Transverse structure of the nucleon
Part 2: Theory background

Alessandro Bacchetta

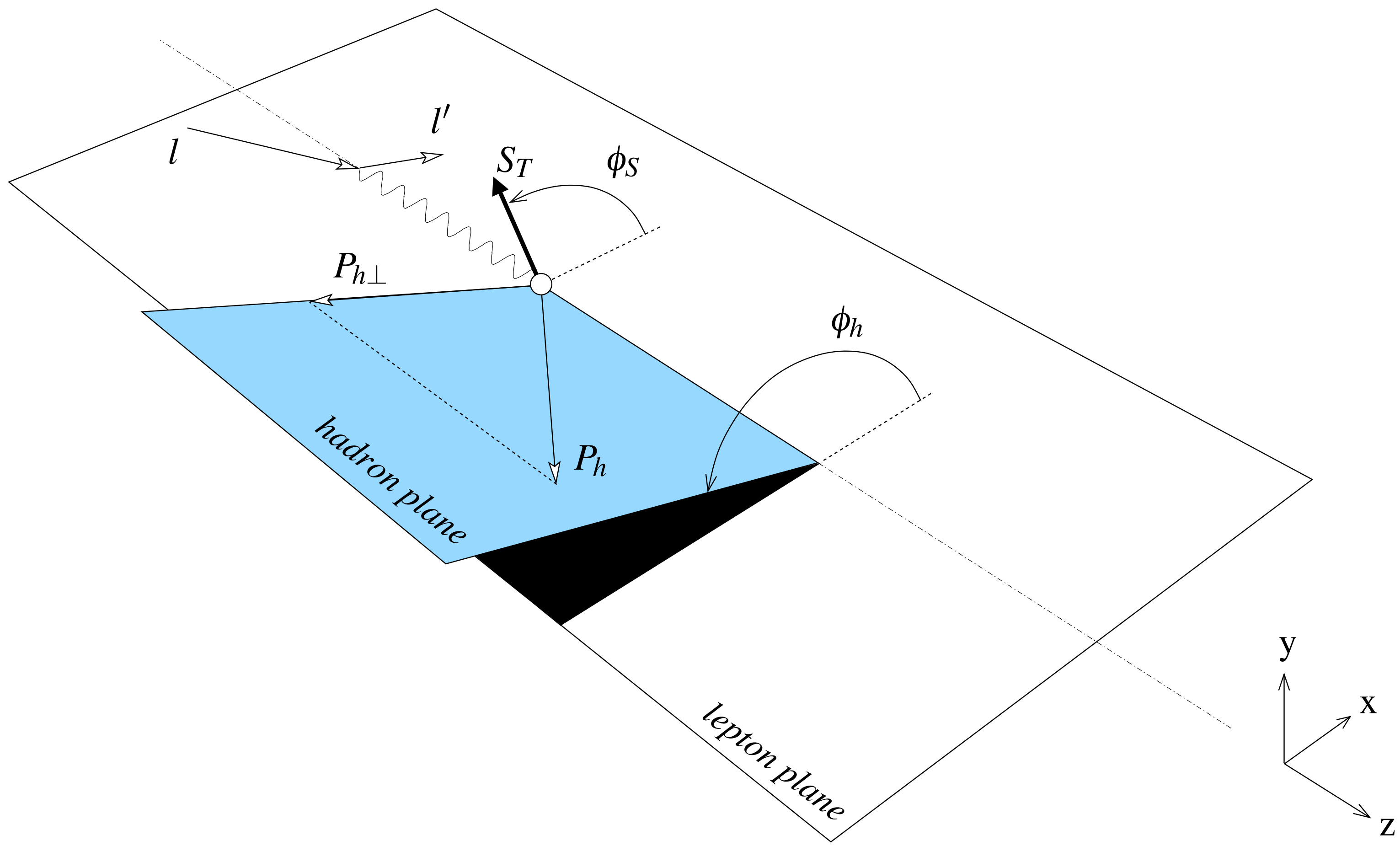
Jefferson Lab
N. Isgur Fellowship

DIS and structure functions

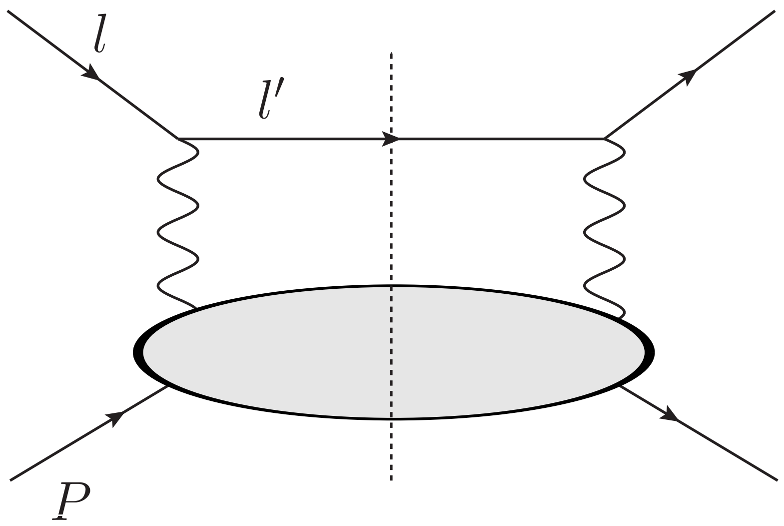
Deep inelastic scattering

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X,$$

$$x_B = \frac{Q^2}{2\,P \cdot q}, \qquad y = \frac{P \cdot q}{P \cdot l}, \qquad z_h = \frac{P \cdot P_h}{P \cdot q}.$$

