Transverse structure of the nucleon
Part 2: Theory background

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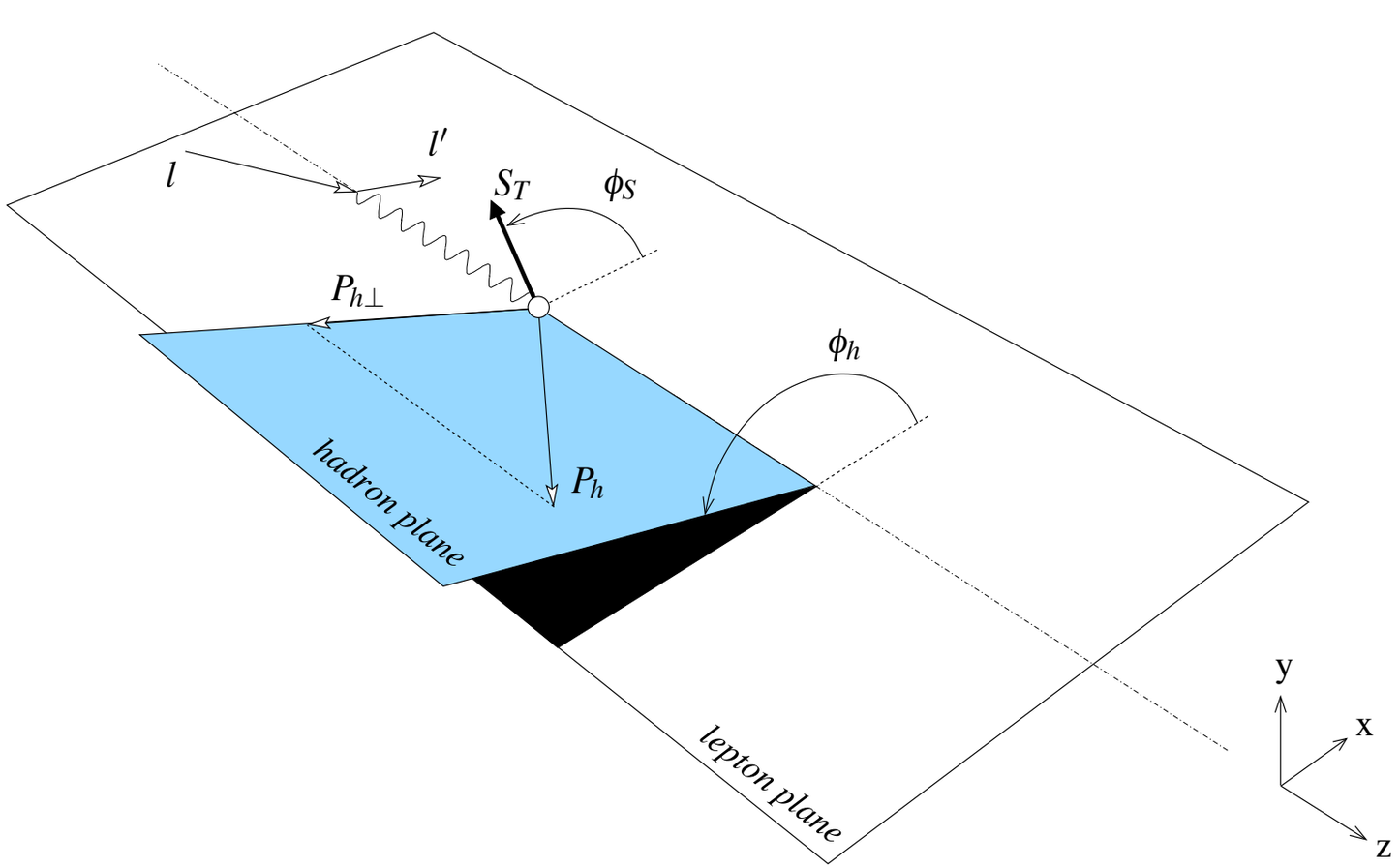
DIS and structure functions

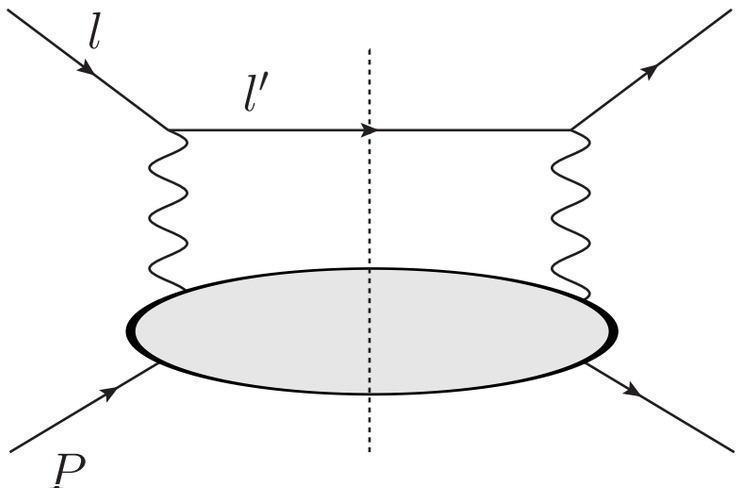
Deep inelastic scattering

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$

$$x_B = \frac{Q^2}{2\,P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z_h = \frac{P\cdot P_h}{P\cdot q}.$$



Inclusive DIS



$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

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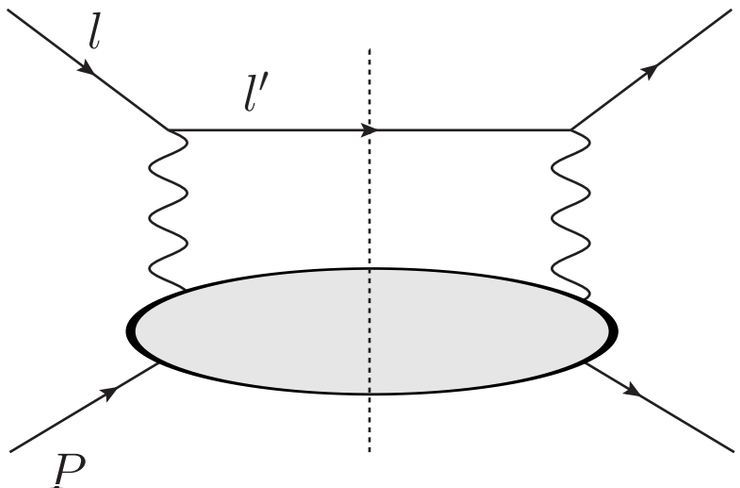

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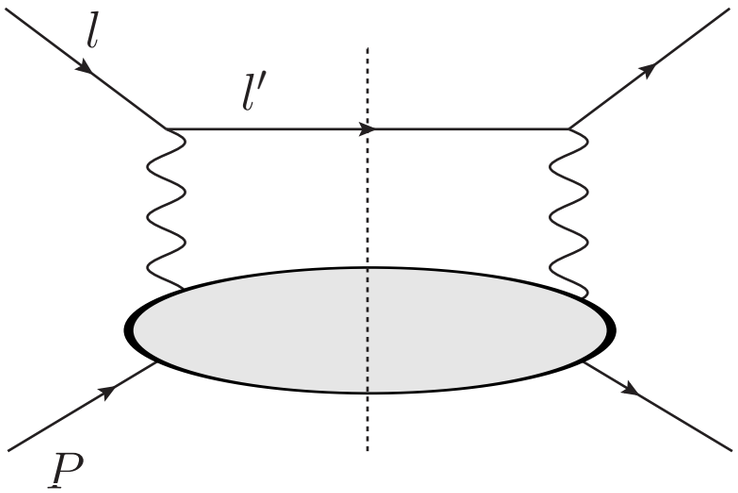
Single-photon-exchange approximation

