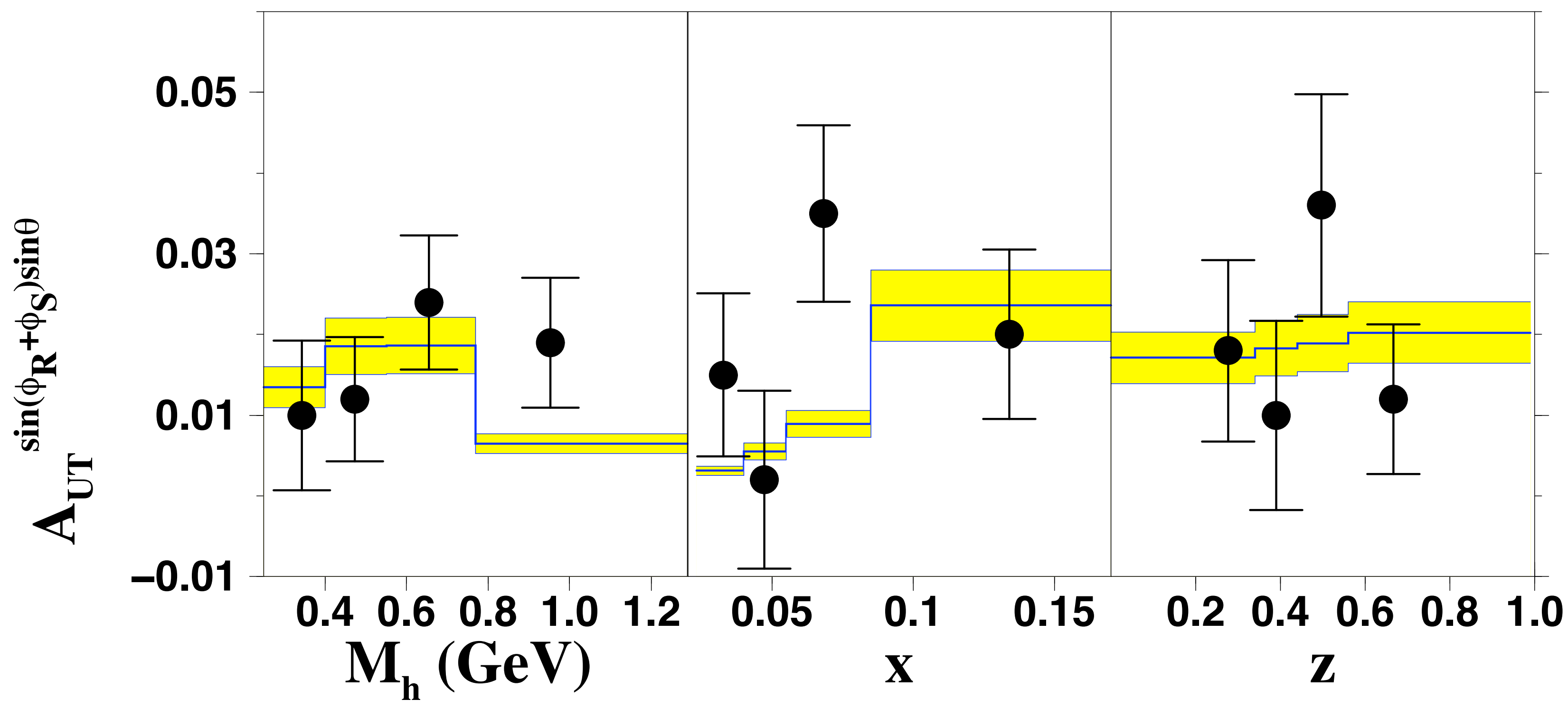
$$\delta_u\Big|_{0.005}^{1} = 0.33^{+0.11}_{-0.08} \pm 0.15$$

$$\delta_u\Big|_{0.005}^{0.3} = 0.33^{+0.11}_{-0.08}$$

$$\delta_u\Big|_{0.001}^{1} = 0.33^{+0.11}_{-0.08}(\text{stat}) \pm 0.15(\text{sys, high } x) \pm 0.14(\text{sys, low } x)$$

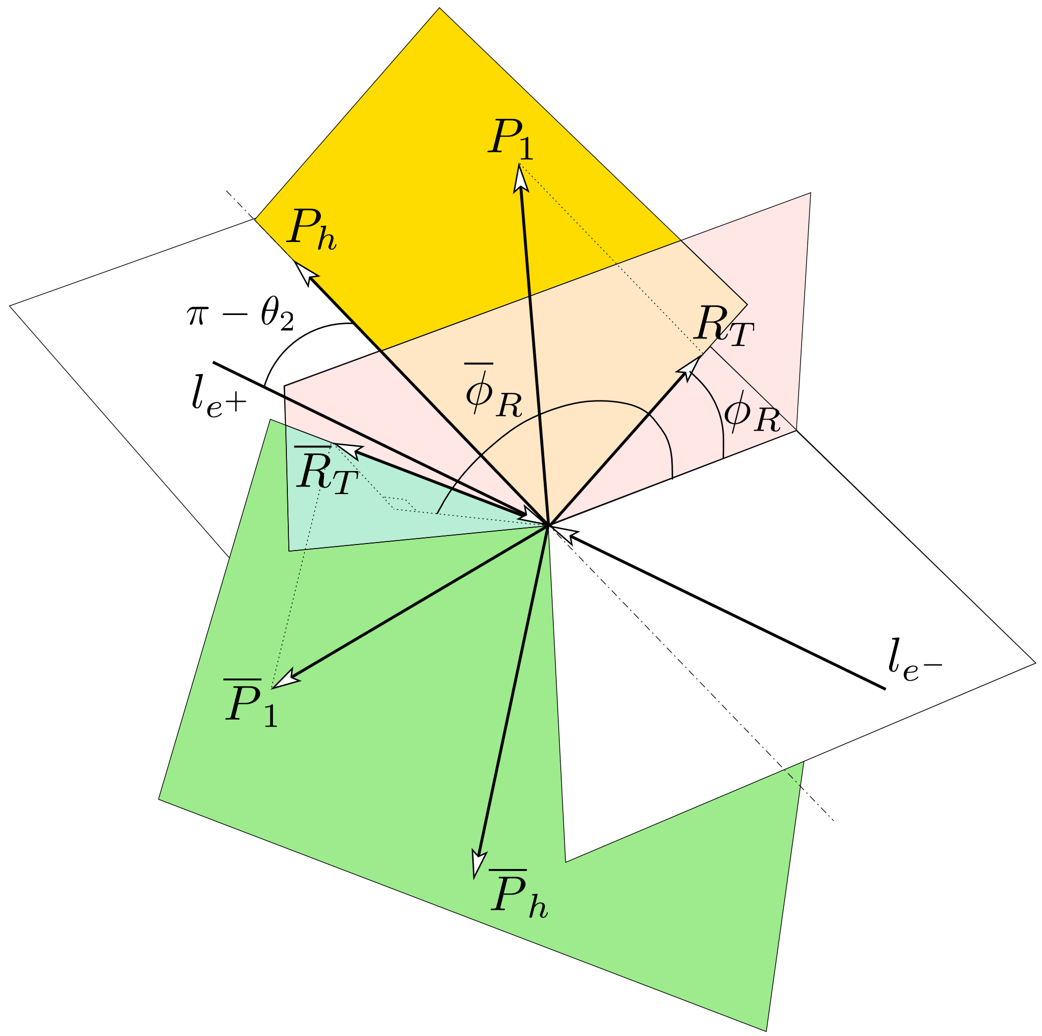
$$\delta_d\Big|_{0.001}^{1} = -0.14^{+0.04}_{-0.06}(\text{stat}) \pm 0.02(\text{sys, high } x) \pm 0.12(\text{sys, low } x)$$