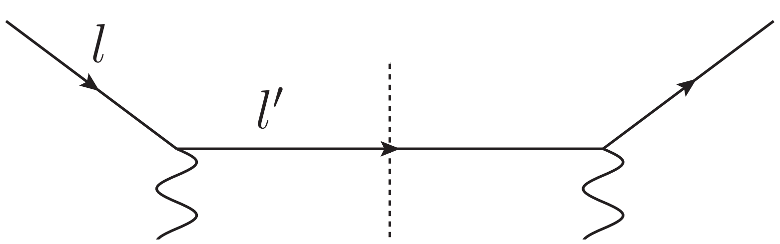


Inclusive DIS



$$\frac{d^3\sigma}{dx_B\,dy\,d\phi_S} = \frac{\alpha^2\,y}{2\,Q^4}\,L_{\mu\nu}(l,l',\lambda_e)\;2MW^{\mu\nu}(q,P,S)$$

Single-photon-exchange approximation



$$\begin{aligned}
L_{\mu\nu} &= \sum_{\lambda_e'}\Big(\bar u(l',\lambda_e')\,\gamma_\mu\,u(l,\lambda_e)\Big)^*\;\Big(\bar u(l',\lambda_e')\,\gamma_\nu\,u(l,\lambda_e)\Big)\\
&= -Q^2 g_{\mu\nu} + 2\left(l_\mu l'_\nu + l'_\mu l_\nu\right) + 2i\,\lambda_e\,\epsilon_{\mu\nu\rho\sigma}\,l^\rho l'^\sigma\\
&= \frac{2Q^2}{y^2}\Big[-\Big(1-y+\frac{y^2}{2}\Big)g_{\perp\mu\nu} + 2(1-y)\hat t_\mu \hat t_\nu\\
&\quad + 2(1-y)\Big(\hat l_\mu \hat l_\nu + \frac{1}{2}g_{\perp\mu\nu}\Big)\\
&\quad - i\lambda_e y\Big(1-\frac{y}{2}\Big)\,\epsilon_{\perp\mu\nu} - i\lambda_e y\sqrt{1-y}\,\hat t_{[\mu}\epsilon_{\perp\nu]\rho}\hat l^\rho + \ldots\Big]
\end{aligned}$$

Lepton tensor

$$l^\mu = \left(\frac{(2-y)Q}{2y},\ \frac{\sqrt{1-y}\,Q}{y},\ 0,\ \frac{Q}{2} \right),$$
$$l'^\mu = \left(\frac{(2-y)Q}{2y},\ \frac{\sqrt{1-y}\,Q}{y},\ 0,\ -\frac{Q}{2} \right)$$

$$L_{\mu\nu}(\lambda_e = 0) = \frac{2Q^2}{y^2} \begin{pmatrix} 2(1-y) & \dots & 0 & 0 \\ \dots & \left(1 - y + \frac{y^2}{2}\right) + (1-y) & 0 & 0 \\ 0 & 0 & \left(1 - y + \frac{y^2}{2}\right) - (1-y) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\epsilon_x^\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \epsilon_y^\mu = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \epsilon_L^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Helicity vs transversity / circular vs linear

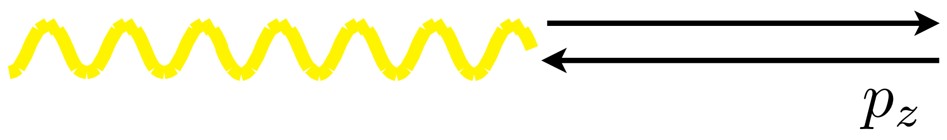
$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\chi_\uparrow = \frac{1}{\sqrt{2}}(\chi_+ + e^{i\phi_s}\chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi_s} \end{pmatrix} \qquad \chi_\downarrow = \frac{1}{\sqrt{2}}(\chi_+ - e^{i(\phi_s+\pi)}\chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(\phi_s+\pi)} \end{pmatrix}$$

$$\epsilon_+^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}, \qquad \epsilon_-^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix},$$

$$\epsilon_x^\mu = \frac{1}{\sqrt{2}}(\epsilon_+^\mu + \epsilon_-^\mu) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \epsilon_y^\mu = \frac{i}{\sqrt{2}}(\epsilon_+^\mu - \epsilon_-^\mu) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Helicity conservation



