Transverse structure of the nucleon
Part 5: A model calculation

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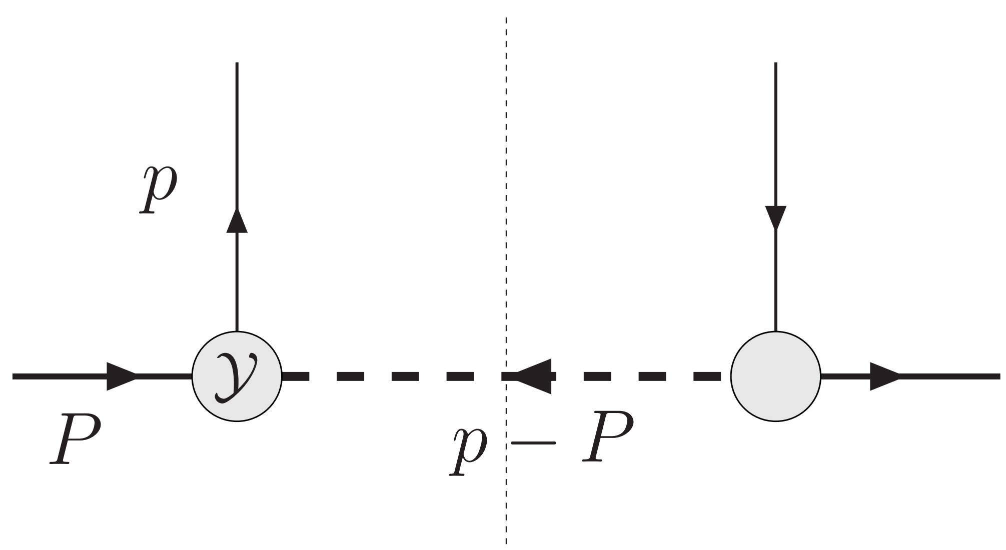
Spectator model

$$\Phi(x, \boldsymbol{p}_T, S) \sim \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \overline{\mathcal{M}}^{(0)}(S)\, \mathcal{M}^{(0)}(S)\Big|_{p^2=\tau(x,\boldsymbol{p}_T)}$$

$$p^2 \equiv \tau(x, \boldsymbol{p}_T) = -\frac{\boldsymbol{p}_T^2 + L_X^2(m^2)}{1-x} + m^2 \ , \quad L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M^2 \ ,$$



Scalar diquark case

$$\mathcal{M}^0 = \langle P - p|\psi(0)|P,S\rangle = \frac{i}{\not{p} - m}\,\mathcal{Y}_s\,U(P,S) = i\,\frac{\not{p} + m}{p^2 - m^2}\;ig_s(p^2)\,\frac{1 + \gamma_5\,\not{S}}{2}\,U(P,S)$$

$$\Phi(x, \boldsymbol{p}_T, S) = \frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\frac{g_s^2(p^2)}{(p^2 - m^2)^2}\left[(\not{p} + m)\,\frac{1 + \gamma_5\,\not{S}}{2}\,(\not{P} + M)\,(\not{p} + m)\right]\Bigg|_{\dots}$$

Calculation of the TMD f_1

$$
\begin{aligned}
f_1(x, \boldsymbol{p}_T^2) &= \Phi^{[\gamma^+]} \\
&= \frac{1}{2} \, \frac{1}{(2\pi)^3} \, \frac{1}{2(1-x)P^+} \, \frac{g_s^2}{(p^2 - m^2)^2} \mathrm{Tr}\left[(\not{p} + m) \, \frac{1 + \gamma_5 \not{S}}{2} \, (\not{P} + M) \, (\not{p} + m) \gamma^+ \right] \\
&= \frac{g_s^2}{16\pi^3} \, \frac{m(m + 2Mx) - p^2 + 2x \, p \cdot P}{(1-x) \, (p^2 - m^2)^2} \\
&= \frac{g_s^2}{16\pi^3} \, \frac{1-x}{(p_T^2 + L^2)^2} \left(p_T^2 + (m + Mx)^2 \right).
\end{aligned}
$$

$$
f_1(x) = \frac{g_s^2 \, (1-x)}{8\pi^2} \left(x \, \frac{\Lambda^2}{L^2} \, \frac{(m+M)^2 - M_s^2}{L^2 + \Lambda^2} + \ln \frac{L^2 + \Lambda^2}{L^2} \right)
$$

