Transverse structure of the nucleon Part 5: A model calculation

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Spectator model pleteness relation and at tree-level we truncate the sum over final states to a single on-shell spectator state with mass

with M the hadron mass.

scalar and axial-vector diquarks.

parton densities, as mentioned for the first time in Ref. [15].

$$
\Phi(x, \mathbf{p}_T, S) \sim \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \overline{\mathcal{M}}^{(0)}(S) \mathcal{M}^{(0)}(S) \Big|_{p^2 = \tau(x, \mathbf{p}_T)}
$$

$$
p^2 \equiv \tau(x, \mathbf{p}_T) = -\frac{\mathbf{p}_T^2 + L_X^2(m^2)}{1-x} + m^2, \quad L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M^2,
$$

 $\overline{1}$

 $\overline{1}$

 $\frac{1}{2}$

while in the Drell–Yan case it runs in the opposite direction through \sim . This fact leads to a sign difference in T-odd \sim

Similarly to Ref. [42], we evaluate the correlator of Eq. (4) in the spectator approximation, i.e. we insert a com-

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, (2) and (2) and (2) and (2) and (2) $\frac{1}{2}$ and (2) $\frac{1}{2}$ and (2) $\frac{1}{2}$

, (8) \sim

 Γ , the calculation of the calculation of T-even leading-twist parton densities. The dashed line indicates both Γ

We assume the spectator to be point-like, with the quantum numbers of a diquark. Hence, the proton can couple to

Scalar diquark case

$$
\mathcal{M}^0 = \langle P - p | \psi(0) | P, S \rangle = \frac{i}{\not p - m} \mathcal{Y}_s U(P, S) = i \frac{\not p + m}{p^2 - m^2} i g_s(p^2) \frac{1 + \gamma_5 \mathcal{S}}{2} U(P, S)
$$

$$
\Phi(x, \mathbf{p}_T, S) = \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \frac{g_s^2(p^2)}{(p^2 - m^2)^2} \left[(\not p + m) \frac{1 + \gamma_5 \not S}{2} (P + M) (\not p + m) \right] \Big|_{...}
$$

Calculation of the TMD *f1*

$$
f_1(x, \mathbf{p}_T^2) = \Phi^{\lceil \gamma^+ \rceil}
$$

= $\frac{1}{2} \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \frac{g_s^2}{(p^2 - m^2)^2} \text{Tr} \left[(\not p + m) \frac{1 + \gamma_5 \n}\{g} (\not p + M) (\not p + m) \gamma^+ \right]$
= $\frac{g_s^2}{16\pi^3} \frac{m(m + 2Mx) - p^2 + 2x p \cdot P}{(1 - x) (p^2 - m^2)^2}$
= $\frac{g_s^2}{16\pi^3} \frac{1 - x}{(p_T^2 + L^2)^2} \left(p_T^2 + (m + Mx)^2 \right).$

$$
f_1(x) = \frac{g_s^2 (1-x)}{8\pi^2} \left(x \frac{\Lambda^2}{L^2} \frac{(m+M)^2 - M_s^2}{L^2 + \Lambda^2} + \ln \frac{L^2 + \Lambda^2}{L^2} \right)
$$

TMDs

$$
f_1^q(x, p_T^2) = \int \frac{d\xi^- d^2\xi_T}{16\pi^3} e^{ip\cdot\xi} \langle P|\bar{\psi}^q(0)\gamma^+ \psi^q(\xi)|P\rangle \Big|_{\xi^+ = 0}
$$

$$
x, p_T, \lambda'_q
$$

$$
P,\,\lambda'_N
$$

TMDs

$$
f_1^q(x, p_T^2) = \int \frac{d\xi^{-} d^2\xi_T}{16\pi^3} e^{ip \cdot \xi} \langle P|\bar{\psi}^q(0)\gamma^{+}\psi^q(\xi)|P\rangle \Big|_{\xi^{+}=0}
$$

$$
x, p_T, \lambda_q \qquad x, p_T, \lambda'_q
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$$

$$
x, p_T, \lambda_q \qquad x, p_T, \lambda'_q
$$

$$
P, \lambda_N
$$

$$
P, \lambda'_N
$$

$$
f_1(x, p_T^2) = \frac{1}{16\pi^2} \left(|\psi_+^+(x, p_T)|^2 + |\psi_-^+(x, p_T)|^2 \right)
$$