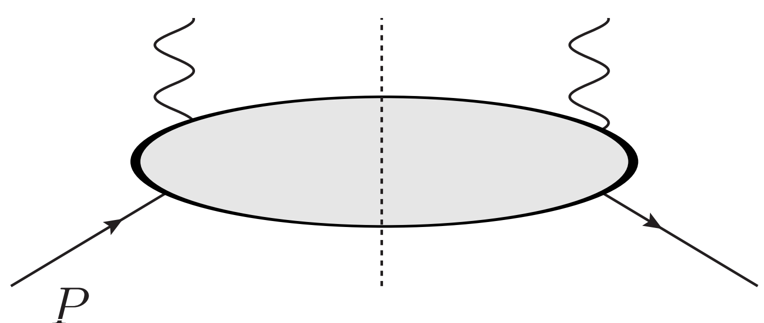
$$\bar{u}_+(-p_z)\gamma^\mu u_+(p_z) \sim (0,1,i,0) \qquad \bar{u}_-(-p_z)\gamma^\mu u_+(p_z) = 0$$

$$\bar{u}^\uparrow(-p_z)\gamma^\mu u^\uparrow(p_z) \sim (0,\cos\phi_s,-\sin\phi_s,0) \qquad \bar{u}^\downarrow(-p_z)\gamma^\mu u^\uparrow(p_z) = 0$$

Structure functions

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$2MW^{\mu\nu} = \frac{1}{x}\left[-g_\perp^{\mu\nu} F_{UU,T} + \hat{t}^\mu \hat{t}^\nu F_{UU,L} + i S_L \epsilon_\perp^{\mu\nu} F_{LL} - i\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho} S_\rho F_{LT}^{\cos\phi_S}\right]$$



$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{\left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1-y)\, F_{UU,L} + S_L \lambda_e\, y\left(1 - \frac{y}{2}\right)\, F_{LL} + |\boldsymbol{S}_T|\lambda_e\, y\sqrt{1-y}\, \cos\phi_S\, F_{LT}^{\cos\phi_s}\right\}$$

Semi-inclusive DIS

$$\frac{d^3\sigma}{dx_B\, dy\, d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$\frac{2E_h\, d^6\sigma}{d^3P_h\, dx_B\, dy\, d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

$$\frac{d^6\sigma}{dx_B\, dy\, dz_h\, d\phi_S\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2\, y}{2\, z_h\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

Structure functions

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$
\begin{aligned}
&\frac{d\sigma}{dx\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\, y\, Q^2}\, \frac{y^2}{2\,(1-\varepsilon)} \Bigg\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \right] \\
&\quad + S_L\, \lambda_e \left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \right] \\
&\quad + S_T \Bigg[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg] + S_T \lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\}
\end{aligned}
$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Light-cone coordinates

Light-cone vectors will be indicated as

$$a^\mu = \left[a^-,\ a^+,\ \boldsymbol{a}_T\right] = \left[\frac{a^0 - a^3}{\sqrt{2}},\ \frac{a^0 + a^3}{\sqrt{2}},\ a^1,\ a^2\right]. \tag{1}$$

The dot-product in light-cone components is

$$a \cdot b = a^+ b^- + a^- b^+ - \boldsymbol{a}_T \cdot \boldsymbol{b}_T \tag{2}$$

The light-cone decomposition of a vector can be written in a Lorentz covariant fashion using two light-like vectors n_+ and n_- satisfying $n_\pm^2 = 0$ and $n_+ \cdot n_- = 1$ and promoting $\boldsymbol{a}_T$ to a four-vector $a_T^\mu = [0, 0, \boldsymbol{a}_T]$ so that

$$a^\mu = a^+ n_+^\mu + a^- n_-^\mu + a_T^\mu, \tag{3}$$

where

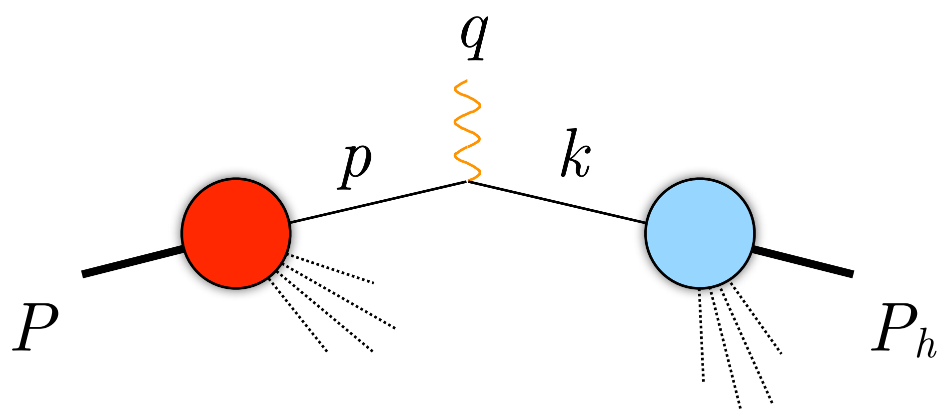
$$a^+ = a \cdot n_-, \qquad a^- = a \cdot n_+, \qquad a_T \cdot n_+ = a_T \cdot n_- = 0. \tag{4}$$