TMDs

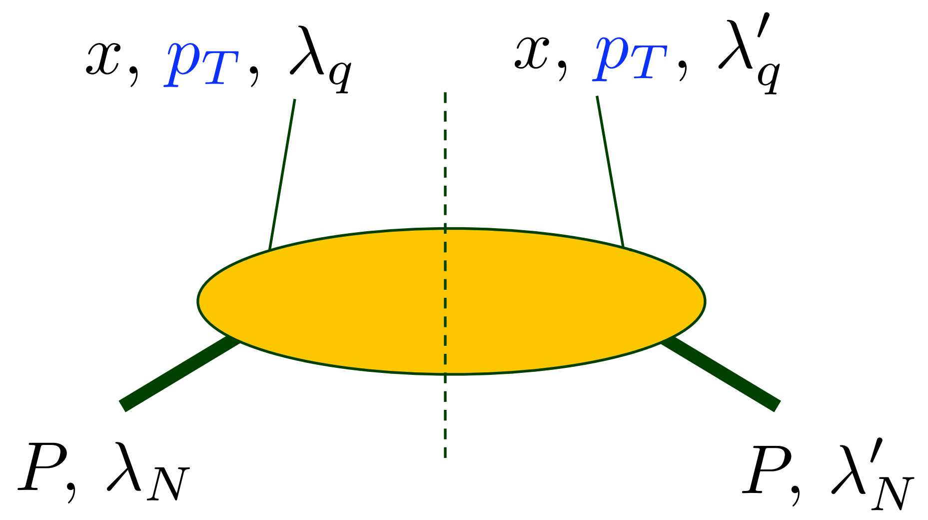
$$f_1^q(x, p_T^2) = \int \frac{d\xi^- d^2\xi_T}{16\pi^3} \, e^{ip\cdot\xi} \langle P|\bar{\psi}^q(0)\gamma^+\psi^q(\xi)|P\rangle\Big|_{\xi^+=0}$$

$x, \, p_T, \, \lambda'_q$

$P, \, \lambda'_N$

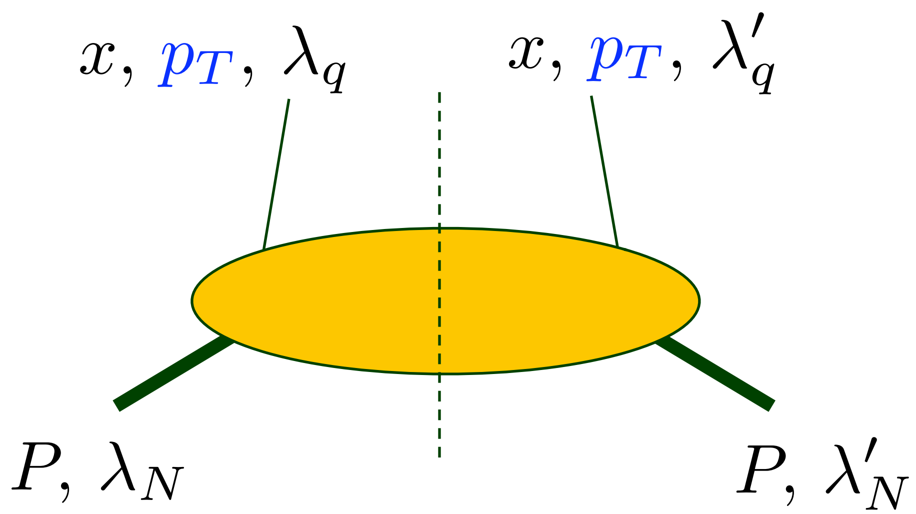
TMDs

$$f_1^q(x, p_T^2) = \int \frac{d\xi^- d^2\xi_T}{16\pi^3} e^{ip\cdot\xi} \langle P|\bar{\psi}^q(0)\gamma^+\psi^q(\xi)|P\rangle\Big|_{\xi^+=0}$$



TMDs

$$f_1^q(x, p_T^2) = \int \frac{d\xi^- d^2\xi_T}{16\pi^3} \, e^{ip\cdot\xi} \langle P|\bar{\psi}^q(0)\gamma^+\psi^q(\xi)|P\rangle\Big|_{\xi^+=0}$$



$$f_1(x, p_T^2) = \frac{1}{16\pi^2}\left(|\psi_+^+(x, p_T)|^2 + |\psi_-^+(x, p_T)|^2\right)$$

Light-cone wave functions

$$\psi^{\lambda_N}_{\lambda_q}(x, \boldsymbol{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}}\,\frac{\bar{u}(p,\lambda_q)}{p^2 - m^2}\,\mathcal{Y}_s\,U(P,\lambda_N)$$

$$U(P,+) = \frac{1}{\sqrt{2^{3/2}\,P^+}}\begin{pmatrix} 0 \\ \sqrt{2}\,P^+ \\ 0 \\ M \end{pmatrix}\,, \qquad U(P,-) = \frac{1}{\sqrt{2^{3/2}\,P^+}}\begin{pmatrix} \sqrt{2}\,P^+ \\ 0 \\ M \\ 0 \end{pmatrix}$$

$$\bar{u}(p,+) = \frac{1}{\sqrt{2^{3/2}\,xP^+}}\begin{pmatrix} p_x - ip_y \\ m \\ 0 \\ \sqrt{2}\,xP^+ \end{pmatrix}\,, \quad \bar{u}(p,-) = \frac{1}{\sqrt{2^{3/2}\,xP^+}}\begin{pmatrix} m \\ -p_x - ip_y \\ \sqrt{2}\,xP^+ \\ 0 \end{pmatrix}$$