Dihadron functions: DIS

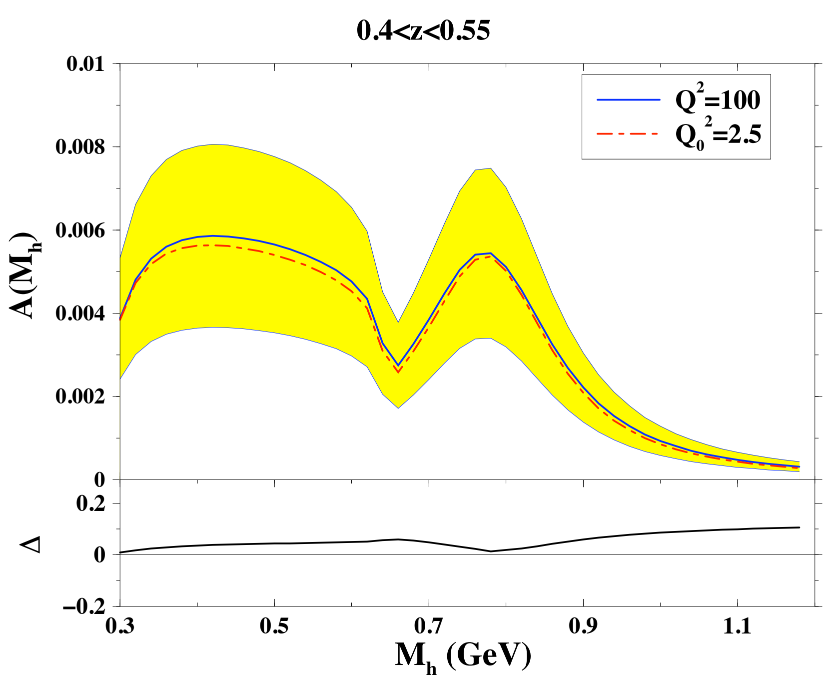


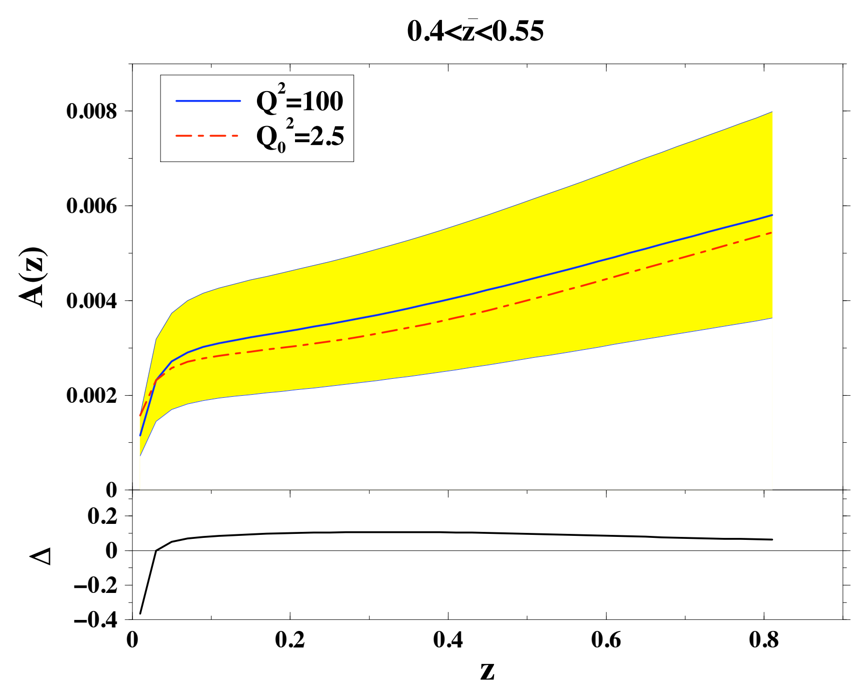
$$A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta} = -\frac{(1-y)}{(1-y+\frac{y^2}{2})}\,\frac{1}{2}\sqrt{1-4\frac{M_\pi^2}{M_{\pi\pi}^2}}\;\frac{\sum_q e_q^2\,h_1^q(x)\,H_{1,q}^{\sphericalangle,sp}(z,M_{\pi\pi})}{\sum_q e_q^2\,f_1^q(x)\,D_{1,q}(z,M_{\pi\pi})}$$