Inclusive DIS



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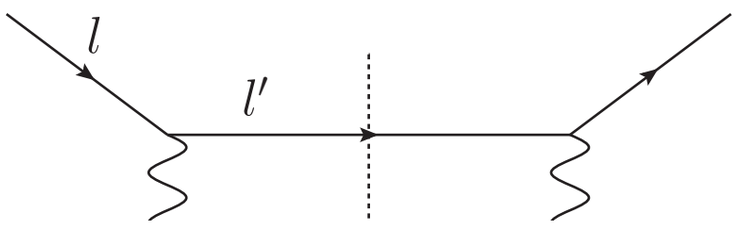
Single-photon-exchange approximation



$$\begin{aligned}
L_{\mu\nu} &= \sum_{\lambda'_e} \Big(\bar{u}(l', \lambda'_e)\, \gamma_\mu\, u(l, \lambda_e)\Big)^* \Big(\bar{u}(l', \lambda'_e)\, \gamma_\nu\, u(l, \lambda_e)\Big) \\
&= -Q^2 g_{\mu\nu} + 2\left(l_\mu l'_\nu + l'_\mu l_\nu\right) + 2i\, \lambda_e\, \epsilon_{\mu\nu\rho\sigma}\, l^\rho l'^\sigma \\
&= \frac{2Q^2}{y^2}\Big[-\Big(1 - y + \frac{y^2}{2}\Big) g_{\perp\mu\nu} + 2(1-y)\hat{t}_\mu \hat{t}_\nu \\
&\quad + 2(1-y)\Big(\hat{l}_\mu \hat{l}_\nu + \frac{1}{2} g_{\perp\mu\nu}\Big) \\
&\quad - i\lambda_e y\Big(1 - \frac{y}{2}\Big)\, \epsilon_{\perp\mu\nu} - i\lambda_e y\sqrt{1-y}\, \hat{t}_{[\mu}\epsilon_{\perp\nu]\rho}\hat{l}^\rho + \ldots\Big]
\end{aligned}$$

Lepton tensor

$$l^\mu = \left(\frac{(2-y)Q}{2y}, \ \frac{\sqrt{1-y}\,Q}{y}, \ 0, \ \frac{Q}{2} \right),$$
$$l'^\mu = \left(\frac{(2-y)Q}{2y}, \ \frac{\sqrt{1-y}\,Q}{y}, \ 0, \ -\frac{Q}{2} \right)$$

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$$L_{\mu\nu}(\lambda_e = 0) = \frac{2Q^2}{y^2} \begin{pmatrix} 2(1-y) & \dots & 0 & 0 \\ \dots & \left(1 - y + \frac{y^2}{2}\right) + (1-y) & 0 & 0 \\ 0 & 0 & \left(1 - y + \frac{y^2}{2}\right) - (1-y) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Lepton tensor

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$$\epsilon_x^\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \epsilon_y^\mu = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \epsilon_L^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Helicity vs transversity / circular vs linear

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\chi_\uparrow = \frac{1}{\sqrt{2}}(\chi_+ + e^{i\phi_s}\chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi_s} \end{pmatrix} \qquad \chi_\downarrow = \frac{1}{\sqrt{2}}(\chi_+ - e^{i(\phi_s+\pi)}\chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(\phi_s+\pi)} \end{pmatrix}$$

Helicity vs transversity / circular vs linear

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$$\epsilon_+^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}, \qquad \epsilon_-^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix},$$

$$\epsilon_x^\mu = \frac{1}{\sqrt{2}}(\epsilon_+^\mu + \epsilon_-^\mu) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \epsilon_y^\mu = \frac{i}{\sqrt{2}}(\epsilon_+^\mu - \epsilon_-^\mu) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Helicity conservation

$$\bar{u}_+(-p_z)\gamma^\mu u_+(p_z) \sim (0,1,i,0) \qquad \bar{u}_-(-p_z)\gamma^\mu u_+(p_z) = 0$$

$$\bar{u}^\uparrow(-p_z)\gamma^\mu u^\uparrow(p_z) \sim (0,\cos\phi_s,-\sin\phi_s,0) \qquad \bar{u}^\downarrow(-p_z)\gamma^\mu u^\uparrow(p_z) = 0$$

Structure functions

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

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$$2MW^{\mu\nu} = \frac{1}{x}\Big[-g_\perp^{\mu\nu} F_{UU,T} + \hat{t}^\mu \hat{t}^\nu F_{UU,L} + iS_L \epsilon_\perp^{\mu\nu} F_{LL} - i\hat{t}^{[\mu} \epsilon_\perp^{\nu]\rho} S_\rho F_{LT}^{\cos\phi_S}\Big]$$

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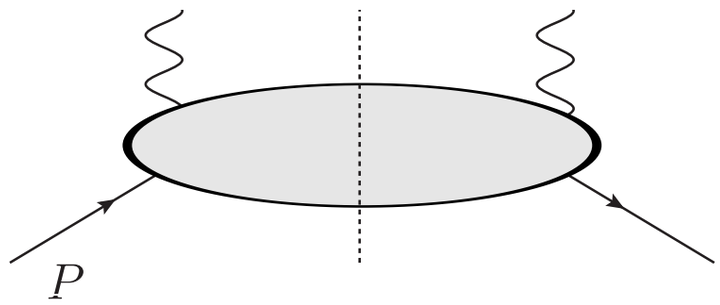

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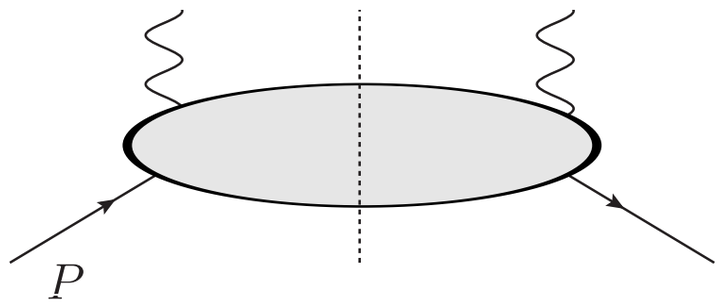

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{\left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1-y)\, F_{UU,L} + S_L \lambda_e\, y\left(1 - \frac{y}{2}\right)\, F_{LL}\right.$$
$$\left. + |\boldsymbol{S}_T| \lambda_e\, y\sqrt{1-y}\, \cos\phi_S\, F_{LT}^{\cos\phi_s}\right\}$$

Semi-inclusive DIS

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

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$$\frac{2E_h d^6\sigma}{d^3 P_h dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

Semi-inclusive DIS

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$$\frac{d^6\sigma}{dx_B dy dz_h d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2\, y}{2\, z_h\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

Structure functions

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] + S_T\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\}
\end{aligned}
$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Structure functions

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&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}\right. \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad \left. + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right. \\
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\end{aligned}
$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Light-cone coordinates

Light-cone vectors will be indicated as

$$a^\mu = \left[a^-,\ a^+,\ \boldsymbol{a}_T\right] = \left[\frac{a^0 - a^3}{\sqrt{2}},\ \frac{a^0 + a^3}{\sqrt{2}},\ a^1,\ a^2\right]. \tag{1}$$

The dot-product in light-cone components is

$$a \cdot b = a^+ b^- + a^- b^+ - \boldsymbol{a}_T \cdot \boldsymbol{b}_T \tag{2}$$

The light-cone decomposition of a vector can be written in a Lorentz covariant fashion using two light-like vectors n_+ and n_- satisfying $n_\pm^2 = 0$ and $n_+ \cdot n_- = 1$ and promoting $\boldsymbol{a}_T$ to a four-vector $a_T^\mu = [0, 0, \boldsymbol{a}_T]$ so that

$$a^\mu = a^+ n_+^\mu + a^- n_-^\mu + a_T^\mu, \tag{3}$$

where

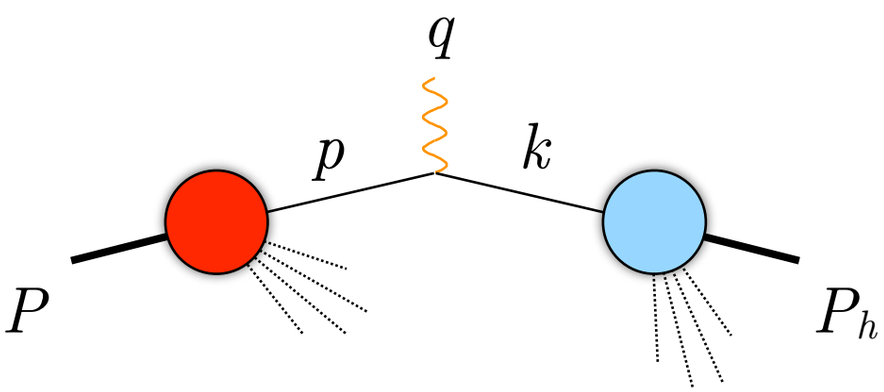
$$a^+ = a \cdot n_-, \qquad a^- = a \cdot n_+, \qquad a_T \cdot n_+ = a_T \cdot n_- = 0. \tag{4}$$

