

Transversity and Transverse-momentum distributions

Alessandro Bacchetta



Some goals of hadronic physics

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- Understand CONFINEMENT

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- Study the STRUCTURE of the proton, e.g.,
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- L

- L

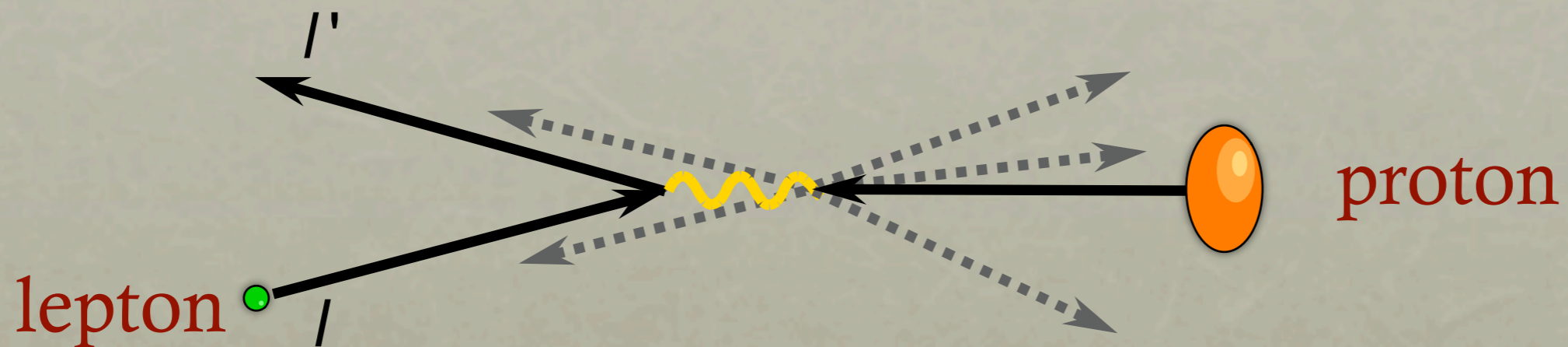
**TMDs and transversity are relevant
for all of these issues**

Parton distribution functions essentials

Deep inelastic scattering (DIS)