Dihadron functions: e^+e^-



$$A(\cos\theta_2, z, M_h^2, \overline{z}, \overline{M}_h^2) = \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,\frac{\pi^2}{32}\,\frac{|\boldsymbol{R}|\,|\overline{\boldsymbol{R}}|}{M_h\,\overline{M}_h}\,\frac{\sum_q e_q^2\, H_{1,q}^{\sphericalangle sp}(z, M_h^2)\,\overline{H}_{1,q}^{\sphericalangle sp}(\overline{z}, \overline{M_h}^2)}{\sum_q e_q^2\, D_{1,q}(z, M_h^2)\,\overline{D}_{1,q}(\overline{z}, \overline{M_h}^2)}$$


0.4<z<0.55
Q^2=100
Q_0^2=2.5
A(M_h)
Δ
M_h (GeV)

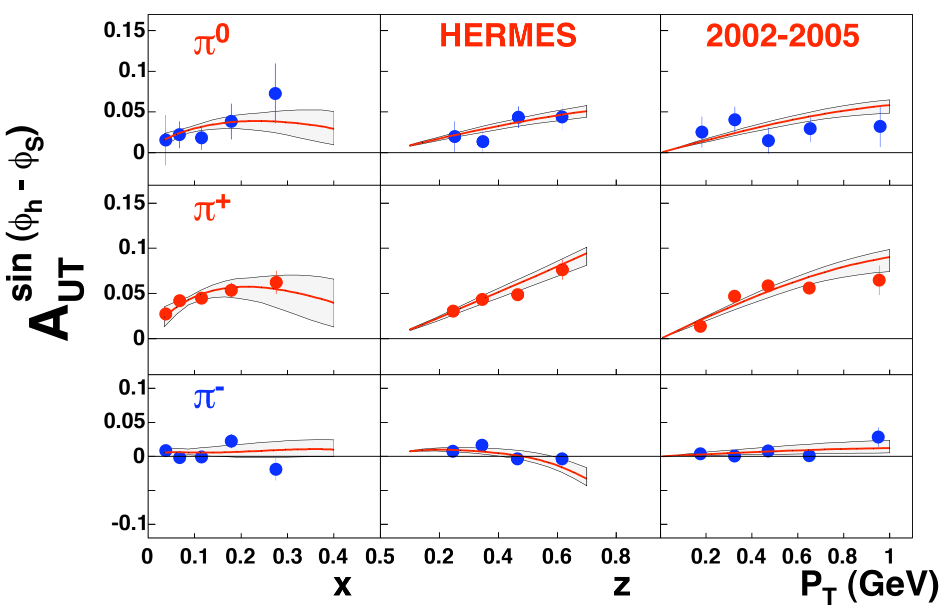

0.4<z̄<0.55
Q^2=100
Q_0^2=2.5
A(z)
Δ
z

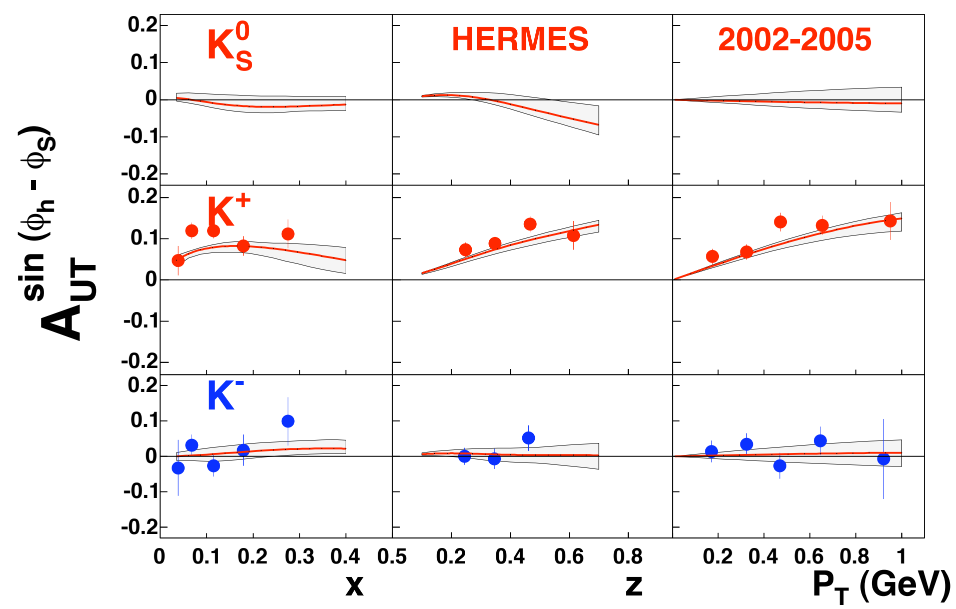
Overview of observables

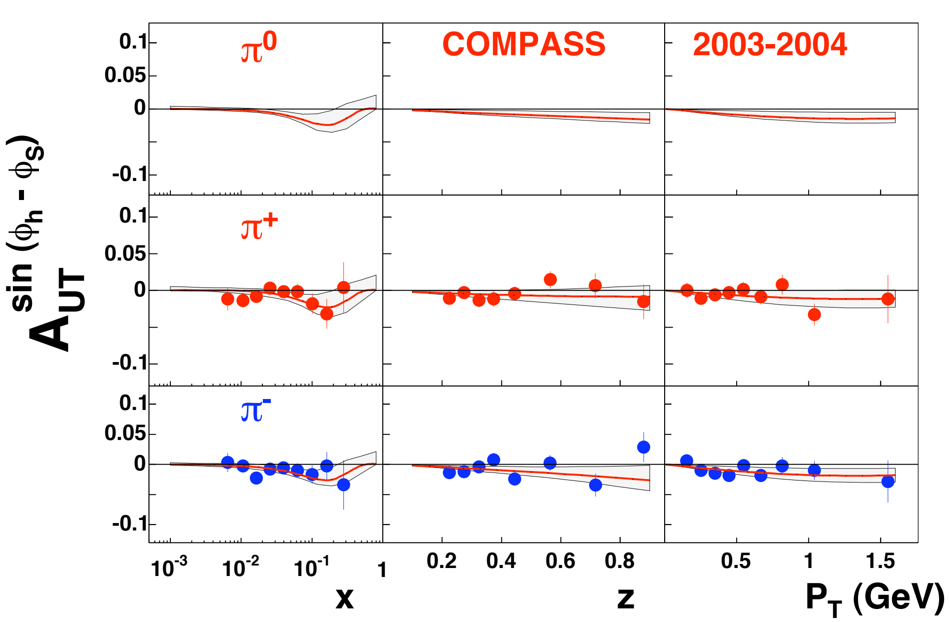
	Process	Experiment	Observable	Grade
	$l\, p^\uparrow \to l\, \pi\, X$	Hermes, Compass, JLab, EIC	$h_1 \otimes H_1^\perp$	★★★★
			$h_1\, \tilde{H}$	★★
	$l\, p^\uparrow \to l\, (\pi\, \pi) X$	Hermes, Compass, JLab@12GeV, EIC	$h_1\, H_1^{\sphericalangle}$	★★★
	$l\, p^\uparrow \to l\, \Lambda\, X$	Hermes, Compass, JLab@12GeV, EIC	$h_1\, H_1$	★
Doubly polarized	$p^\uparrow (p/\bar{p})^\uparrow \to l\, \bar{l}\, X$	Rhic	$h_1\, \bar{h}_1$	★
		JParc		★★★
		Pax		★★★★★
	$p^\uparrow (p/\bar{p})^\uparrow \to \pi X$	Rhic	$h_1\, h_1\, D_1$	★★
		JParc		★★★
		Pax		★★★★
Singly polarized	$(p/\bar{p}/\pi) p^\uparrow \to (\pi\pi) X$	Rhic, JParc Compass, Panda	$f_1\, h_1\, H_1^{\sphericalangle}$	★★★
	$(p/\bar{p}/\pi) p^\uparrow \to \Lambda\, X$	Rhic, JParc Compass, Panda	$f_1\, h_1\, H_1$	★
	$(\pi/\bar{p}) p^\uparrow \to l\, \bar{l}\, X$	Compass, JParc, Panda	$h_1^\perp \otimes h_1$	★★
	$(p/\bar{p}/\pi) p^\uparrow \to j(j/\gamma) X$	Rhic, Compass, JParc, Panda	$h_1^\perp \otimes h_1$	★★
	$\to \pi(j/\gamma) X$		$f_1 \otimes h_1 \otimes H_1^\perp$	★
	$\to (\pi/j/\gamma) X$		$h_1^\perp \otimes h_1 \otimes D_1$	★