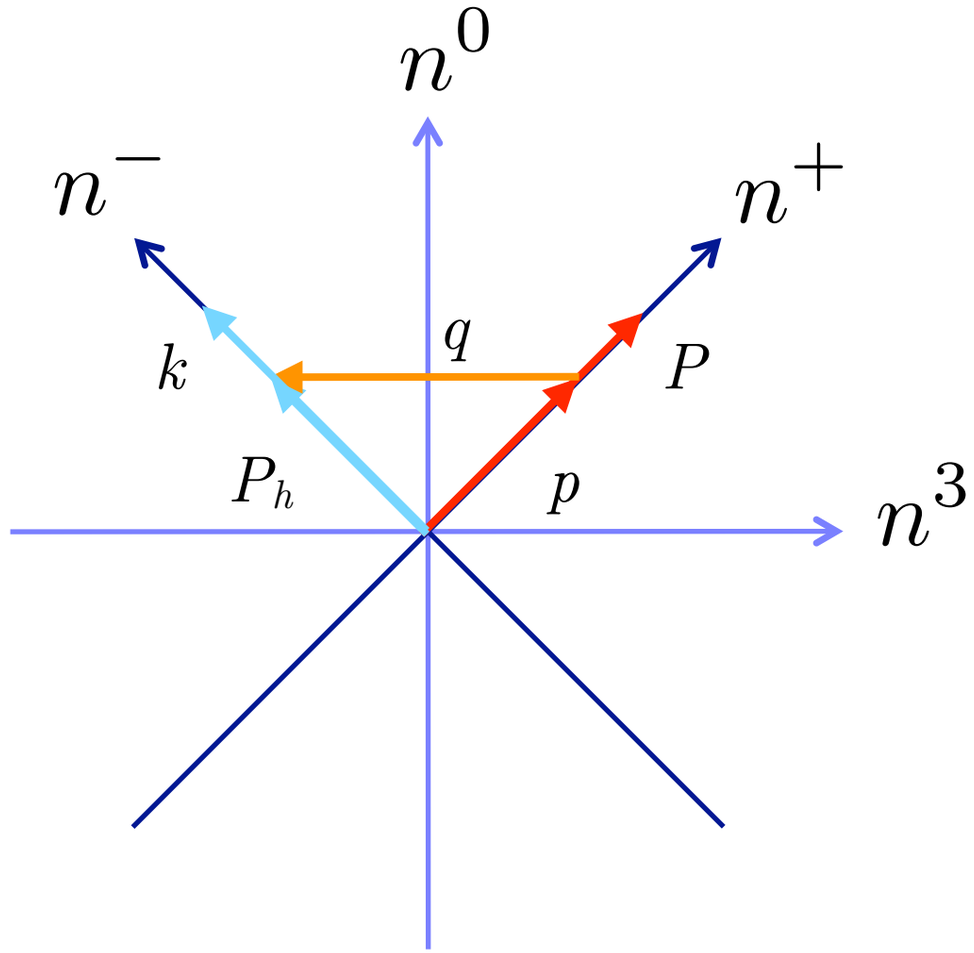
Note that

$$a_T \cdot b_T = -\boldsymbol{a}_T \cdot \boldsymbol{b}_T \tag{5}$$

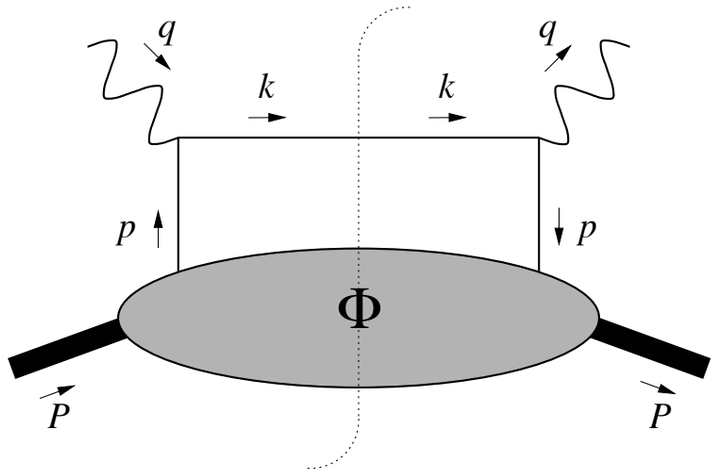
Light-cone coordinates

Q^2 much higher than any other scalar product





Correlation functions in DIS



$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2 \, \frac{1}{2} \mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$

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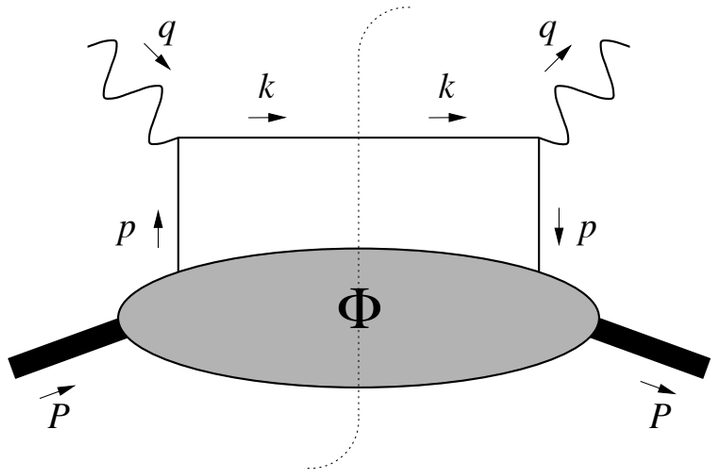

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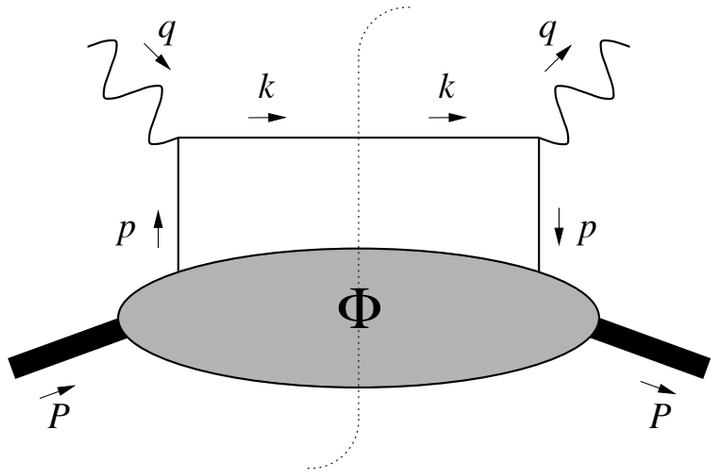
$$\begin{aligned}\Phi_{ij}(x,S) &= \int d^2\boldsymbol{p}_T dp^- \; \Phi_{ij}(p,P,S)\Big|_{p^+=xP^+} \\ &= \int \frac{d\xi^-}{2\pi} \; e^{ip\cdot\xi} \left\langle P,S\right| \overline{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi) \left|P,S\right\rangle\Big|_{\xi^+=\boldsymbol{\xi}_T=0}\end{aligned}$$