Light-cone wave functions

$$\psi^{\lambda_N}_{\lambda_q}(x, \boldsymbol{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}}\, \frac{\bar{u}(p, \lambda_q)}{p^2 - m^2}\, \mathcal{Y}_s\, U(P, \lambda_N)$$

$$\begin{aligned}
\psi^+_+(x, \boldsymbol{p}_T) &= (m + xM)\, \phi/x && (L_z = 0),\\
\psi^+_-(x, \boldsymbol{p}_T) &= -(p_x + ip_y)\, \phi/x && (L_z = +1),\\
\psi^-_+(x, \boldsymbol{p}_T) &= -\left[\psi^+_-(x, \boldsymbol{p}_T)\right]^* && (L_z = -1),\\
\psi^-_-(x, \boldsymbol{p}_T) &= \psi^+_+(x, \boldsymbol{p}_T) && (L_z = 0),\\
\phi(x, \boldsymbol{p}_T^2) &= -\frac{g_s}{\sqrt{1-x}}\, \frac{x(1-x)}{\boldsymbol{p}_T^2 + L_s^2(m^2)}
\end{aligned}$$

Parallel and antiparallel helicity distributions

$$f_1^+(x, \boldsymbol{p}_T^2) = \frac{1}{16\pi^2} |\psi_+^+(x, p_T)|^2 = \frac{g_s^2}{16\pi^3} \, \frac{1-x}{(p_T^2 + L^2)^2} \, (m + Mx)^2$$

$$f_1^-(x, \boldsymbol{p}_T^2) = \frac{1}{16\pi^2} |\psi_-^+(x, p_T)|^2 = \frac{g_s^2}{16\pi^3} \, \frac{1-x}{(p_T^2 + L^2)^2} \, p_T^2$$

All other T-even TMDs

$$g_{1L}(x,\boldsymbol{p}_T) = \frac{1}{16\pi^3}\left(|\psi_+^+|^2 - |\psi_-^+|^2\right),$$

$$\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{S}}_T}{M}\,g_{1T}(x,\boldsymbol{p}_T) = \frac{1}{16\pi^3}\left(|\psi_+^\uparrow|^2 - |\psi_-^\uparrow|^2\right),$$

$$\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{S}}_{qT}}{M}\,h_{1L}^\perp(x,\boldsymbol{p}_T) = \frac{1}{16\pi^3}\left(|\psi_\uparrow^+|^2 - |\psi_\downarrow^+|^2\right),$$

$$\hat{\boldsymbol{S}}_T\cdot\hat{\boldsymbol{S}}_{qT}\;h_{1T}(x,\boldsymbol{p}_T) + \frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{S}}_T}{M}\,\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{S}}_{qT}}{M}\,h_{1T}^\perp(x,\boldsymbol{p}_T) = \frac{1}{16\pi^3}\left(|\psi_\uparrow^\uparrow|^2 - |\psi_\downarrow^\uparrow|^2\right)$$

Results for scalar and vector diquark case

$$g_{1L}^{q(s)}(x,\boldsymbol{p}_T^2) = \frac{g_s^2}{(2\pi)^3}\,\frac{[(m+xM)^2-\boldsymbol{p}_T^2]\,(1-x)^3}{2\,(\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2))^4}\,,$$

$$g_{1L}^{q(a)}(x,\boldsymbol{p}_T^2) = \frac{g_a^2}{(2\pi)^3}\,\frac{[\boldsymbol{p}_T^2\,(1+x^2)-(m+xM)^2\,(1-x)^2]\,(1-x)}{2\,(\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2))^4}\,,$$

$$g_{1T}^{q(s)}(x,\boldsymbol{p}_T^2) = \frac{g_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2))^4}\,,$$

$$g_{1T}^{q(a)}(x,\boldsymbol{p}_T^2) = \frac{g_a^2}{(2\pi)^3}\,\frac{xM\,(m+xM)\,(1-x)^2}{(\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2))^4}\,,$$

$$h_{1L}^{\perp\,q(s)}(x,\boldsymbol{p}_T^2) = -\frac{g_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2))^4}\,,$$

$$h_{1L}^{\perp\,q(a)}(x,\boldsymbol{p}_T^2) = \frac{g_a^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^2}{(\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2))^4}\,,$$

$$h_{1T}^{q(s)}(x,\boldsymbol{p}_T^2) = \frac{g_s^2}{(2\pi)^3}\,\frac{[\boldsymbol{p}_T^2+(m+xM)^2]\,(1-x)^3}{2\,(\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2))^4}\,,$$

$$h_{1T}^{q(a)}(x,\boldsymbol{p}_T^2) = -\frac{g_a^2}{(2\pi)^3}\,\frac{\boldsymbol{p}_T^2\,x(1-x)}{(\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2))^4}\,,$$