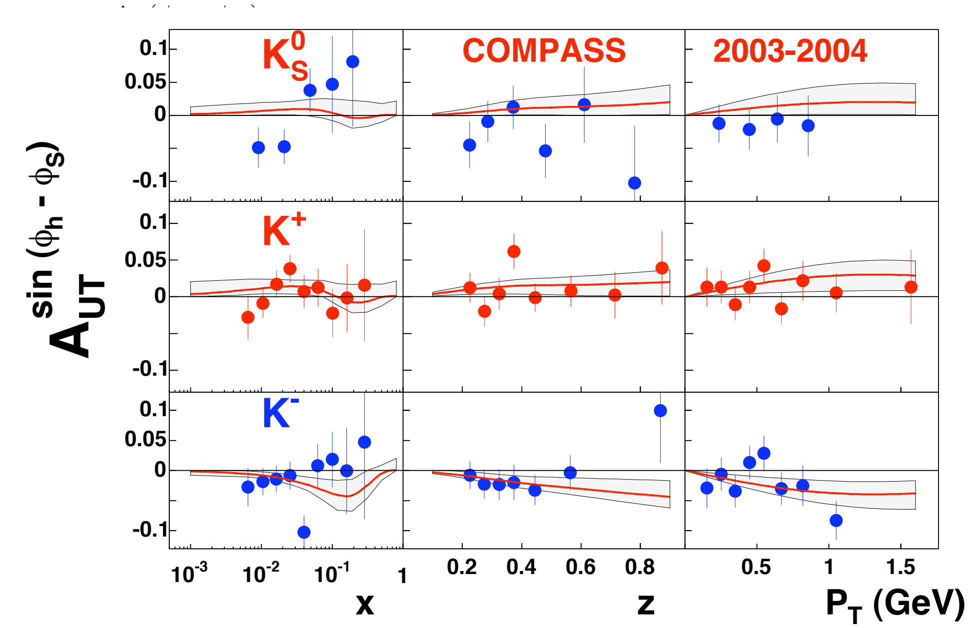
Sivers

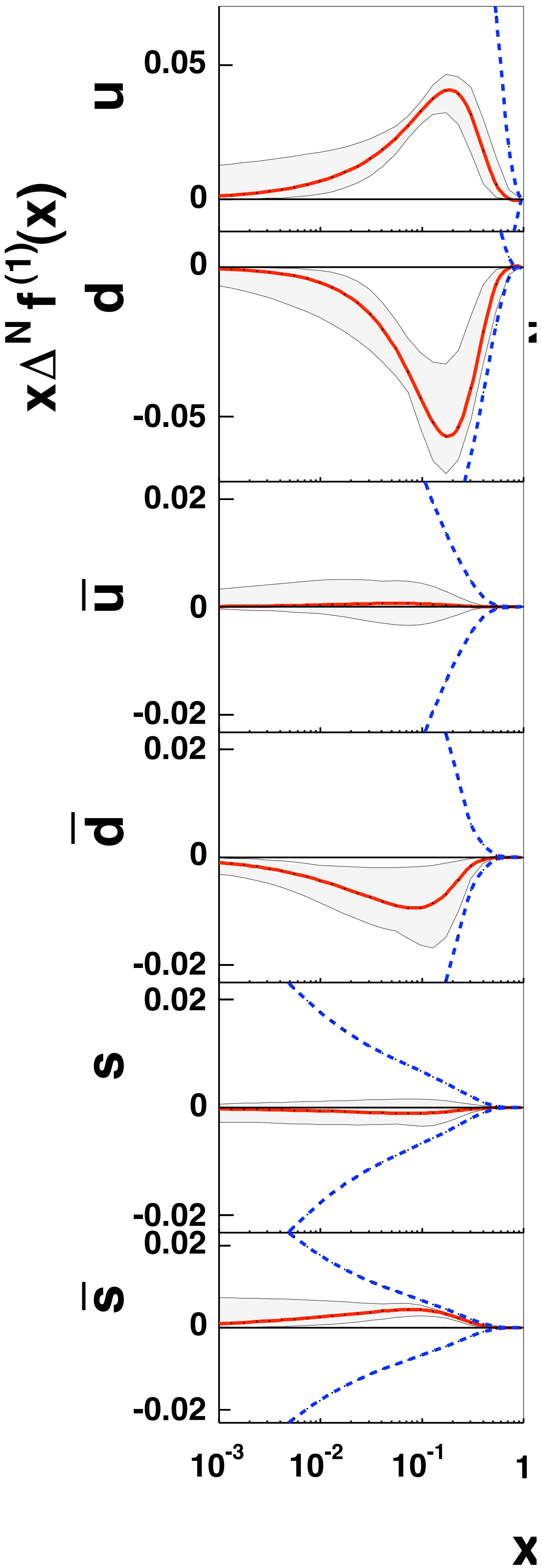
Data


π^0
HERMES
2002-2005
π^+
π^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)


K_S^0
HERMES
2002-2005
K^+
K^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)


π^0
COMPASS
2003-2004
π^+
π^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)


K_S^0
COMPASS
2003-2004
K^+
K^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)