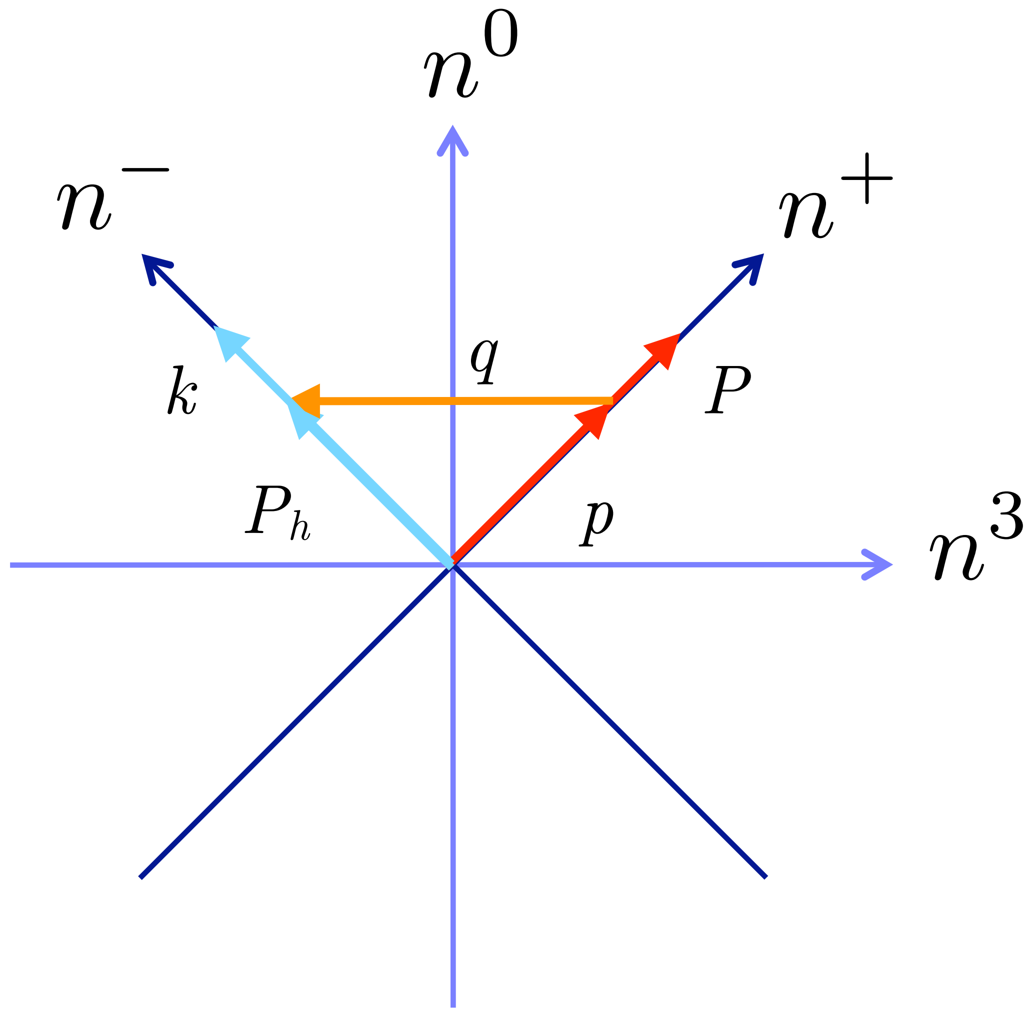
Note that

$$a_T \cdot b_T = -\boldsymbol{a}_T \cdot \boldsymbol{b}_T \tag{5}$$

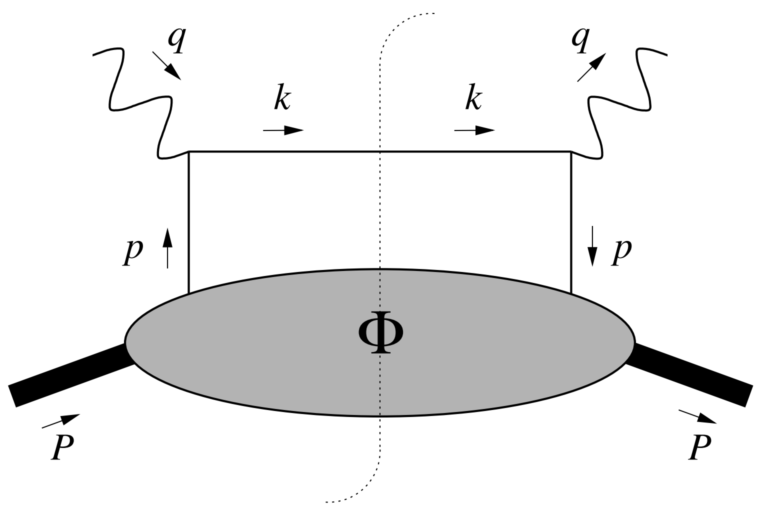
Light-cone coordinates

Q^2 much higher than any other scalar product





Correlation functions in DIS



$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2\, \frac{1}{2}\mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$