$$h_{1T}^{\perp\,q(s)}(x,\boldsymbol{p}_T^2) = -\frac{g_s^2}{(2\pi)^3}\,\frac{M^2\,(1-x)^3}{(\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2))^4}\,,$$

$$h_{1T}^{\perp\,q(a)}(x,\boldsymbol{p}_T^2) = 0\,.$$

$$h_1^{q(s)}(x,\boldsymbol{p}_T^2) = h_{1T}^{q(s)}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T^2}{2M^2}\,h_{1T}^{\perp\,q(s)}(x,\boldsymbol{p}_T^2) = \frac{g_s^2}{(2\pi)^3}\,\frac{(m+xM)^2\,(1-x)^3}{2\,[\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2)]^4}$$

$$h_1^{q(a)}(x,\boldsymbol{p}_T^2) = -\frac{g_a^2}{(2\pi)^3}\,\frac{\boldsymbol{p}_T^2\,x(1-x)}{[\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2)]^4}\,.$$

Unpolarized and helicity distribution: fit

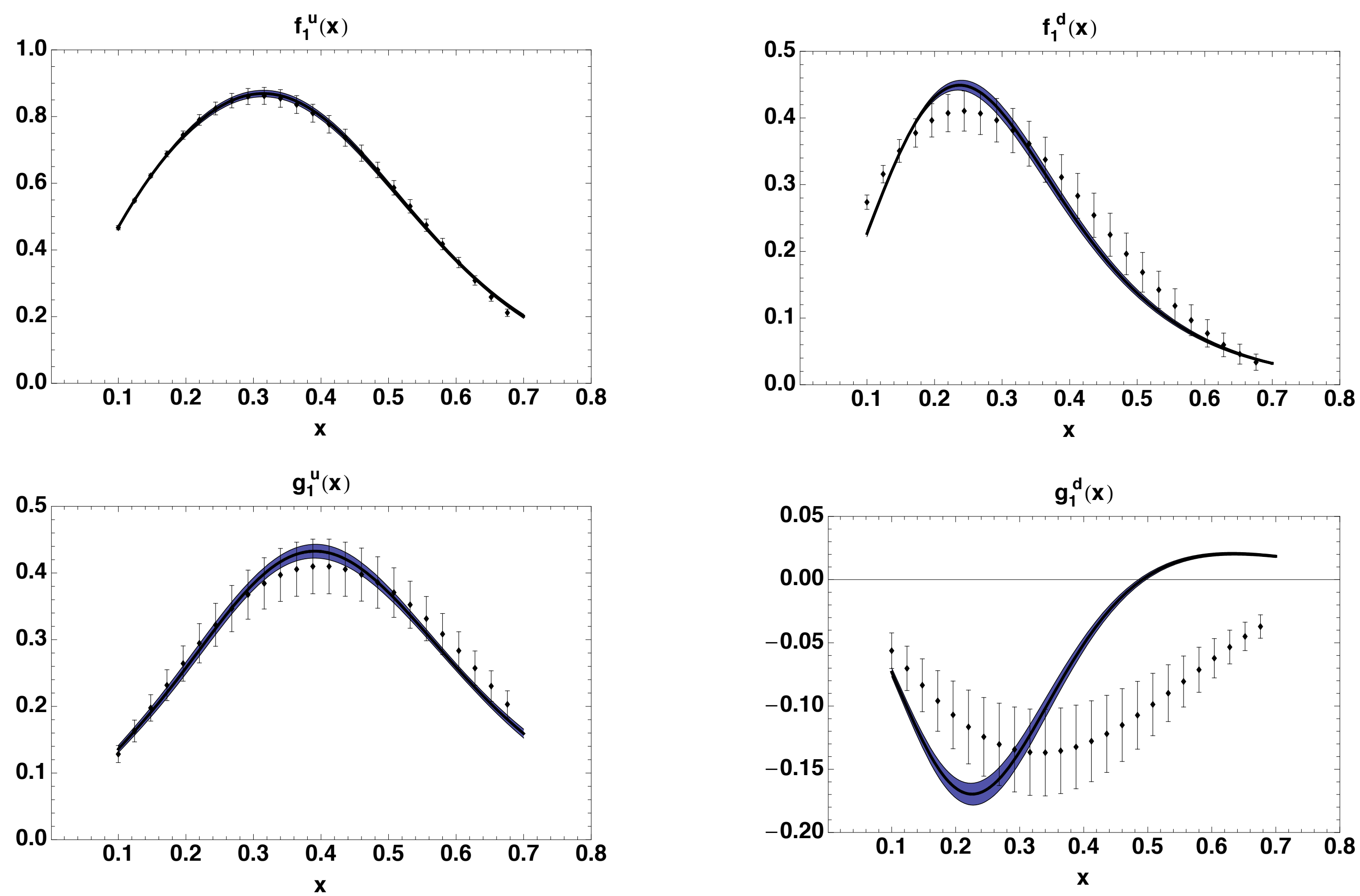


FIG. 4: The distribution functions $f_1(x)$ (above) and $g_1(x)$ (below) for the up quark (left panel) and the down quark (right panel). Data are a selection of 25 equidistant points in $0.1 \leq x \leq 0.7$ from the parametrizations of Ref. [68] (ZEUS2002) and Ref. [69] (GRSV2000) at LO, respectively (we assigned a constant relative error of 10% to g_1^u and 25% to g_1^d based on comparisons with similar fits [70]). The curves represent the best fit (χ^2/d.o.f. = 3.88) obtained with our spectator model. The statistical uncertainty bands correspond to $\Delta\chi^2 = 1$.