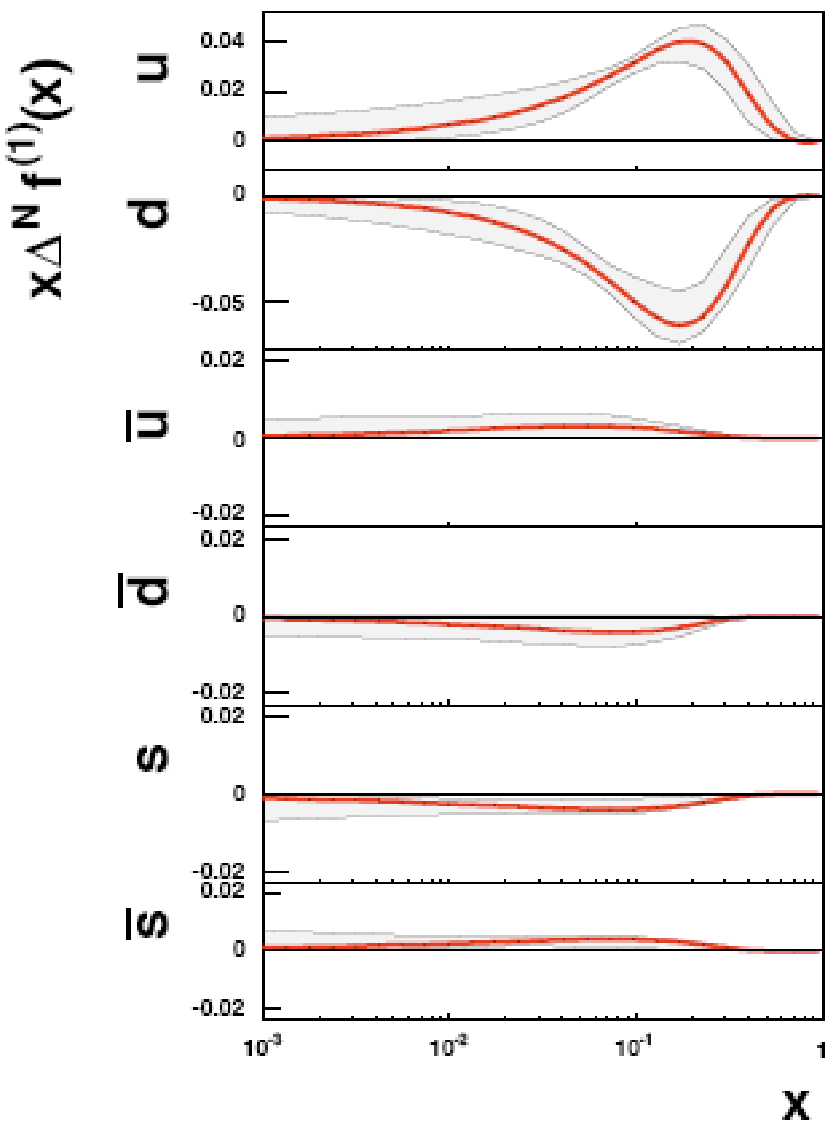


Sivers functions

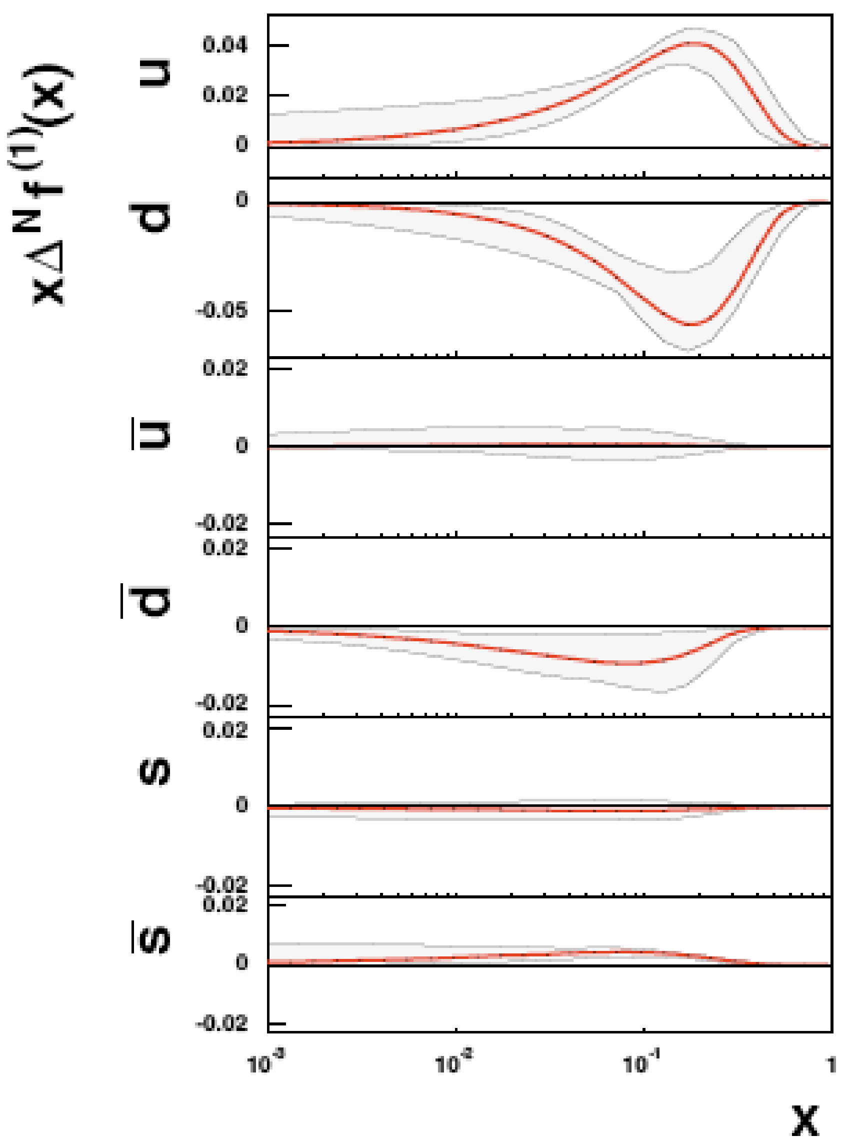
- Data from HERMES, COMPASS
- 96 data points (some correlations -- cf. 467 points for Δq fits)
- no sys errors
- $\chi^2 \approx 1.0$
- Statistical uncertainty only ($\Delta\chi^2 \approx 17$)

Sivers function - Torino

“Symmetric sea”



Free fit



Anselmino et al., 0805.2677

Sivers function: Bochum