$$-(l - l')^2 = Q^2 = \text{virtuality of photon}$$

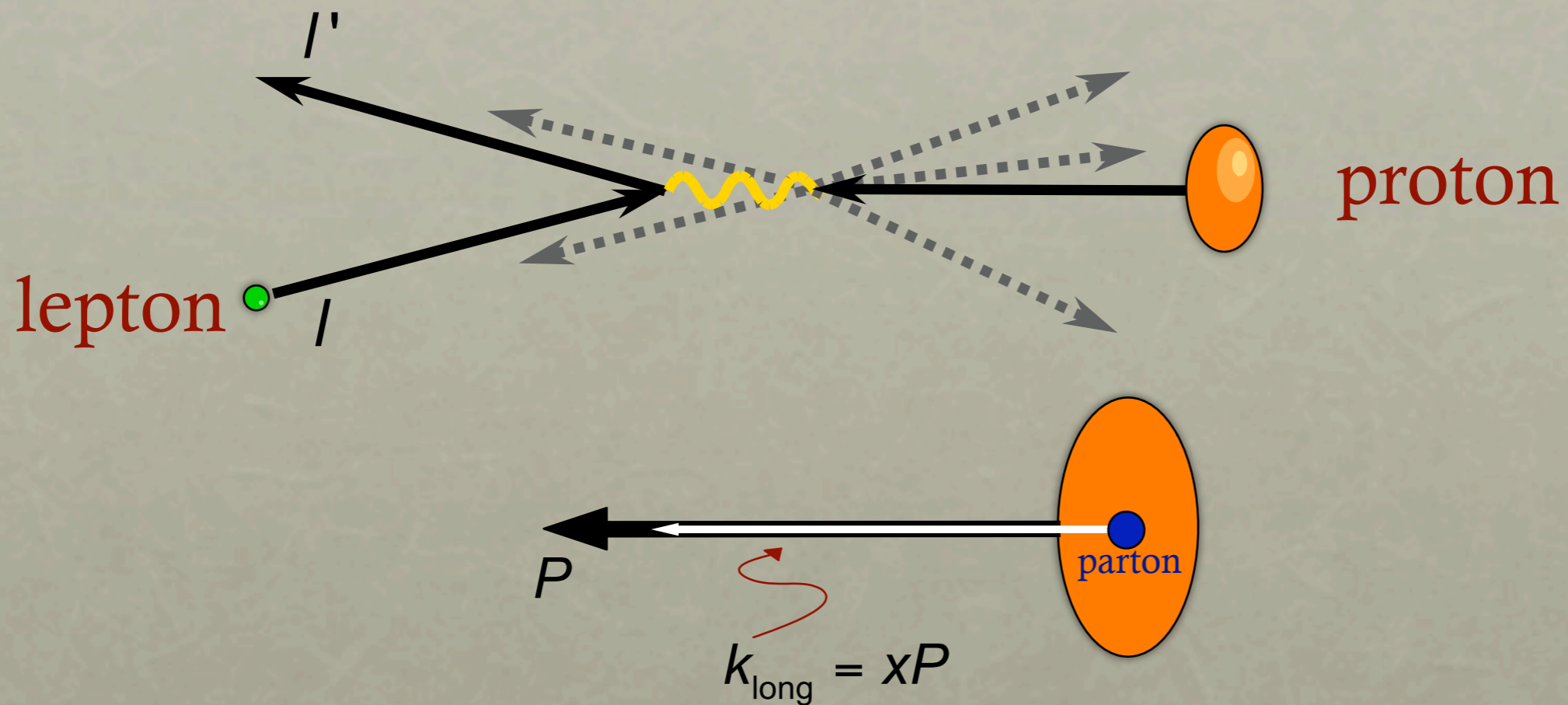
$$x = \frac{Q^2}{2P \cdot (l - l')}$$