Diquark	M_X (GeV)	Λ_X (GeV)	c_X
Scalar s (uu)	0.822 ± 0.053	0.609 ± 0.038	0.847 ± 0.111
Axial-vector a (ud)	1.492 ± 0.173	0.716 ± 0.074	1.061 ± 0.085
Axial-vector a' (uu)	0.890 ± 0.008	0.376 ± 0.005	0.880 ± 0.008

Unpolarized and helicity distribution: fit

A.B., F. Conti, M. Radici, arXiv:0807.0323

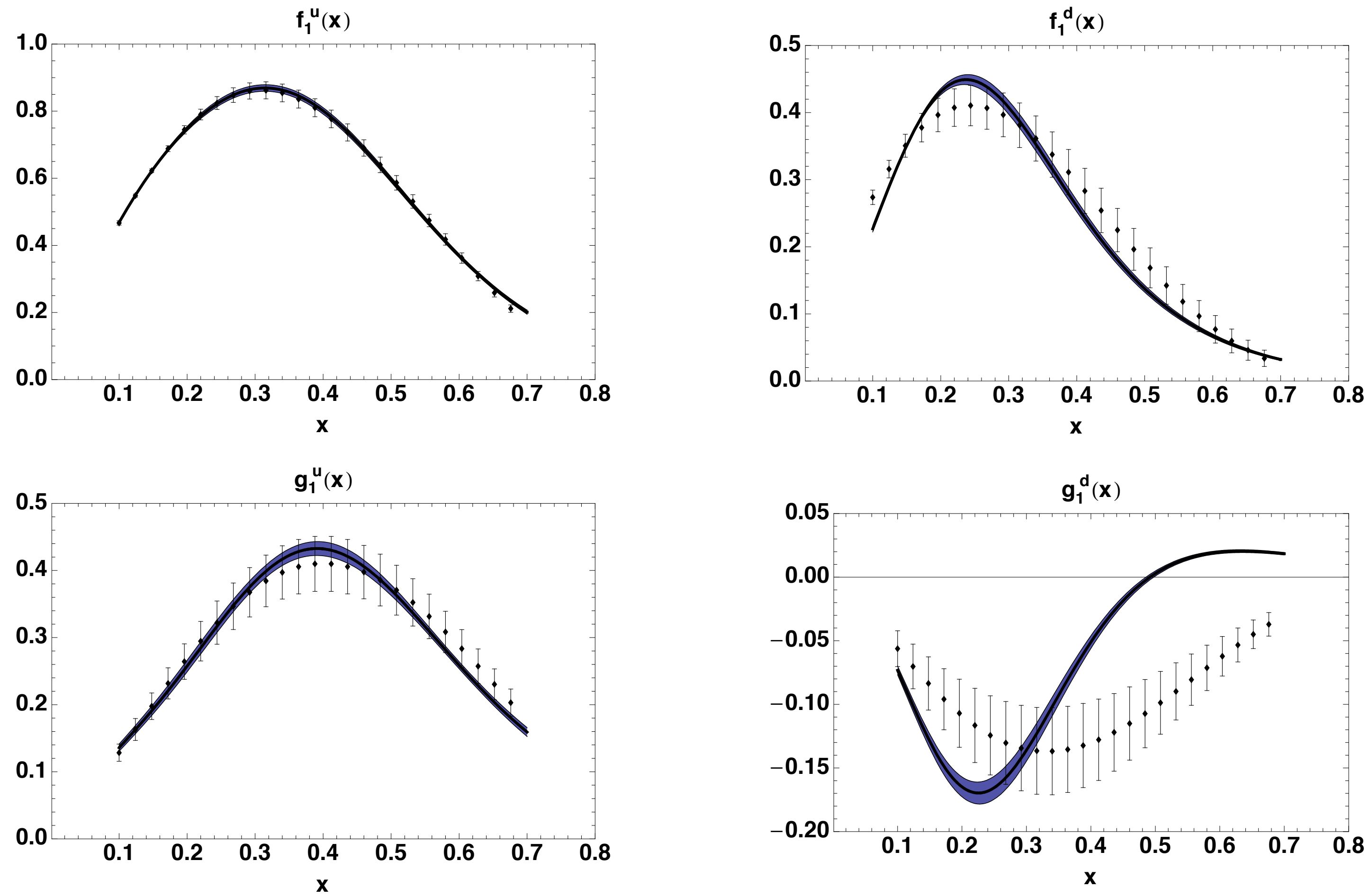


FIG. 4: The distribution functions $f_1(x)$ (above) and $g_1(x)$ (below) for the up quark (left panel) and the down quark (right panel). Data are a selection of 25 equidistant points in $0.1 \leq x \leq 0.7$ from the parametrizations of Ref. [68] (ZEUS2002) and Ref. [69] (GRSV2000) at LO, respectively (we assigned a constant relative error of 10% to g_1^u and 25% to g_1^d based on comparisons with similar fits [70]). The curves represent the best fit (χ^2/d.o.f. = 3.88) obtained with our spectator model. The statistical uncertainty bands correspond to $\Delta\chi^2 = 1$.

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Some nice features

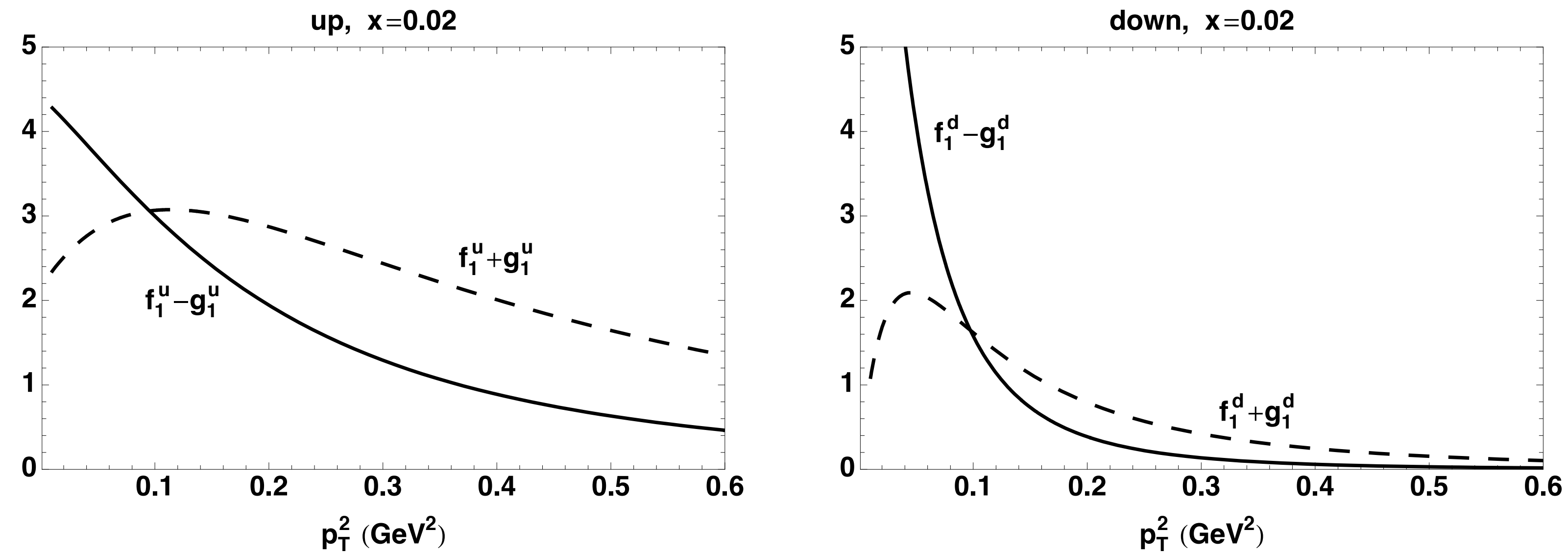


FIG. 6: The $\boldsymbol{p}_T^2$ dependence of the distributions $f_1(x,\boldsymbol{p}_T^2)-g_1(x,\boldsymbol{p}_T^2)$ (solid line) and $f_1(x,\boldsymbol{p}_T^2)+g_1(x,\boldsymbol{p}_T^2)$ (dashed line) and for up (left panel) and down quark (right panel), at $x=0.02$. The difference in their behavior is due to the different role played in the two combinations by wavefunctions with nonzero orbital angular momentum.