Correlation functions in DIS



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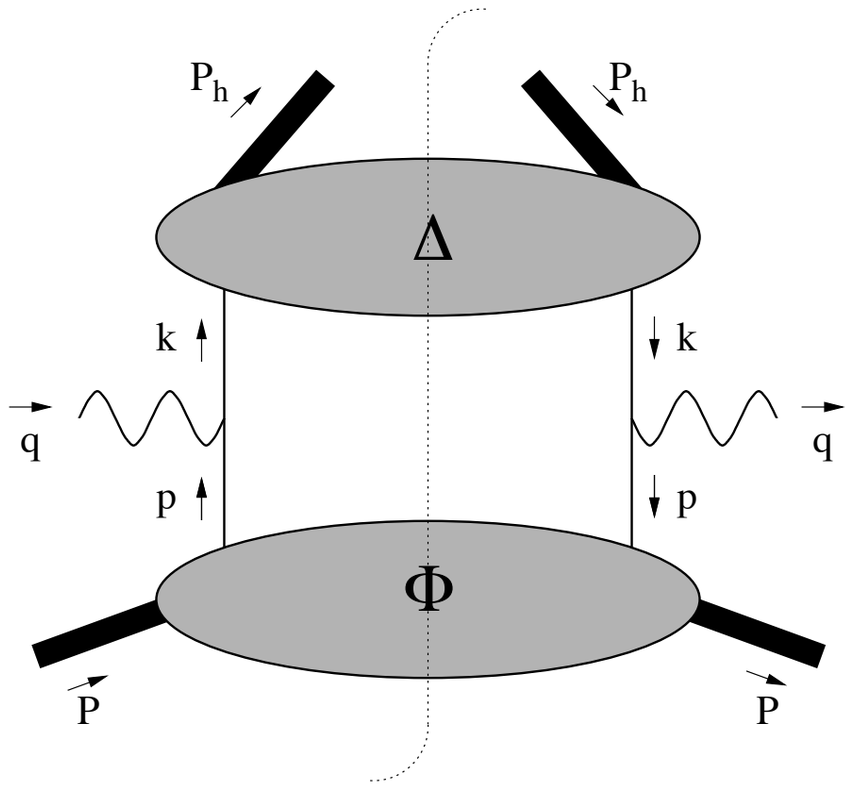
$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi \; e^{ip\cdot\xi}\left\langle P,S\right|\overline{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\left|P,S\right\rangle$$

Correlation functions in SIDIS

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{I}\Big[\mathrm{Tr}(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu)\Big]$$



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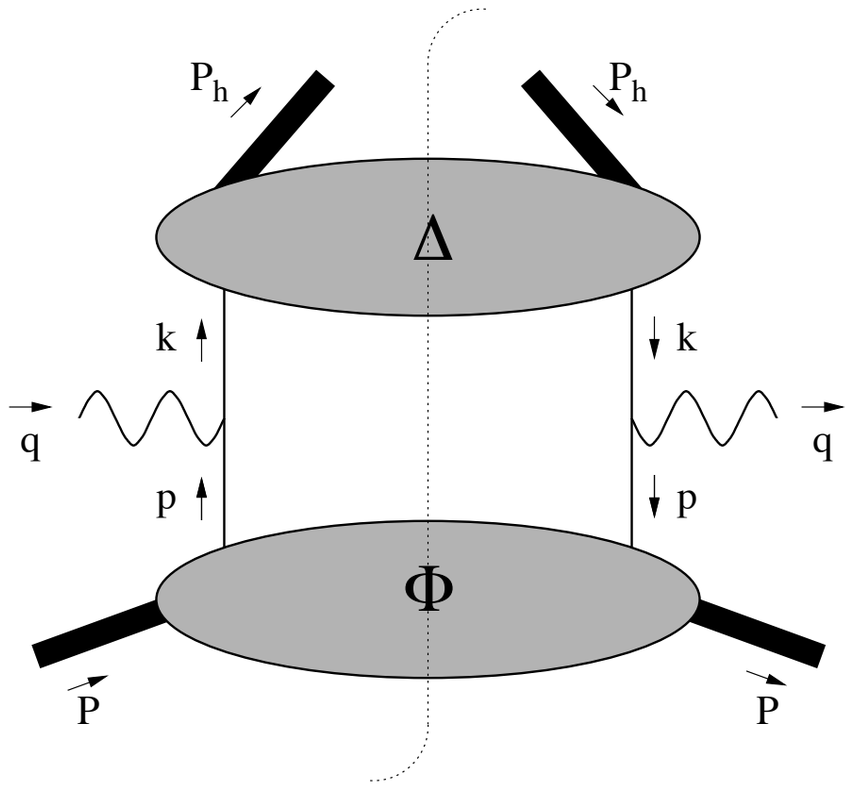

$$\mathcal{I}\Big[\cdots\Big] \equiv \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T-\frac{\boldsymbol{P}_{h\perp}}{z}-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big]$$

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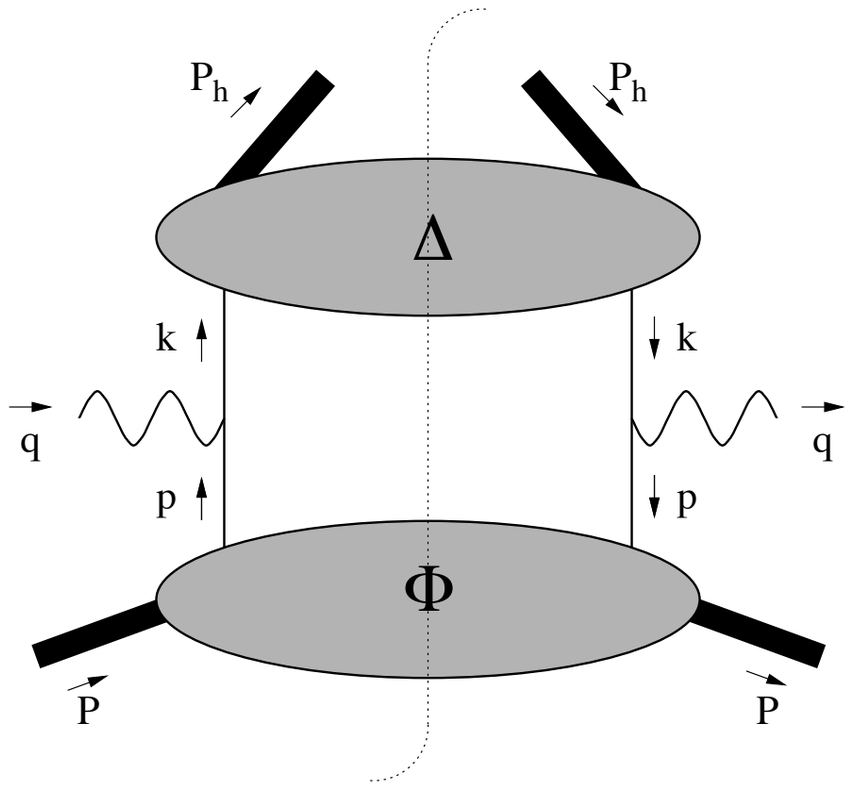

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Only at low transverse momentum $\boldsymbol{P}_{h\perp}^2 \ll Q^2$

Integrated vs unintegrated correlators

$$\Phi_{ij}(x,S) = \int d^2\boldsymbol{p}_T\ \Phi_{ij}(x,\boldsymbol{p}_T)$$

$$= \int \frac{d\xi^-}{2\pi}\ e^{ip\cdot\xi} \left\langle P,S\right| \overline{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi) \left|P,S\right\rangle \Big|_{\xi^+=\boldsymbol{\xi}_T=0}$$

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$$\Phi_{ij}(x,\boldsymbol{p}_T,S) = \int dp^-\ \Phi(p,P,S)\Big|_{p^+=xP^+}$$
$$= \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\ e^{ip\cdot\xi}\left\langle P,S\right|\overline{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\left|P,S\right\rangle\Big|_{\xi^+=0}$$

Correlation functions

$p,\ P,\ S$

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\end{aligned}$$

$$\tilde{p}^\nu = \delta^{\nu\mu} p_\mu$$

For an unpolarized target, the most general decomposition is

$$\Phi(p,P) = M\,A_1\,\mathbf{1} + A_2\ \not{P} + A_3\ \not{p} + \frac{A_4}{M}\,\sigma_{\mu\nu}P^\mu p^\nu$$

where the amplitudes A_i are real scalar functions $A_i = A_i(p\cdot P, p^2)$ with dimension $1/[m]^4$.