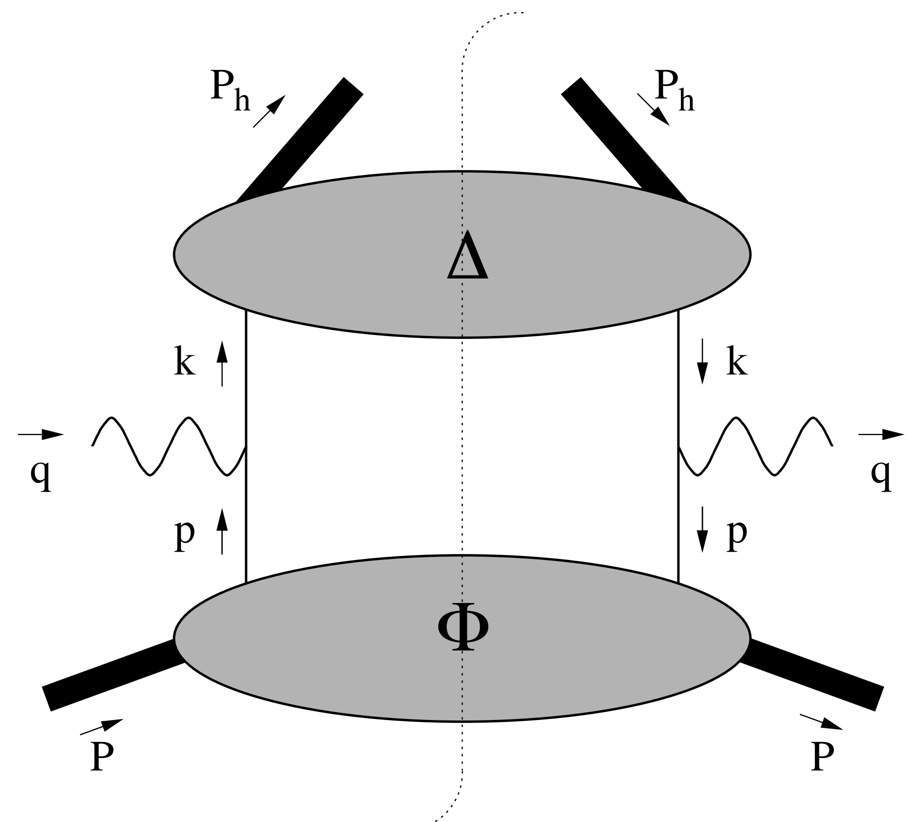
$$\Phi_{ij}(x,S) = \int d^2\boldsymbol{p}_T dp^- \;\Phi_{ij}(p,P,S)\bigg|_{p^+=xP^+}$$

$$= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\big\langle P,S\big|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,\big|P,S\big\rangle\bigg|_{\xi^+=\boldsymbol{\xi}_T=0}$$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\big\langle P,S\big|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,\big|P,S\big\rangle$$

Correlation functions in SIDIS

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{I}\Big[\mathrm{Tr}(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu)\Big]$$



$$\mathcal{I}\Big[\cdots\Big] \equiv \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T-\frac{\boldsymbol{P}_{h\perp}}{z}-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big]$$

Only at low transverse momentum $\boldsymbol{P}_{h\perp}^2 \ll Q^2$

Integrated vs unintegrated correlators

$$\begin{aligned}\Phi_{ij}(x,S) &= \int d^2\boldsymbol{p}_T\ \Phi_{ij}(x,\boldsymbol{p}_T) \\ &= \int \frac{d\xi^-}{2\pi}\ e^{ip\cdot\xi}\langle P,S\big|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,\big|P,S\rangle\Big|_{\xi^+=\boldsymbol{\xi}_T=0}\end{aligned}$$

$$\begin{aligned}\Phi_{ij}(x,\boldsymbol{p}_T,S) &= \int dp^-\ \Phi(p,P,S)\Big|_{p^+=xP^+} \\ &= \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\ e^{ip\cdot\xi}\langle P,S\big|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,\big|P,S\rangle\Big|_{\xi^+=0}\end{aligned}$$

Correlation functions

$$p,\ P,\ S$$

$$\mathbf{1},\ \gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ i\sigma^{\mu\nu}\gamma_5 \qquad\qquad \sigma^{\mu\nu} \equiv \frac{i}{2}\,[\gamma^\mu, \gamma^\nu]\,.$$

$$\text{Hermiticity:} \qquad \Phi(p,P,S) = \gamma^0\,\Phi^\dagger(p,P,S)\,\gamma^0, \qquad \text{(1a)}$$

$$\text{parity:} \qquad \Phi(p,P,S) = \gamma^0\,\Phi(\tilde{p},\tilde{P},-\tilde{S})\,\gamma^0 \qquad \text{(1b)}$$

$$\tilde{p}^\nu = \delta^{\nu\mu} p_\mu$$

For an unpolarized target, the most general decomposition is

$$\Phi(p,P) = M\,A_1\,\mathbf{1} + A_2\ \not{P} + A_3\ \not{p} + \frac{A_4}{M}\,\sigma_{\mu\nu}P^\mu p^\nu$$

where the amplitudes A_i are real scalar functions $A_i = A_i(p\cdot P, p^2)$ with dimension $1/[m]^4$.