Light-cone wave functions

$$
\psi^{\lambda_N}_{\lambda_q}(x, \mathbf{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}} \frac{\bar{u}(p, \lambda_q)}{p^2 - m^2} \mathcal{Y}_s U(P, \lambda_N)
$$

$$
U(P,+) = \frac{1}{\sqrt{2^{3/2} P^+}} \begin{pmatrix} 0 \\ \sqrt{2} P^+ \\ 0 \\ M \end{pmatrix} , \qquad U(P,-) = \frac{1}{\sqrt{2^{3/2} P^+}} \begin{pmatrix} \sqrt{2} P^+ \\ 0 \\ M \\ 0 \end{pmatrix}
$$

$$
\bar{u}(p,+) = \frac{1}{\sqrt{2^{3/2} x P^+}} \begin{pmatrix} p_x - ip_y \\ m \\ 0 \\ \sqrt{2} x P^+ \end{pmatrix} , \quad \bar{u}(p,-) = \frac{1}{\sqrt{2^{3/2} x P^+}} \begin{pmatrix} m \\ -p_x - ip_y \\ \sqrt{2} x P^+ \\ 0 \end{pmatrix}
$$

Light-cone wave functions

$$
\psi^{\lambda_N}_{\lambda_q}(x, \mathbf{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}} \frac{\bar{u}(p, \lambda_q)}{p^2 - m^2} \mathcal{Y}_s U(P, \lambda_N)
$$

$$
\psi_{+}^{+}(x, \mathbf{p}_{T}) = (m + xM) \phi/x
$$
\n
$$
\psi_{-}^{+}(x, \mathbf{p}_{T}) = -(p_{x} + ip_{y}) \phi/x
$$
\n
$$
\psi_{+}^{-}(x, \mathbf{p}_{T}) = -[\psi_{-}^{+}(x, \mathbf{p}_{T})]^{*}
$$
\n
$$
\psi_{-}^{-}(x, \mathbf{p}_{T}) = \psi_{+}^{+}(x, \mathbf{p}_{T})
$$
\n
$$
\psi_{-}^{-}(x, \mathbf{p}_{T}) = \psi_{+}^{+}(x, \mathbf{p}_{T})
$$
\n
$$
\phi(x, \mathbf{p}_{T}^{2}) = -\frac{g_{s}}{\sqrt{1 - x}} \frac{x(1 - x)}{\mathbf{p}_{T}^{2} + L_{s}^{2}(m^{2})}
$$
\n
$$
(L_{z} = 0),
$$

Parallel and antiparallel helicity distributions

$$
f_1^+(x, \mathbf{p}_T^2) = \frac{1}{16\pi^2} |\psi_+^+(x, p_T)|^2 = \frac{g_s^2}{16\pi^3} \frac{1-x}{(p_T^2 + L^2)^2} (m + Mx)^2
$$

$$
f_1^-(x, \mathbf{p}_T^2) = \frac{1}{16\pi^2} |\psi_+^+(x, p_T)|^2 = \frac{g_s^2}{16\pi^3} \frac{1-x}{(p_T^2 + L^2)^2} p_T^2
$$

All other T-even TMDs

$$
g_{1L}(x, \mathbf{p}_T) = \frac{1}{16\pi^3} \left(|\psi_+^+|^2 - |\psi_-^+|^2 \right),
$$

\n
$$
\frac{\mathbf{p}_T \cdot \hat{\mathbf{S}}_T}{M} g_{1T}(x, \mathbf{p}_T) = \frac{1}{16\pi^3} \left(|\psi_+^{\dagger}|^2 - |\psi_-^{\dagger}|^2 \right),
$$

\n
$$
\frac{\mathbf{p}_T \cdot \hat{\mathbf{S}}_q}{M} h_{1L}^{\dagger}(x, \mathbf{p}_T) = \frac{1}{16\pi^3} \left(|\psi_+^+|^2 - |\psi_+^+|^2 \right),
$$

\n
$$
\hat{\mathbf{S}}_T \cdot \hat{\mathbf{S}}_q T h_{1T}(x, \mathbf{p}_T) + \frac{\mathbf{p}_T \cdot \hat{\mathbf{S}}_T}{M} \frac{\mathbf{p}_T \cdot \hat{\mathbf{S}}_q T}{M} h_{1T}^{\dagger}(x, \mathbf{p}_T) = \frac{1}{16\pi^3} \left(|\psi_+^{\dagger}|^2 - |\psi_+^{\dagger}|^2 \right)
$$

Results for scalar and vector diquark case \overline{D} ector dic ! ark cas 16 nd $\overline{16}$ ا پ $\frac{1}{2}$ $\frac{1}{2}$

M

 $\frac{1}{\sqrt{2}}$

! " [|]ψ⁺

² [−] [|]ψ⁺

M h
M h⊥n h

2 #

16π3
16π3

 $\overline{}$

^p^T · ^Sˆ^T

 $\frac{1}{\sqrt{2}}$

The above results and along the above results and α and β and β are positivity bounds β .

 $\sqrt{2}$)

Note that the functions give the functions give the functions give the functions $\mathcal{L}_\mathbf{1}$

(2π)³

 $(2\pi)^3$ $|\mathbf{p}_{\rm T}^2 + L$

 $\mathcal{L} \subset \mathcal{L}$

 $\mathcal{L}(\mathcal{A})$

 $[p_T^2 + L_a^2 (\Lambda_a^2$

a)] $\mathcal{A}(\mathcal{A}) = \mathcal{A}(\mathcal{A}) = \mathcal{A}(\math$

¹^L arise from the interference of LCWFs with [|]Lz[|] = 1 and ^L^z = 0. The function ^h[⊥]

 $\frac{a}{(2a)^2}$.