Parton distribution functions

Parton distribution functions

If we keep only the leading terms in $1/P^+$ (leading twist)

$$\Phi(p,P) \approx P^+ \left(A_2 + xA_3\right) \not{n}_+ + P^+ \frac{i}{2M} \left[\not{n}_+, \not{p}_T\right] A_4,$$

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Here we introduced the parton distribution functions

$$f_1(x,p_T^2) = 2P^+ \int dp^- \left(A_2 + xA_3\right), \qquad h_1^\perp(x,p_T^2) = 2P^+ \int dp^- \left(-A_4\right).$$

Parton distribution functions

$$\Phi^{[\Gamma]} = \frac{1}{2}\mathrm{Tr}[\Phi\,\Gamma]$$

$$\Phi^{[\gamma^+]} = f_1(x, p_T^2)\,,$$

$$\Phi^{[i\sigma^{\alpha+}\gamma_5]} = -\frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, h_1^\perp(x, p_T^2)$$

Collinear PDFs and TMDs

$$\Phi(x,p_T) = \frac{1}{2}\left\{ f_1\,\not{n}_+ - f_{1T}^{\perp}\,\frac{\epsilon_T^{\rho\sigma}\,p_{T\rho}S_{T\sigma}}{M}\,\not{n}_+ + S_L\,g_{1L}\gamma_5\,\not{n}_+ - g_{1T}\,\frac{p_T\cdot S_T}{M}\gamma_5\,\not{n}_+ \right.$$
$$\left. + h_{1T}\,\frac{\left[\not{S}_T,\,\not{n}_+\right]\gamma_5}{2} + h_{1s}^{\perp}\,\frac{\left[\not{p}_T,\,\not{n}_+\right]\gamma_5}{2M} + i\,h_1^{\perp}\,\frac{\left[\not{p}_T,\,\not{n}_+\right]}{2M} \right\}$$

Collinear PDFs and TMDs

$$\Phi(x) = \frac{1}{2}\left\{ f_1 \, \not{n}_+ + S_L \, g_1 \gamma_5 \, \not{n}_+ + h_1 \, \frac{\left[\not{S}_T, \, \not{n}_+ \right] \gamma_5}{2} \right\}$$

$$\Phi(x, p_T) = \frac{1}{2}\left\{ f_1 \, \not{n}_+ - f_{1T}^{\perp} \, \frac{\epsilon_T^{\rho\sigma} \, p_{T\rho} S_{T\sigma}}{M} \, \not{n}_+ + S_L \, g_{1L} \gamma_5 \, \not{n}_+ - g_{1T} \, \frac{p_T \cdot S_T}{M} \gamma_5 \, \not{n}_+ \right.$$
$$\left. + h_{1T} \, \frac{\left[\not{S}_T, \, \not{n}_+ \right] \gamma_5}{2} + h_{1s}^{\perp} \, \frac{\left[\not{p}_T, \, \not{n}_+ \right] \gamma_5}{2M} + i \, h_1^{\perp} \, \frac{\left[\not{p}_T, \, \not{n}_+ \right]}{2M} \right\}$$

Dirac matrices: an unusual representation

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\gamma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Good/Bad and Right/Left projectors

$$\mathcal{P}^+ = \gamma^-\gamma^+/2, \qquad \mathcal{P}_R = (1+\gamma_5)/2,$$
$$\mathcal{P}^- = \gamma^+\gamma^-/2, \qquad \mathcal{P}_L = (1-\gamma_5)/2$$

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The correlator as probability density matrix

$$\Phi\gamma^0 = \begin{pmatrix} f_1 & i\frac{(p_x+ip_y)}{M}h_1^\perp & 0 & 0 \\ -i\frac{(p_x-ip_y)}{M}h_1^\perp & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \left|P\right\rangle\left\langle P\right| \psi_j^\dagger$$

AB, M. Boglione, A. Henneman, P.J. Mulders, PRL 85 (00)

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- Probabilistic interpretation

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- Need of orbital angular momentum

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In *x*-transversity basis

$$\Phi\gamma^0 = \begin{pmatrix} f_1 - \frac{p_y}{M} h_1^\perp & i\frac{p_x}{M} h_1^\perp & 0 & 0 \\ -i\frac{p_x}{M} h_1^\perp & f_1 + \frac{p_y}{M} h_1^\perp & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \left|P\right\rangle\!\left\langle P\right| \psi_j^\dagger$$

TMDs and their probabilistic interpretation

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quark pol.

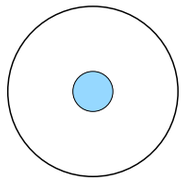
nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

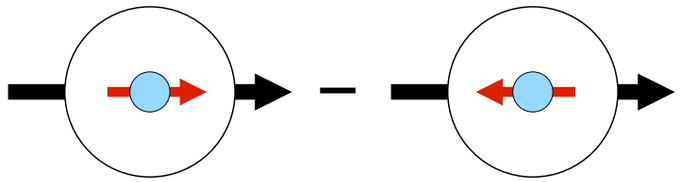
TMDs in black survive transverse-momentum integration

TMDs in red are T-odd

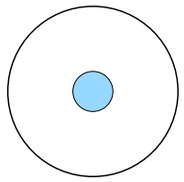
TMDs and their probabilistic interpretation

$f_1 =$ 

$g_1 =$

$h_1 =$ 

TMDs and their probabilistic interpretation

$f_1 =$ 

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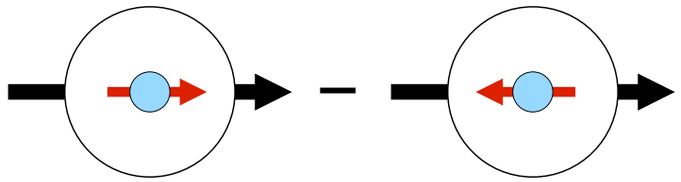
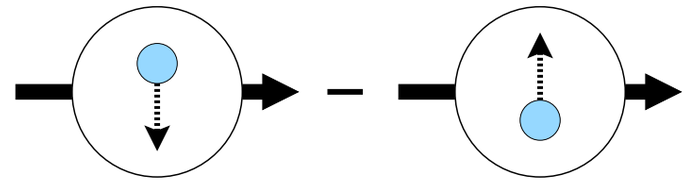
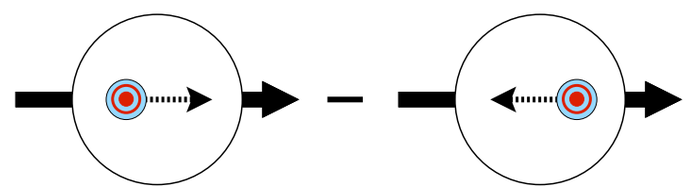
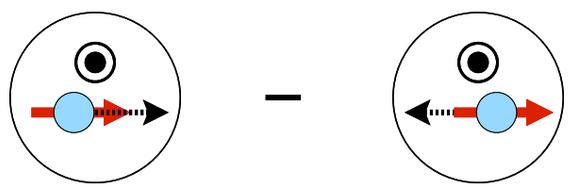
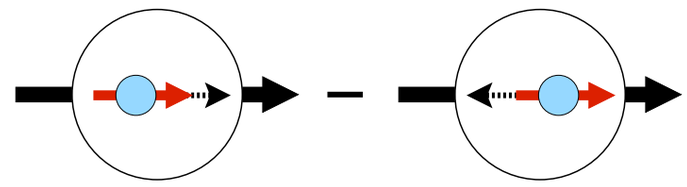
$h_1 =$ 

$f_{1T}^{\perp} =$ 

$h_1^{\perp} =$

$g_{1T} =$ 

$h_{1L}^{\perp} =$ 

$h_{1T}^{\perp} =$ 

A short note on g_T

$$\Phi(x, S_x)\gamma^0 = \frac{1}{2}\left\{ f_1\gamma^- + S_L\, g_1\gamma_5\gamma^- - h_1\, S_T\gamma_5\gamma^1\gamma^- \right\}\gamma^0$$
$$+ \frac{M}{2P^+}\left\{ e - g_T\, S_T\gamma_5\gamma^1 + h_L\, S_L\, \frac{[\gamma^-, \gamma^+]\gamma_5}{2} \right\}\gamma^0$$

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My impression is that it's relevant for the Bakker-Trueman-Leader transverse angular momentum sum-rule

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http://www.ts.infn.it/eventi/transversitySR/

Unpolarized sector

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(x h\, H_1^\perp + \frac{M_h}{M}\, f_1 \frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(x f^\perp D_1 + \frac{M_h}{M}\, h_1^\perp \frac{\tilde{H}}{z}\right)\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^\perp H_1^\perp\right],$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\, e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