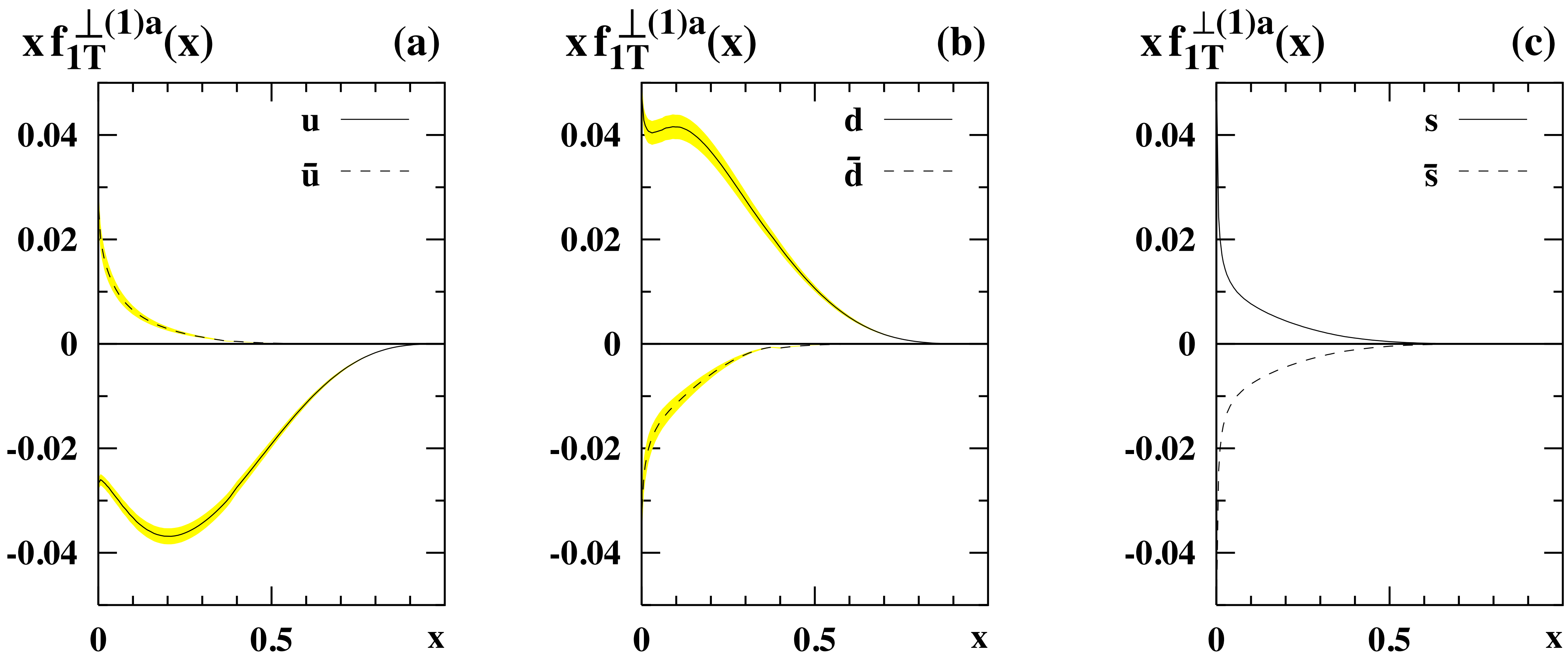


FIGURE 7. The $x f_{1T}^{\perp(1)a}(x)$ vs. x as extracted from preliminary HERMES and COMPASS data [10, 11]. (a) The flavours u and $\bar{u}$. (b) The flavours d and $\bar{d}$. (c) The flavours s and $\bar{s}$ that were fixed to $\pm$ positivity bounds (17) for reasons explained in Sec. 7, see also Eqs. (18, 19). The shaded areas in (a) and (b) show the respective 1-σ-uncertainties.