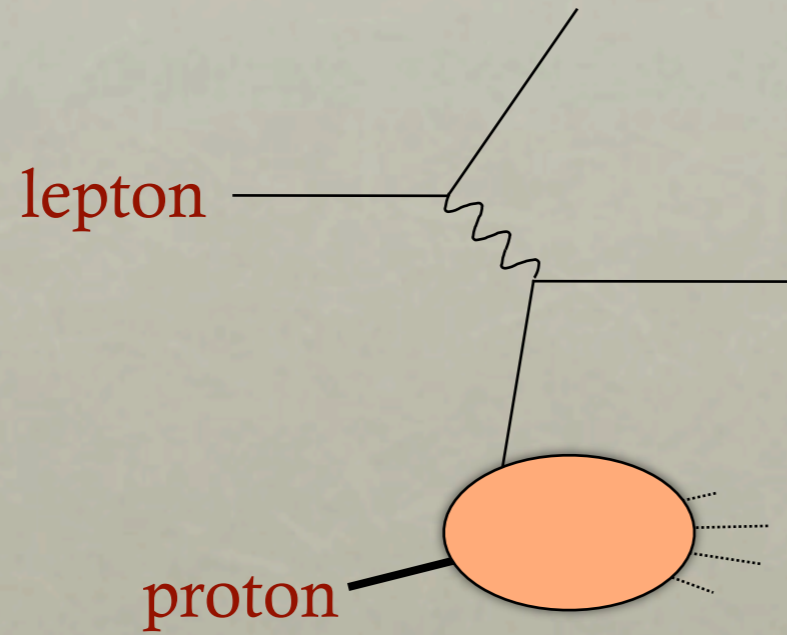
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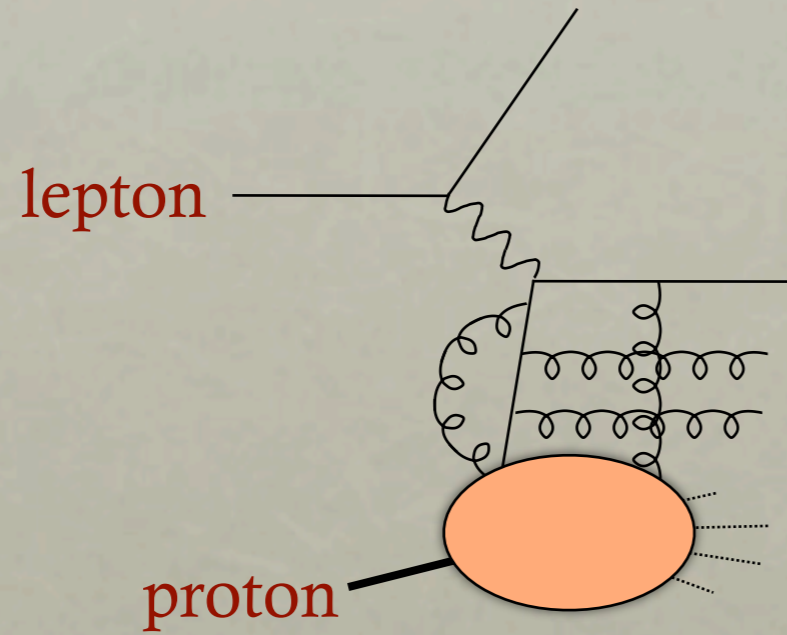