Parton distribution functions

If we keep only the leading terms in $1/P^+$ (leading twist)

$$\Phi(p,P) \approx P^+ \left(A_2 + x A_3\right) \not{n}_+ + P^+ \frac{i}{2M} \left[\not{n}_+, \not{p}_T\right] A_4,$$

$$\Phi(x, p_T) \equiv \int dp^- \Phi(p,P) = \frac{1}{2}\left\{ f_1 \not{n}_+ + i h_1^\perp \frac{\left[\not{p}_T, \not{n}_+\right]}{2M} \right\}.$$

Here we introduced the parton distribution functions

$$f_1(x, p_T^2) = 2P^+ \int dp^- \left(A_2 + x A_3\right), \qquad h_1^\perp(x, p_T^2) = 2P^+ \int dp^- \left(-A_4\right).$$

Parton distribution functions

$$\Phi^{[\Gamma]} = \frac{1}{2}\mathrm{Tr}[\Phi\,\Gamma]$$

$$\Phi^{[\gamma^+]} = f_1(x, p_T^2)\,,$$

$$\Phi^{[i\sigma^{\alpha+}\gamma_5]} = -\frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, h_1^{\perp}(x, p_T^2)$$

Collinear PDFs and TMDs

$$\Phi(x) = \frac{1}{2}\left\{ f_1\, \not{n}_+ + S_L\, g_1 \gamma_5\, \not{n}_+ + h_1\, \frac{\left[\not{S}_T,\, \not{n}_+\right]\gamma_5}{2} \right\}$$

$$\Phi(x, p_T) = \frac{1}{2}\left\{ f_1\, \not{n}_+ - f_{1T}^{\perp}\, \frac{\epsilon_T^{\rho\sigma}\, p_{T\rho} S_{T\sigma}}{M}\, \not{n}_+ + S_L\, g_{1L} \gamma_5\, \not{n}_+ - g_{1T}\, \frac{p_T \cdot S_T}{M} \gamma_5\, \not{n}_+ \right.$$
$$\left. + h_{1T}\, \frac{\left[\not{S}_T,\, \not{n}_+\right]\gamma_5}{2} + h_{1s}^{\perp}\, \frac{\left[\not{p}_T,\, \not{n}_+\right]\gamma_5}{2M} + i\, h_1^{\perp}\, \frac{\left[\not{p}_T,\, \not{n}_+\right]}{2M} \right\}$$

Dirac matrices: an unusual representation

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\gamma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Good/Bad and Right/Left projectors

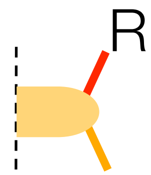
$$\mathcal{P}^+ = \gamma^-\gamma^+/2, \qquad \mathcal{P}_R = (1+\gamma_5)/2,$$
$$\mathcal{P}^- = \gamma^+\gamma^-/2, \qquad \mathcal{P}_L = (1-\gamma_5)/2$$

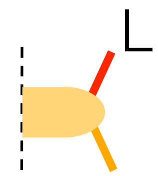
Good

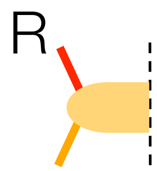
$$\mathcal{P}_R\mathcal{P}^+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathcal{P}_L\mathcal{P}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

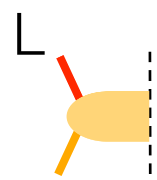
$$\mathcal{P}_R\mathcal{P}^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \mathcal{P}_L\mathcal{P}^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The correlator as probability density matrix







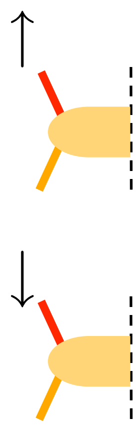


$$\Phi\gamma^0 = \begin{pmatrix} f_1 & i\frac{(p_x+ip_y)}{M}h_1^\perp & 0 & 0 \\ -i\frac{(p_x-ip_y)}{M}h_1^\perp & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \,|P\rangle\langle P|\, \psi_j^\dagger$$

- Probabilistic interpretation
- Positivity bounds
- Need of orbital angular momentum

AB, M. Boglione, A. Henneman, P.J. Mulders, PRL 85 (00)

In *x*-transversity basis



$$\Phi\gamma^0 = \begin{pmatrix} f_1 - \frac{p_y}{M} h_1^\perp & i\frac{p_x}{M} h_1^\perp & 0 & 0 \\ -i\frac{p_x}{M} h_1^\perp & f_1 + \frac{p_y}{M} h_1^\perp & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \big| P \big\rangle \big\langle P \big| \psi_j^\dagger$$