Arnold et al. , 0805.2137

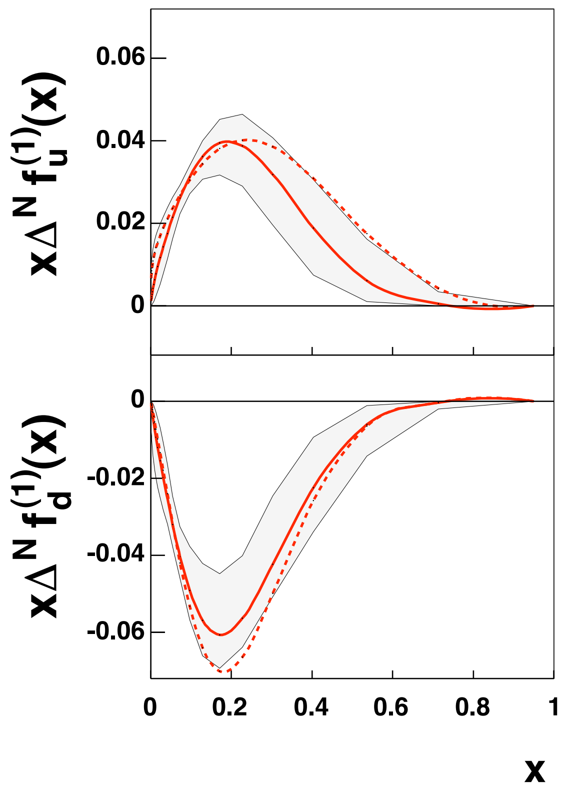
Model statement

$$(1-x) f_{1T}^{\perp q}(x) = -\frac{3}{2} M C_F \alpha_S \, E^q(x,0,0)$$

$$\int_0^1 dx (1-x) f_{1T}^{\perp q}(x) = -\frac{3}{2} M C_F \alpha_S \, \kappa^q$$

Burkardt, Hwang, PRD69 (04)
Lu, Schmidt, PRD75 (07)
A.B., F. Conti, M. Radici, arXiv:0807.0323