Longitudinally polarized beam or/and target

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{G}^\perp}{z}\right)+\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{E}}{z}\right)\right],$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh_L H_1^\perp + \frac{M_h}{M}\,g_{1L}\frac{\tilde{G}^\perp}{z}\right)+\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf_L^\perp D_1 - \frac{M_h}{M}\,h_{1L}^\perp\frac{\tilde{H}}{z}\right)\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}\,h_{1L}^\perp H_1^\perp\right],$$

$$F_{LL} = \mathcal{C}\left[g_{1L}D_1\right],$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe_L H_1^\perp - \frac{M_h}{M}\,g_{1L}\frac{\tilde{D}^\perp}{z}\right)-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_L^\perp D_1 + \frac{M_h}{M}\,h_{1L}^\perp\frac{\tilde{E}}{z}\right)\right],$$

Transversely polarized beam

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right) + \boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right) - 4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2 M_h} h_{1T}^{\perp} H_1^{\perp}\right],$$

$$\begin{aligned} F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\mathcal{C}\Bigg\{ & \left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z}\right) \\ & - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\frac{\tilde{D}^{\perp}}{z}\right)\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\mathcal{C}\Bigg\{ & \frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2 - \boldsymbol{p}_T^2}{2M^2}\left(x f_T^{\perp} D_1 - \frac{M_h}{M} h_{1T}^{\perp}\frac{\tilde{H}}{z}\right) \\ & - \frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\frac{\tilde{G}^{\perp}}{z}\right) + \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\frac{\tilde{D}^{\perp}}{z}\right)\right]\Bigg\}, \end{aligned}$$

Trasversely pol. target and long. pol. beam

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\Bigg\{-\left(x g_T D_1 + \frac{M_h}{M}\, h_1 \frac{\tilde{E}}{z}\right) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^\perp}{z}\right) + \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp\,\frac{\tilde{G}^\perp}{z}\right)\right]\Bigg\},$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\Bigg\{-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\left(x g_T^\perp D_1 + \frac{M_h}{M}\, h_{1T}^\perp \frac{\tilde{E}}{z}\right) + \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^\perp}{z}\right) - \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp\,\frac{\tilde{G}^\perp}{z}\right)\right]\Bigg\}$$

The gauge link

Need of a gauge link

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\left\langle P,S\right|\overline{\psi}_j(0)\,\psi_i(\xi)\left|P,S\right\rangle$$

Need of a gauge link

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\left\langle P,S\right|\overline{\psi}_j(0)\,\psi_i(\xi)\left|P,S\right\rangle$$

$$\psi(\xi) \to e^{i\alpha(\xi)}\,\psi(\xi)$$

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Need of a gauge link

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\left\langle P,S\right|\bar{\psi}_j(0)\,\psi_i(\xi)\left|P,S\right\rangle$$

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$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)}\,U(\xi_1,\xi_2)\,e^{-i\alpha(\xi_2)}.$$

Need of a gauge link

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\left\langle P,S\right|\bar{\psi}_j(0)\,\psi_i(\xi)\left|P,S\right\rangle$$

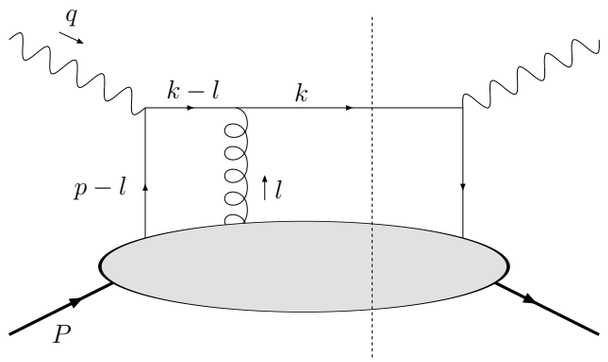
$$\psi(\xi) \to e^{i\alpha(\xi)}\,\psi(\xi)$$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\left\langle P,S\right|\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\left|P,S\right\rangle$$

$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)}\,U(\xi_1,\xi_2)\,e^{-i\alpha(\xi_2)}.$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right]$$

Birth of the gauge link

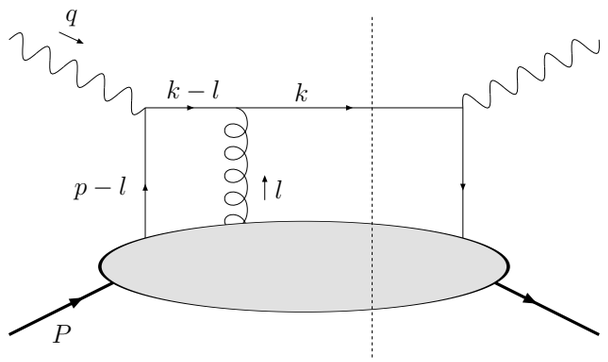


(a)

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \frac{\not{k}-\not{l}+m}{(k-l)^2-m^2+i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

Birth of the gauge link



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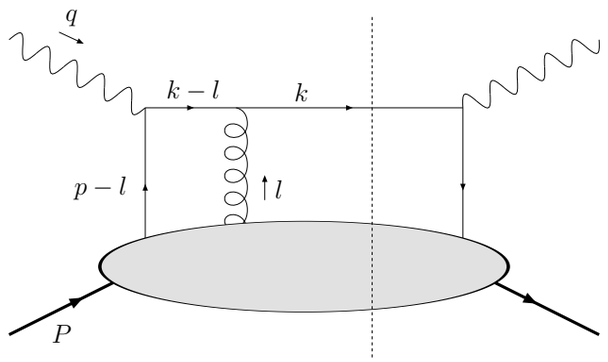

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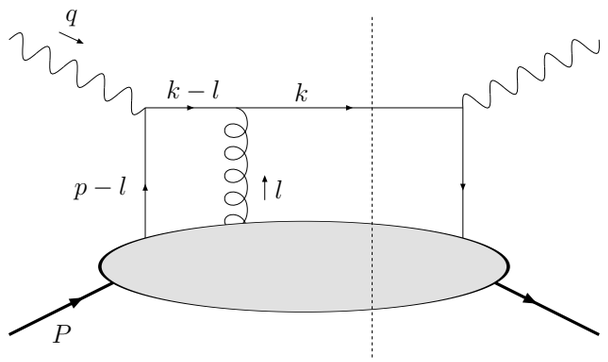
Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

Birth of the gauge link



(a)

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha\,\frac{\not k-\not l+m}{(k-l)^2-m^2+i\epsilon}\,\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

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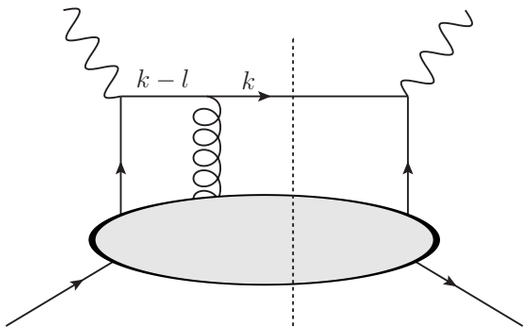
$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-}\mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\eta^+=0;\;\boldsymbol{\eta}_T=\boldsymbol{\xi}_T}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

Birth of the gauge link



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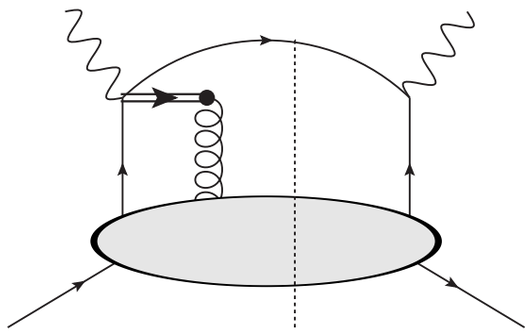
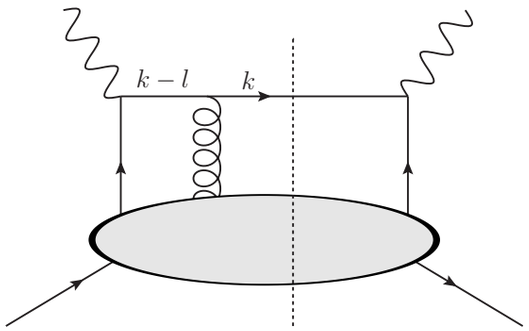