TMDs and their probabilistic interpretation

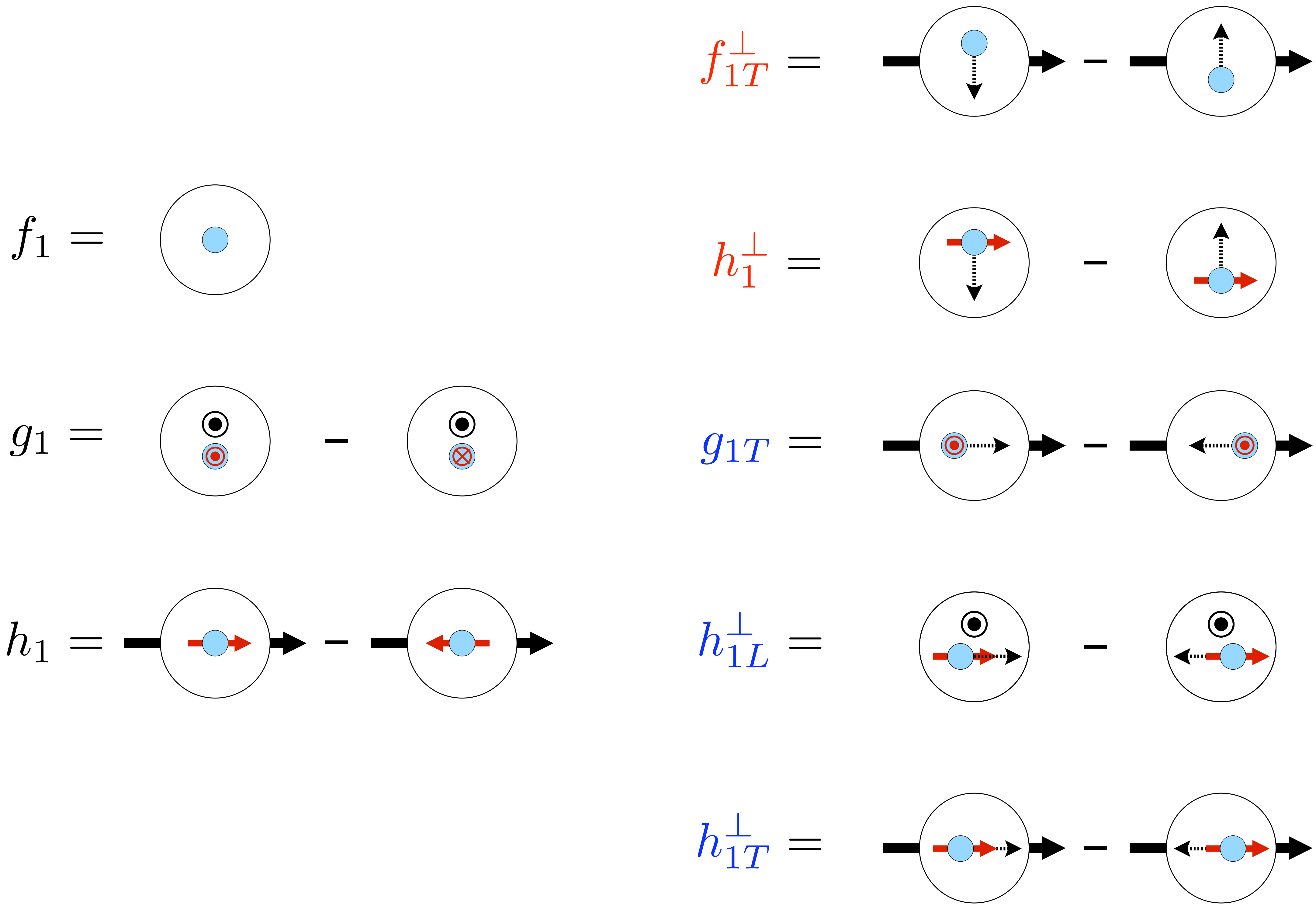
quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

TMDs in black survive transverse-momentum integration
TMDs in red are T-odd

TMDs and their probabilistic interpretation



A short note on g_T

$$\Phi(x, S_x)\gamma^0 = \frac{1}{2}\left\{f_1\gamma^- + S_L\, g_1\gamma_5\gamma^- - h_1\, S_T\gamma_5\gamma^1\gamma^-\right\}\gamma^0$$
$$+ \frac{M}{2P^+}\left\{e - g_T\, S_T\gamma_5\gamma^1 + h_L\, S_L\, \frac{[\gamma^-, \gamma^+]\gamma_5}{2}\right\}\gamma^0$$

$$\gamma_5\gamma^-\gamma^0 = \sqrt{2}\begin{pmatrix}\sigma^3 & 0\\ 0 & 0\end{pmatrix}$$
$$\gamma_5\gamma^1\gamma^-\gamma^0 = \sqrt{2}\begin{pmatrix}\sigma^1 & 0\\ 0 & 0\end{pmatrix}$$
$$\gamma_5\gamma^1\gamma^0 = \sqrt{2}\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}$$