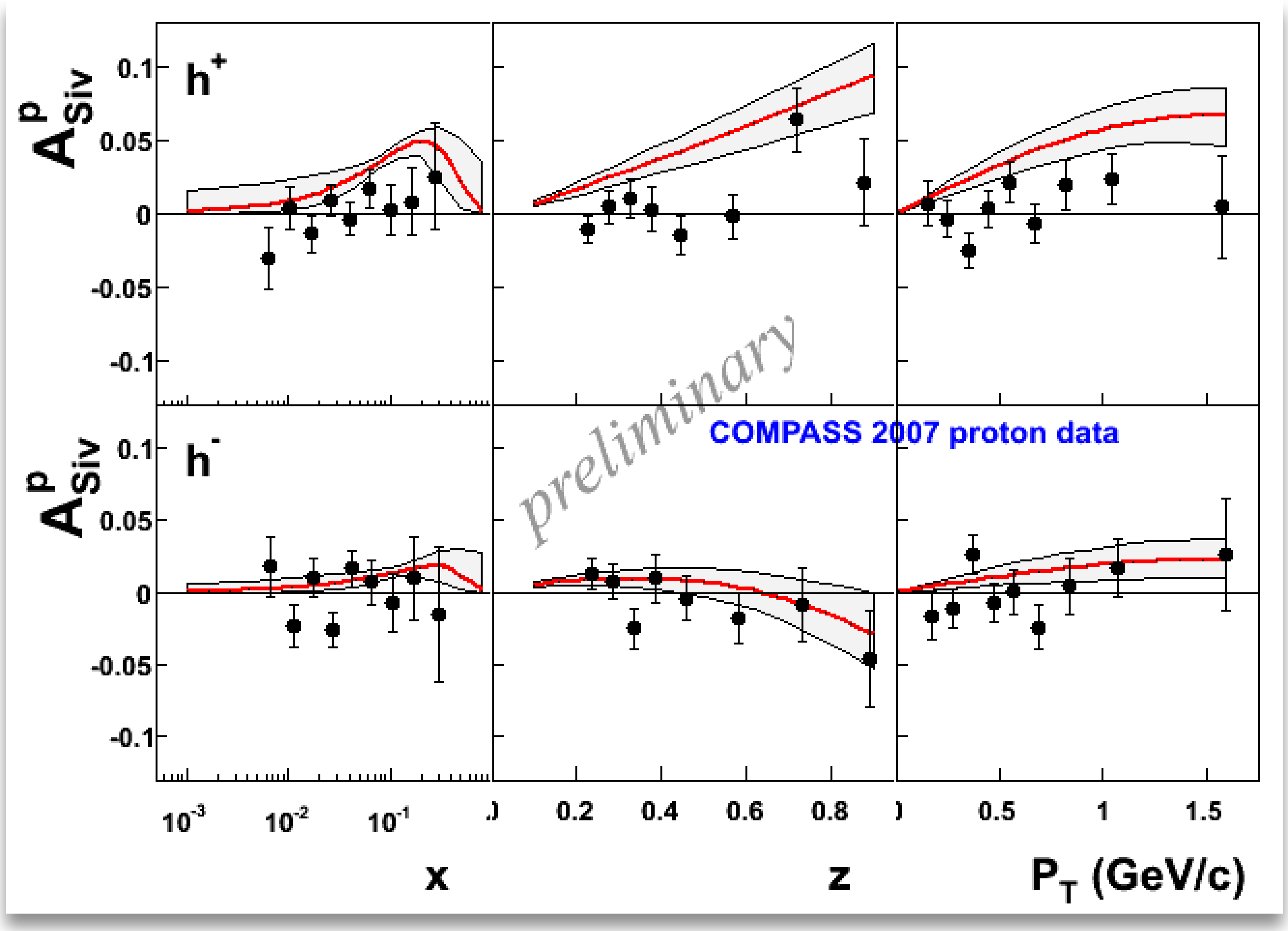
$$k^u = 1.67$$
$$k^d = -2.03$$



Anselmino et al., 0805.2677,
Arnold et al. , 0805.2137

The relation is not general

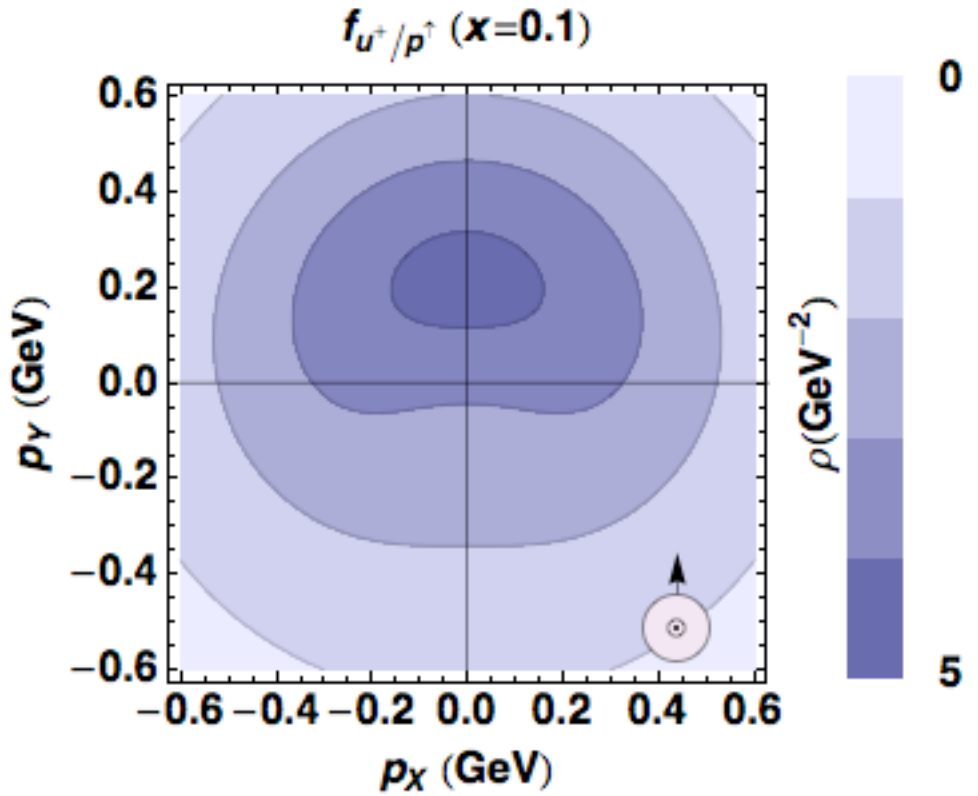
Sivers: COMPASS proton



data: S. Levorato, Transversity 08
prediction: Anselmino et al., 0805.2677

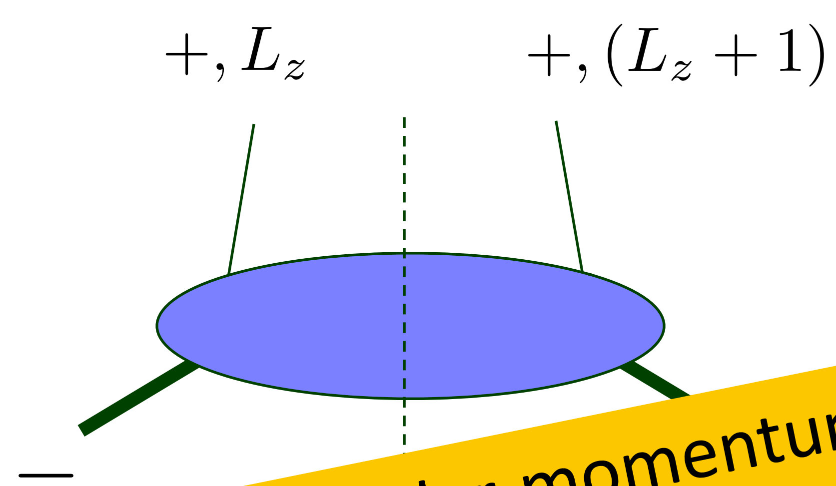
More...

g1T: another interesting function

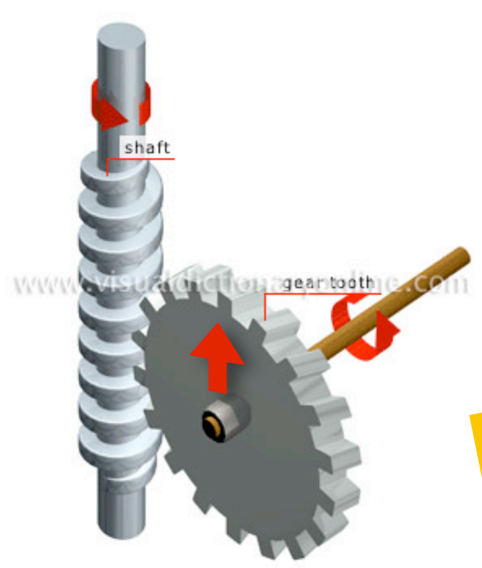


$$g_{1T} = \frac{1}{16\pi^3}\mathrm{Re}\big[(\psi_+^+)^*\psi_+^- - (\psi_-^+)^*\psi_-^-\big]$$

$$f_{1T}^{\perp} = \frac{1}{16\pi^3}\mathrm{Im}\big[(\psi_+^+)^*\psi_+^- + (\psi_-^+)^*\psi_-^-\big]$$



Worm gear



Another way to access angular-momentum information without final-state interactions