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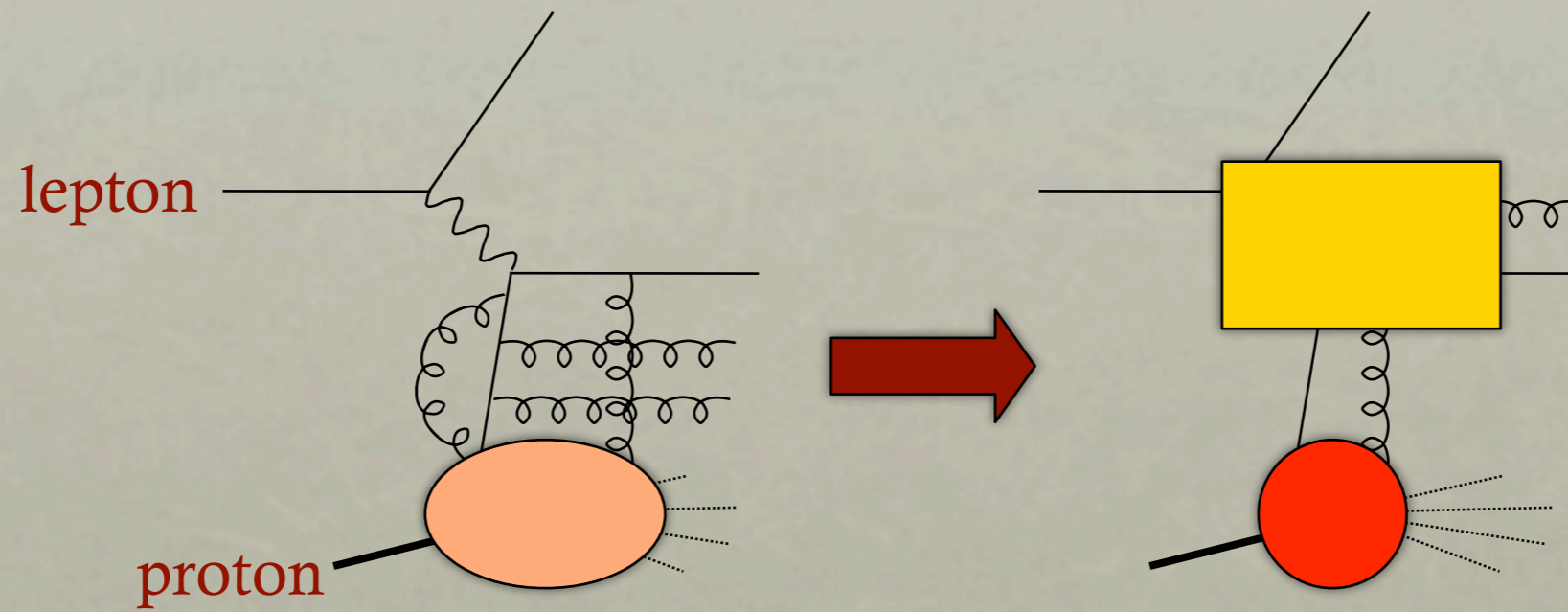
Factorization