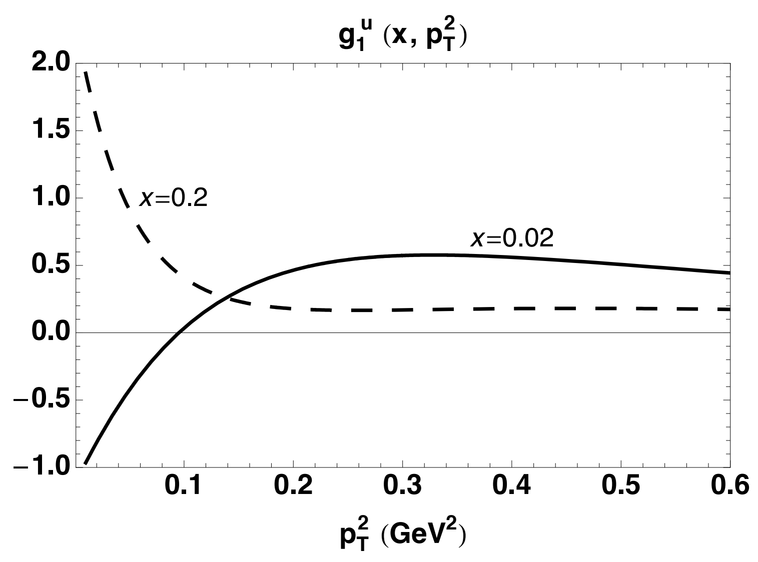
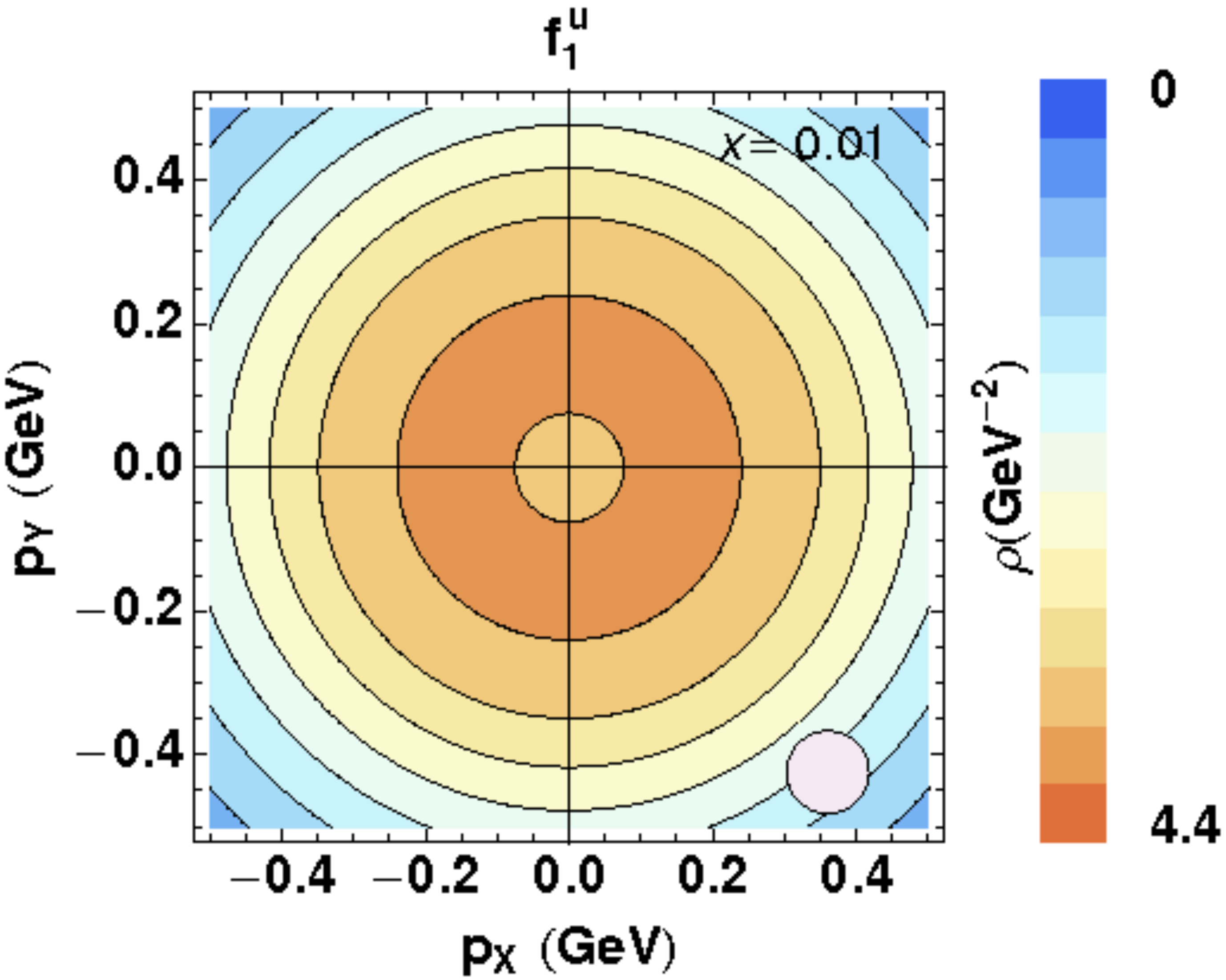