[p²

 $(2\pi)^3$

$$
g_{1L}^{q(s)}(x, p_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{[(m+xM)^2 - p_T^2](1-x)^3}{2(p_T^2 + L_s^2(\Lambda_s^2))^4},
$$

\n
$$
g_{1L}^{q(s)}(x, p_\tau^2) = \frac{g_s^2}{(2\pi)^3} \frac{[p_T^2(1+x^2) - (m+xM)^2(1-x)^2](1-x)}{2(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
g_{1T}^{q(s)}(x, p_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{M(m+xM)(1-x)^3}{(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
g_{1T}^{q(s)}(x, p_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{M(m+xM)(1-x)^2}{(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
h_{1L}^{\perp q(s)}(x, p_T^2) = -\frac{g_s^2}{(2\pi)^3} \frac{M(m+xM)(1-x)^3}{(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
h_{1L}^{\perp q(s)}(x, p_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{M(m+xM)(1-x)^2}{(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
h_{1T}^{q(s)}(x, p_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{p_T^2 + (m+xM)^2[(1-x)^3]}{2(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
h_{1T}^{q(s)}(x, p_\tau^2) = -\frac{g_s^2}{(2\pi)^3} \frac{p_T^2 x(1-x)}{(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
h_{1T}^{\perp q(s)}(x, p_T^2) = -\frac{g_s^2}{(2\pi)^3} \frac{M^2 (1-x)^3}{(p_T^2 + L_a^2(\Lambda_a^2))^4},
$$

\n
$$
h_{1T}^{\perp q(s)}(x, p_T^2) = 0.
$$

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Unpolarized and helicity distribution: fit I ILUU C \mathcal{A} is more important point of view it is more important to satisfy the momentum sum rule, from the mome

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FIG. 4: The distribution functions $f_1(x)$ (above) and $g_1(x)$ (below) for the up quark (left panel) and the down quark (right panel). Data are a selection of 25 equidistant points in $0.1 \le x \le 0.7$ from the parametrizations of Ref. [68] (ZEUS2002) and Ref. [69] (GRSV2000) at LO, respectively (we assigned a constant relative error of 10% to g_1^u and 25% to g_1^d based on comparisons with similar fits [70]). The curves represent the best fit $(\chi^2/\text{d.o.f.} = 3.88)$ obtained with our spectator model. The statistical uncertainty bands correspond to $\Delta \chi^2 = 1$.

TABLE I: Results for the model parameters with dipolar nucleon-quark-diquark form factor and light-cone transverse polariza-

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TABLE I: Results for the model parameters with dipolar nucleon-quark-diquark form factor and light-cone transverse polariza-

Some nice features

 \overline{a}

FIG. 6: The p_T^2 dependence of the distributions $f_1(x, p_T^2) - g_1(x, p_T^2)$ (solid line) and $f_1(x, p_T^2) + g_1(x, p_T^2)$ (dashed line) and for up (left panel) and down quark (right panel), at $x = 0.02$. The difference in their behavior is due to the different role played in the two combinations by wavefunctions with nonzero orbital angular momentum.

FIG. 7: The p_T^2 dependence of the helicity distribution $g_1^u(x, p_T^2)$. Different lines correspond to different values of x.

with the maxima in the maxima in the correct position and a somewhat too small result for the up α

Cool possibilities

Cool possibilities

Generalized parton distribution functions

$$
H(x,\xi,\Delta_T^2) = \int \frac{d^2p_T}{16\pi^2} \left[\psi_+^{+*} \left(\frac{x+\xi}{1+\xi}, p_T - \frac{1-x}{1+\xi} \frac{\Delta_T}{2} \right) \psi_+^{+} \left(\frac{x-\xi}{1-\xi}, p_T + \frac{1-x}{1-\xi} \frac{\Delta_T}{2} \right) \right. \\ \left. + \psi_-^{+*} \left(\frac{x+\xi}{1+\xi}, p_T - \frac{1-x}{1+\xi} \frac{\Delta_T}{2} \right) \psi_+^{+} \left(\frac{x-\xi}{1-\xi}, p_T + \frac{1-x}{1-\xi} \frac{\Delta_T}{2} \right) \right]
$$

$$
H(x,0,\Delta_T^2) = \int \frac{d^2p_T}{16\pi^2} \left[\psi_+^{+*}(x,p_T) \psi_+^+(x,p_T + (1-x)\Delta_T) + \psi_-^{+*}(x,p_T) \psi_-^+(x,p_T + (1-x)\Delta_T) \right]
$$