$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2\;\frac{1}{2}\mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$

$$\Phi^{(a)}(x,S) \sim \left\langle P,S\right|\overline{\psi}(0)\,(-ig)\int_{\infty^-}^{\xi^-}\mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)\left|P,S\right\rangle$$

Birth of the gauge link



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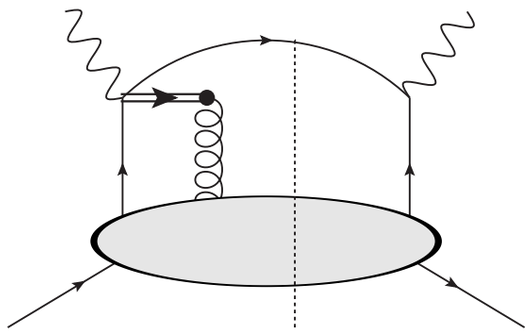

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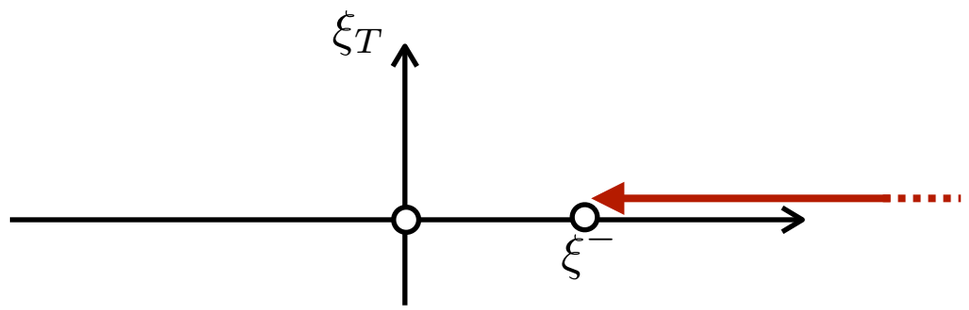
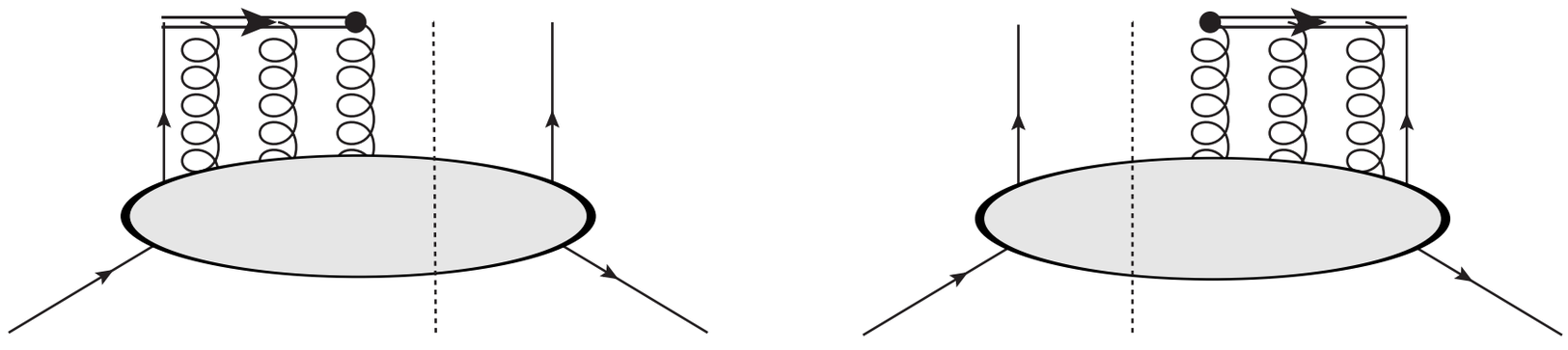
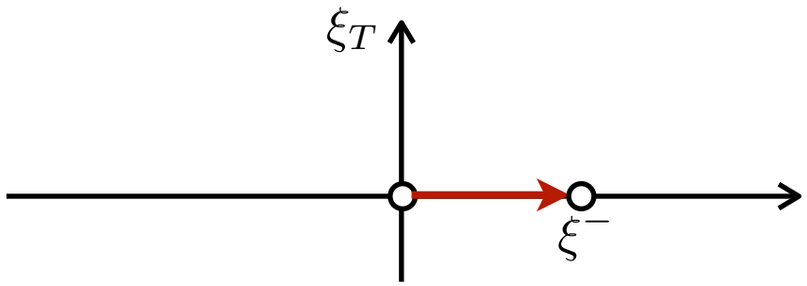


Shape of the gauge link

$$\Phi(x,S) \sim \left\langle P,S\right| \overline{\psi}(0)\, U_{[0,\infty^-]}\, U_{[\infty^-,\xi^-]} \psi(\xi) \left|P,S\right\rangle$$





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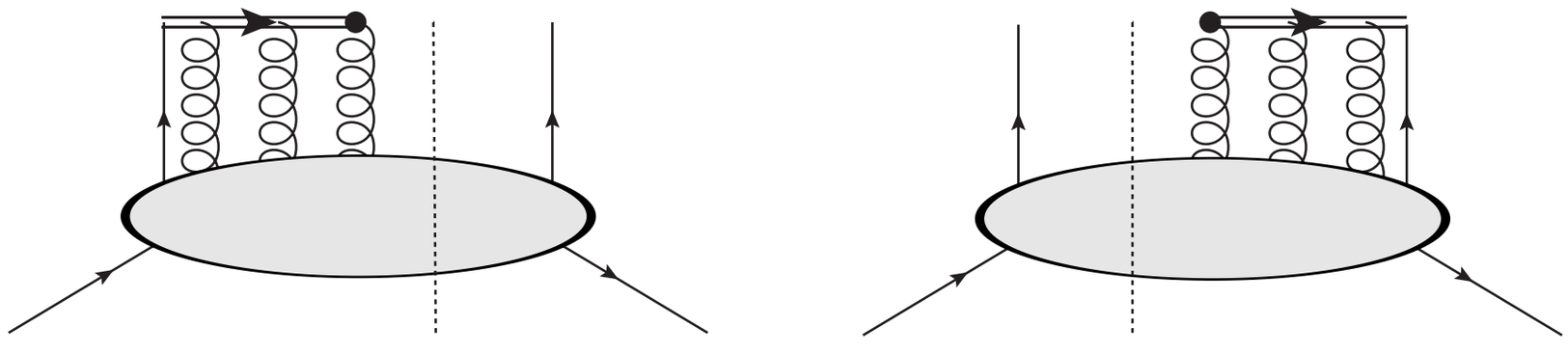
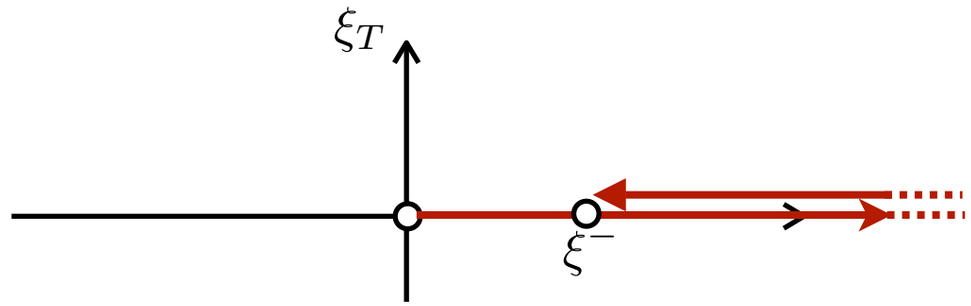
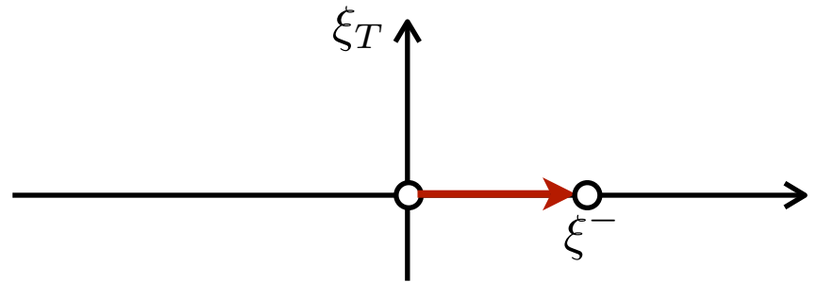
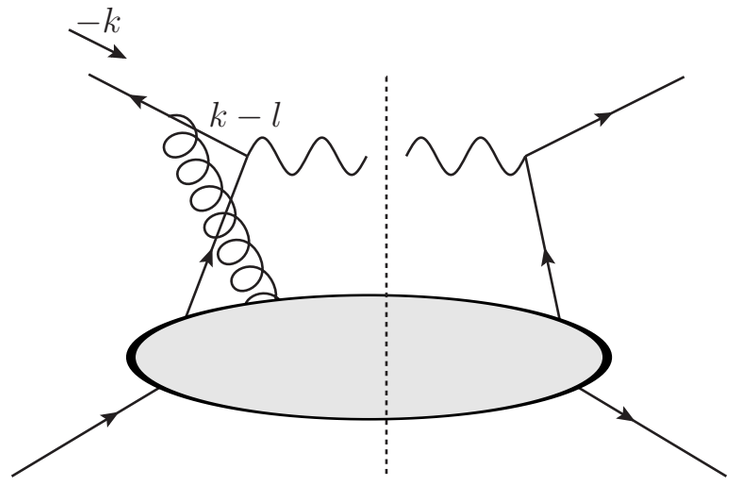