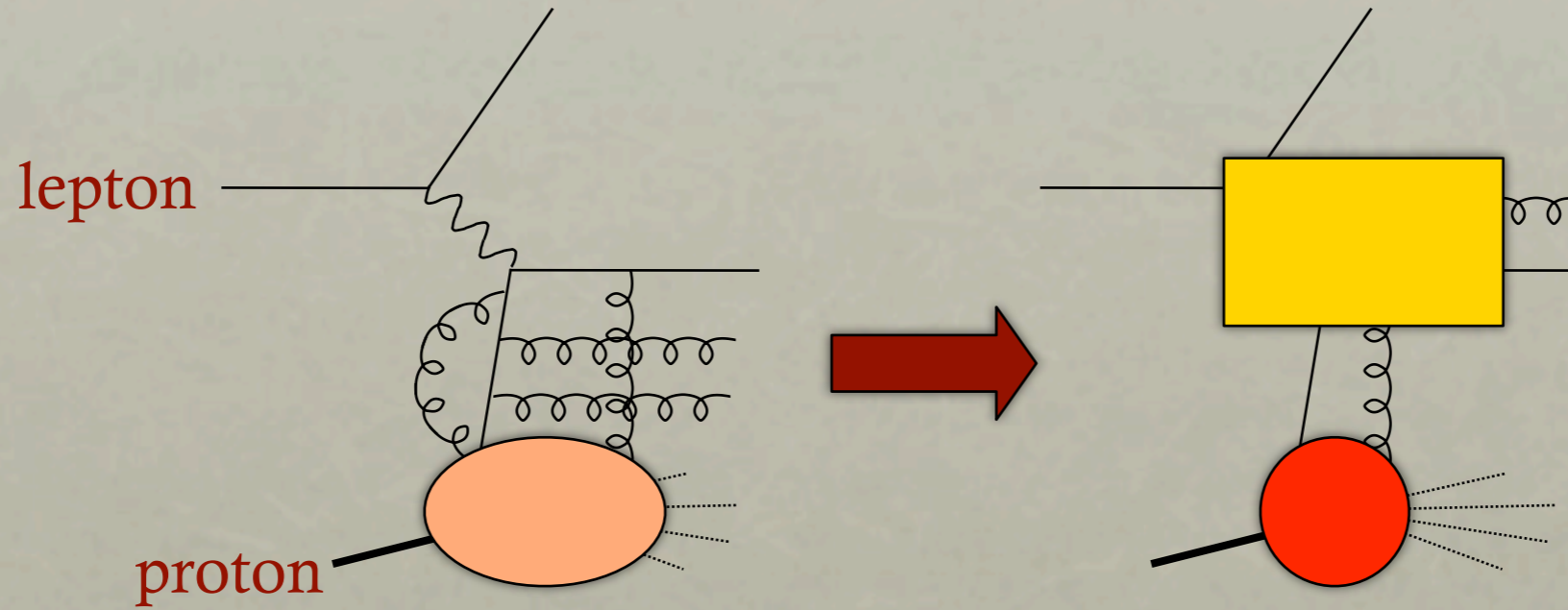
Factorization



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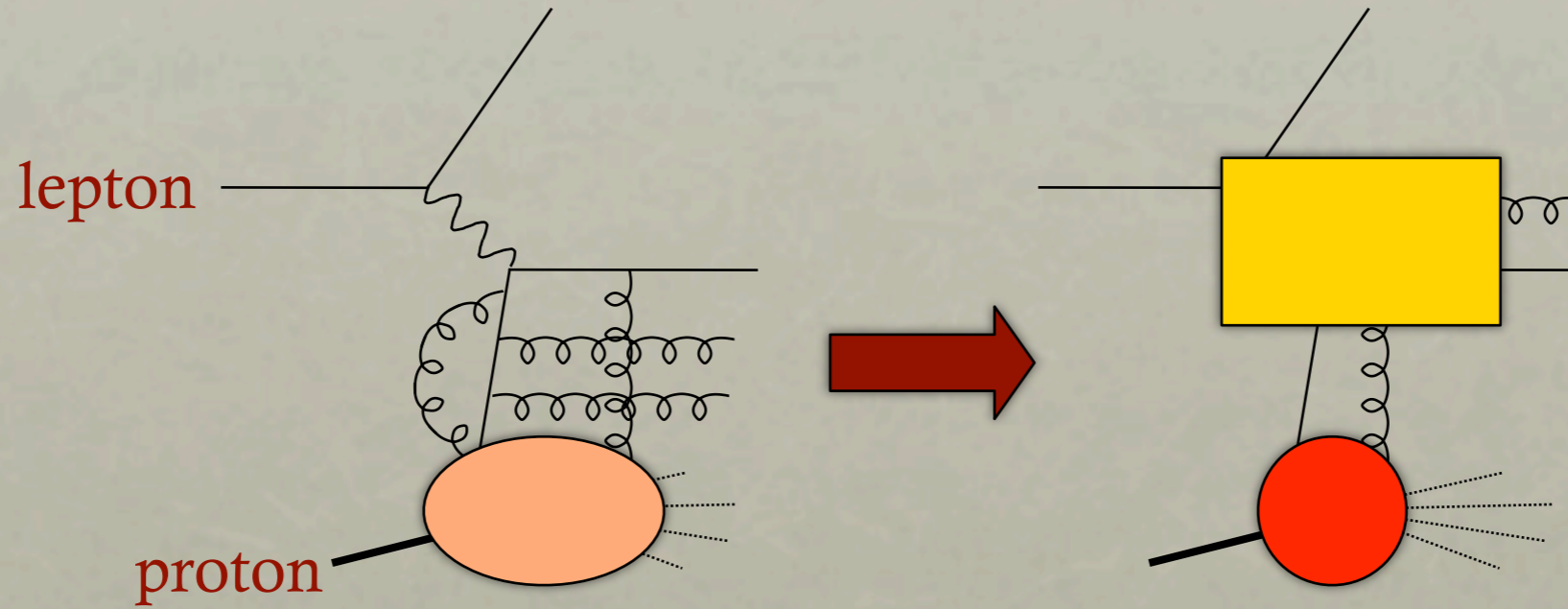