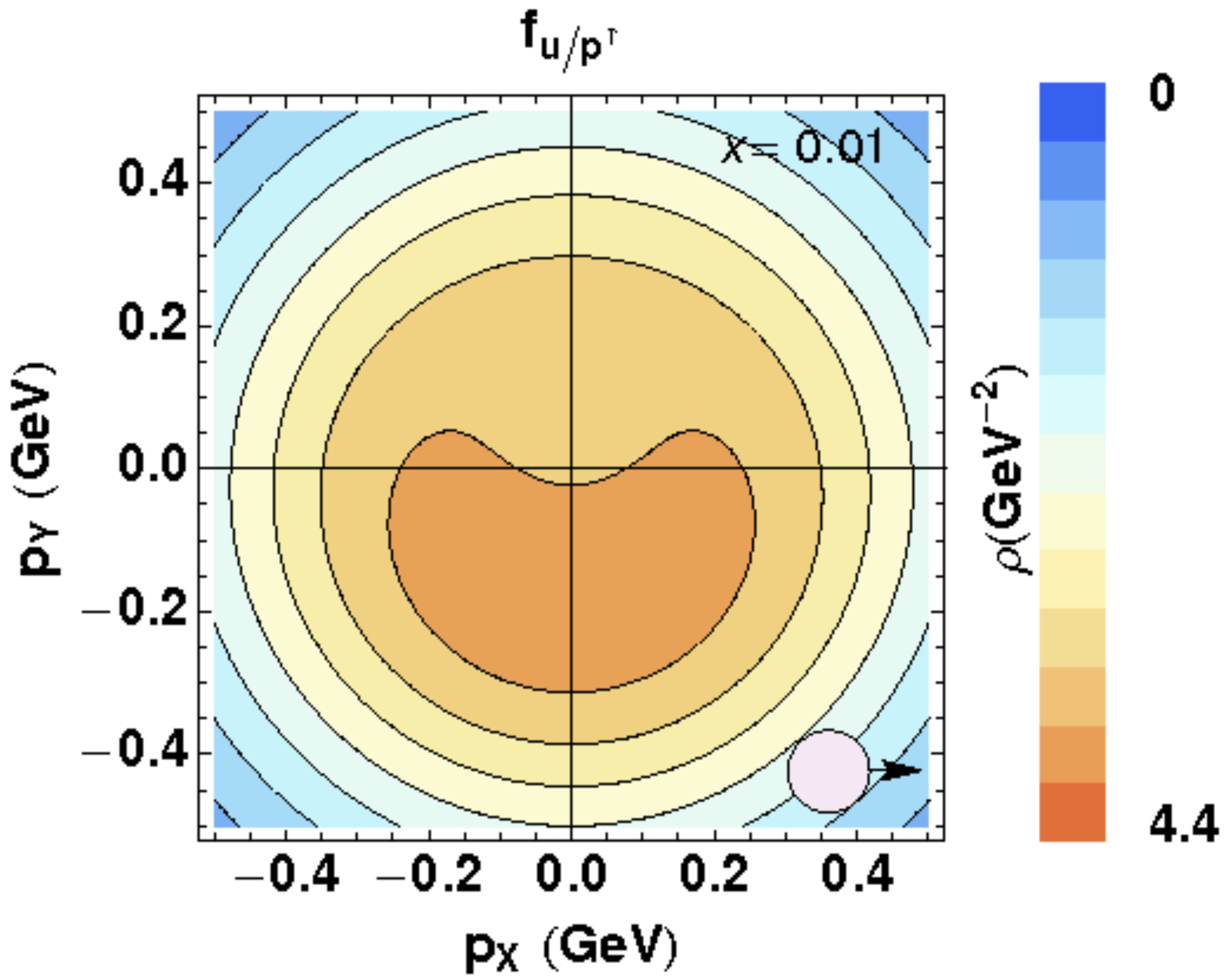