Gauge link in Drell-Yan



$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \frac{\not{k}-\not{l}+m}{(k-l)^2-m^2+i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle\Big|_{\eta^+=0}$$

$$i\frac{\not{k}-\not{l}+m}{(k-l)^2-m^2+i\epsilon} \approx i\frac{-(-k)^-\gamma^+}{2l^+(-k)^- +i\epsilon} \approx \frac{i}{2}\frac{\gamma^+}{-l^+ -i\epsilon}$$

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{-\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\eta^+=0;\;\boldsymbol{\eta}_T=\boldsymbol{\xi}_T}$$

Collins, PLB 536 (02)

Gauge link for TMDs

$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{8\pi^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi)|P\rangle \Big|_{\xi^+=0}$$

Gauge link for TMDs

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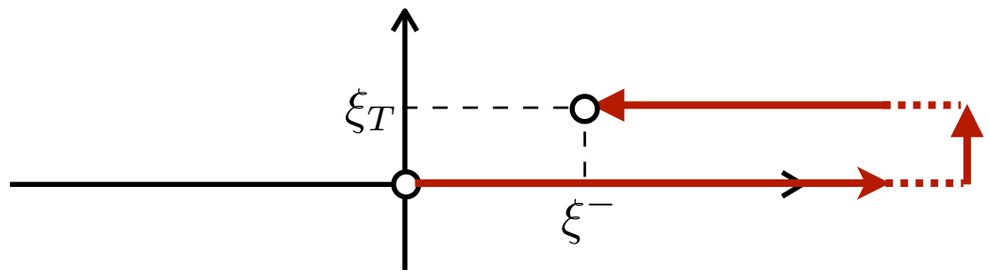
SIDIS



$U_{[+]}$

Gauge link for TMDs

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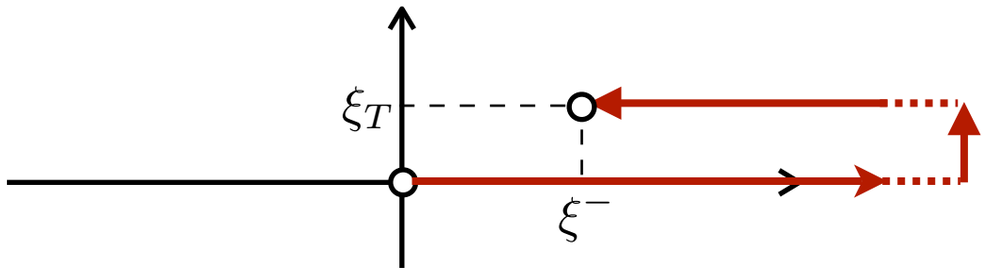
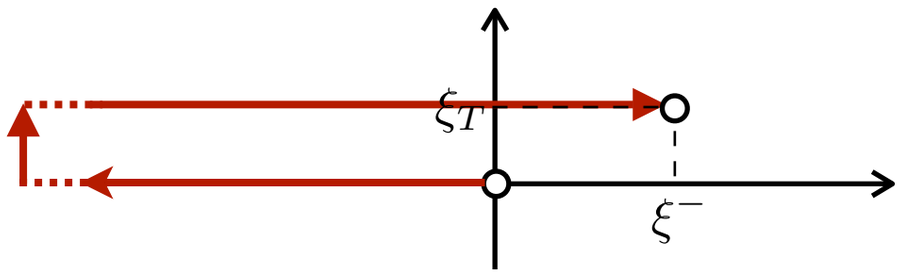
SIDIS



$U_{[+]}$

Drell-Yan

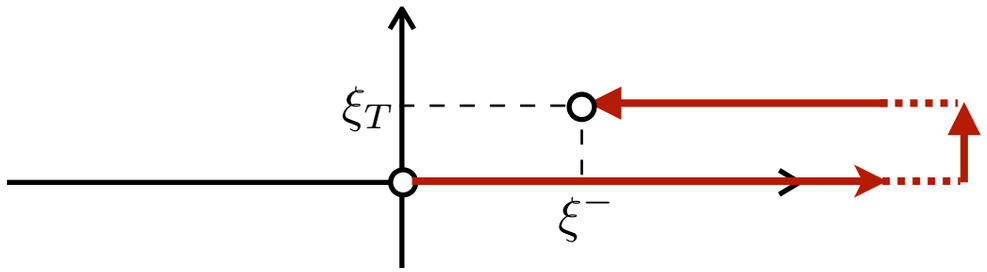


$U_{[-]}$

Gauge link for TMDs

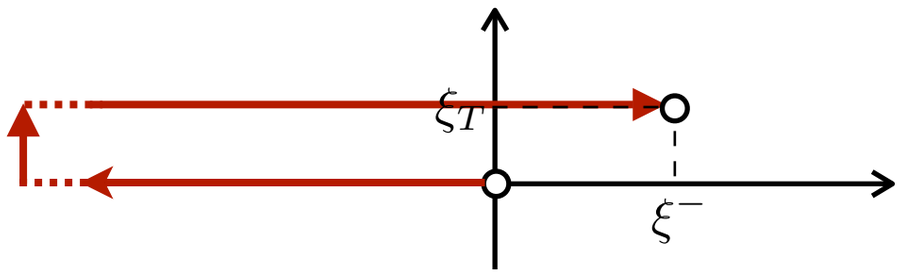
$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{8\pi^3}\, e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi)|P\rangle \Big|_{\xi^+=0}$$

SIDIS



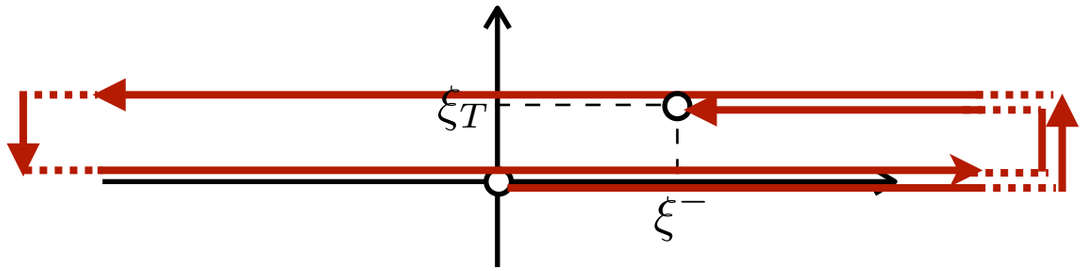
$U_{[+]}$

Drell-Yan



$U_{[-]}$

pp to hadrons



$U_{[\square]} U_{[+]}$

+ several others

Bomhof, Mulders, Pijlman, PLB 596 (04)