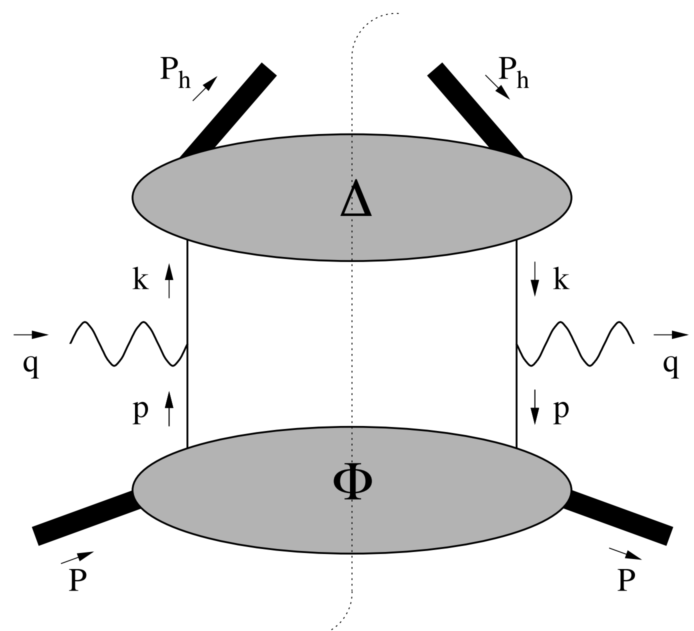
My impression is that it's relevant for the Bakker-Trueman-Leader transverse angular momentum sum-rule

http://www.ts.infn.it/eventi/transversitySR/

Correlation functions in SIDIS

$$\frac{d^6\sigma}{dx_B dy dz_h d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2\, y}{2\, z_h\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

$$2MW^{\mu\nu}(q, P, S, P_h) = 2z_h\, \mathcal{I}\Big[\mathrm{Tr}(\Phi(x_B, \boldsymbol{p}_T, S)\, \gamma^\mu\, \Delta(z_h, \boldsymbol{k}_T)\, \gamma^\nu)\Big]$$



$$\mathcal{I}\Big[\cdots\Big] \equiv \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\, \delta^{(2)}\Big(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T\Big)\, \Big[\cdots\Big] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\, \delta^{(2)}\Big(\boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z} - \boldsymbol{k}_T\Big)\, \Big[\cdots\Big]$$

Only at low transverse momentum $\boldsymbol{P}_{h\perp}^2 \ll Q^2$

Structure functions

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+ S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}\right.$$

$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$\left. + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right.$$

$$\left. + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\Bigg\}$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Unpolarized sector

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\,\frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\,\frac{\tilde{H}}{z}\right)\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\big)\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\big) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\,h_1^{\perp}\,H_1^{\perp}\right],$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,\delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\,w(\boldsymbol{p}_T,\boldsymbol{k}_T)\,f^a(x,p_T^2)\,D^a(z,k_T^2),$$

Longitudinally polarized beam or/and target

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe\,H_1^{\perp}+\frac{M_h}{M}\,f_1\frac{\tilde{G}^{\perp}}{z}\Big)+\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg^{\perp}D_1+\frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_L H_1^{\perp}+\frac{M_h}{M}\,g_{1L}\frac{\tilde{G}^{\perp}}{z}\Big)+\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_L^{\perp}D_1-\frac{M_h}{M}\,h_{1L}^{\perp}\frac{\tilde{H}}{z}\Big)\Big],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\Big[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}h_{1L}^{\perp}H_1^{\perp}\Big],$$

$$F_{LL} = \mathcal{C}\big[g_{1L}D_1\big],$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe_L H_1^{\perp}-\frac{M_h}{M}\,g_{1L}\frac{\tilde{D}^{\perp}}{z}\Big)-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg_L^{\perp}D_1+\frac{M_h}{M}\,h_{1L}^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

Transversely polarized beam

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right) + \boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right) - 4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2 M_h}\, h_{1T}^{\perp} H_1^{\perp}\right],$$

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{\left(x f_T D_1 - \frac{M_h}{M}\, h_1 \frac{\tilde{H}}{z}\right) - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2M M_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\left(x f_T^{\perp} D_1 - \frac{M_h}{M}\, h_{1T}^{\perp}\frac{\tilde{H}}{z}\right) - \frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2M M_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) + \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

Trasversely pol. target and long. pol. beam

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\Big\{-\Big(x g_T D_1 + \frac{M_h}{M}\,h_1 \frac{\tilde{E}}{z}\Big) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\Big) + \Big(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\Big)\Big]\Big\},$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\Big\{-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\Big(x g_T^{\perp} D_1 + \frac{M_h}{M}\,h_{1T}^{\perp}\frac{\tilde{E}}{z}\Big) + \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\Big) - \Big(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\Big)\Big]\Big\}$$