Partonic scattering amplitude



Distribution amplitude

Factorization



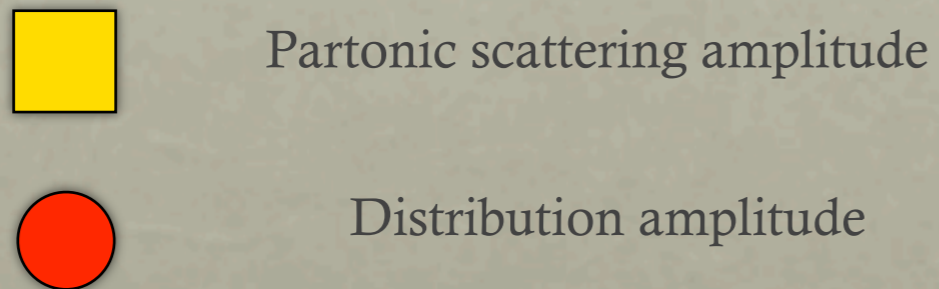
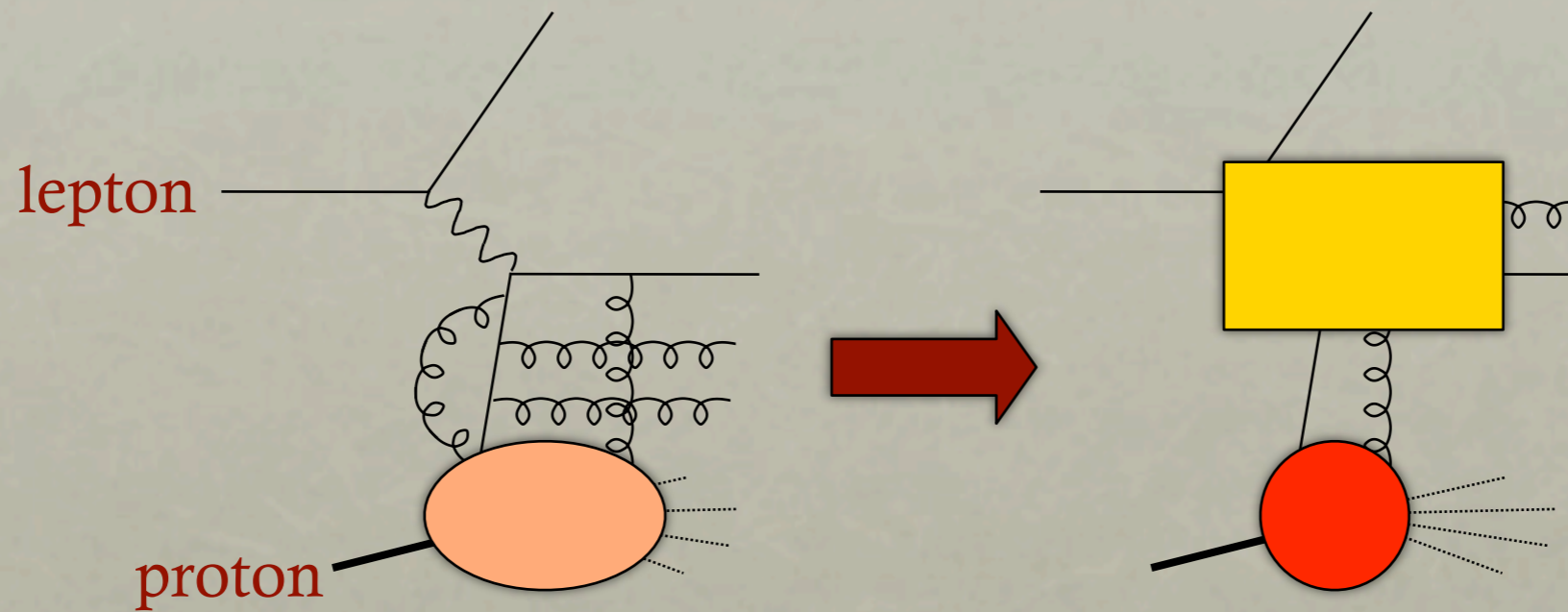
Partonic scattering amplitude