FIG. 7: The $\boldsymbol{p}_T^2$ dependence of the helicity distribution $g_1^u(x,\boldsymbol{p}_T^2)$. Different lines correspond to different values of x.

Cool possibilities



Cool possibilities



Generalized parton distribution functions

$$H(x,\xi,\Delta_T^2) = \int \frac{d^2p_T}{16\pi^2}\,\Big[\psi_+^{+*}\Big(\frac{x+\xi}{1+\xi}, p_T - \frac{1-x}{1+\xi}\frac{\Delta_T}{2}\Big)\psi_+^{+}\Big(\frac{x-\xi}{1-\xi}, p_T + \frac{1-x}{1-\xi}\frac{\Delta_T}{2}\Big)$$

$$+\,\psi_-^{+*}\Big(\frac{x+\xi}{1+\xi}, p_T - \frac{1-x}{1+\xi}\frac{\Delta_T}{2}\Big)\psi_-^{+}\Big(\frac{x-\xi}{1-\xi}, p_T + \frac{1-x}{1-\xi}\frac{\Delta_T}{2}\Big)\Big]$$

$$H(x,0,\Delta_T^2) = \int \frac{d^2p_T}{16\pi^2}\,\Big[\psi_+^{+*}\big(x, p_T\big)\psi_+^{+}\big(x, p_T + (1-x)\Delta_T\big)$$

$$+\,\psi_-^{+*}\big(x, p_T\big)\psi_-^{+}\big(x, p_T + (1-x)\Delta_T\big)\Big]$$