Distribution amplitude

$$d\sigma = H \otimes f$$

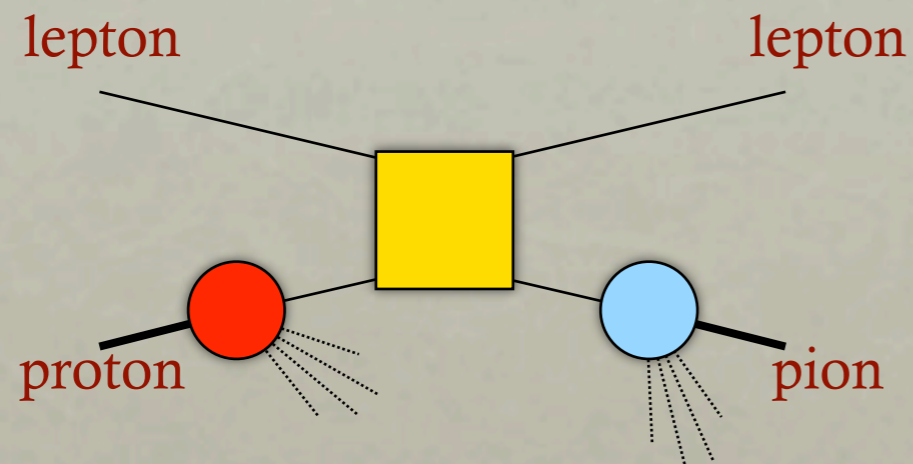
Factorization



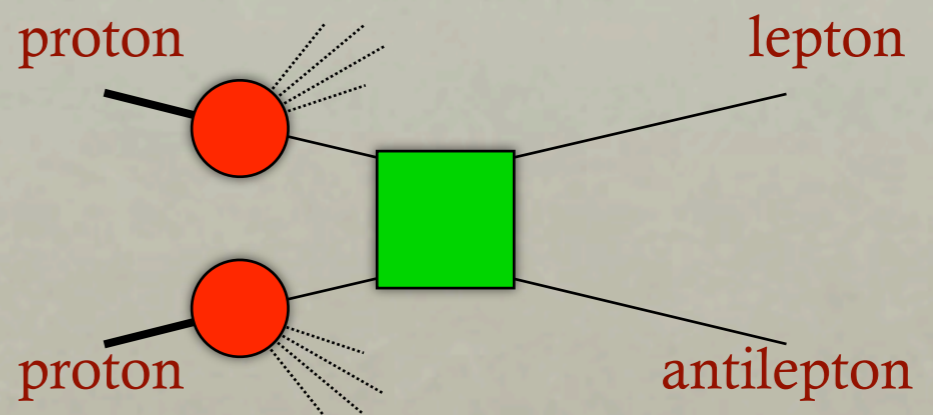
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Key result of QCD

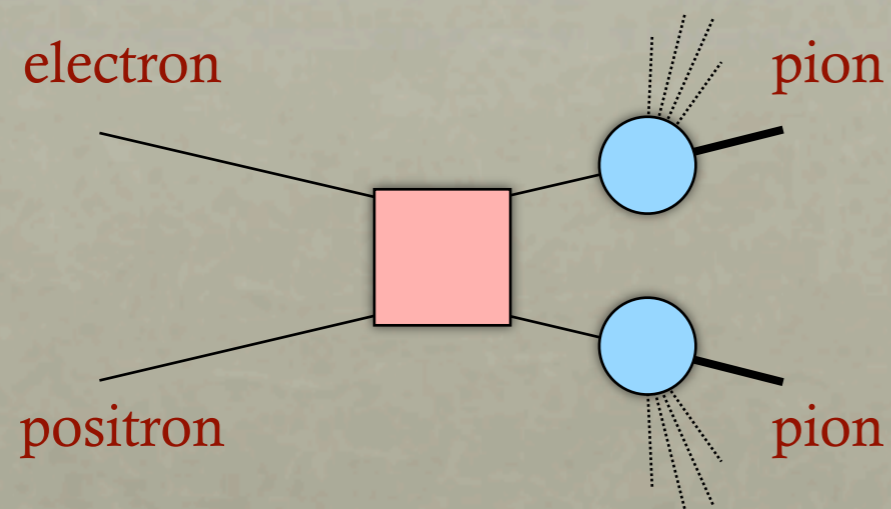
Universality



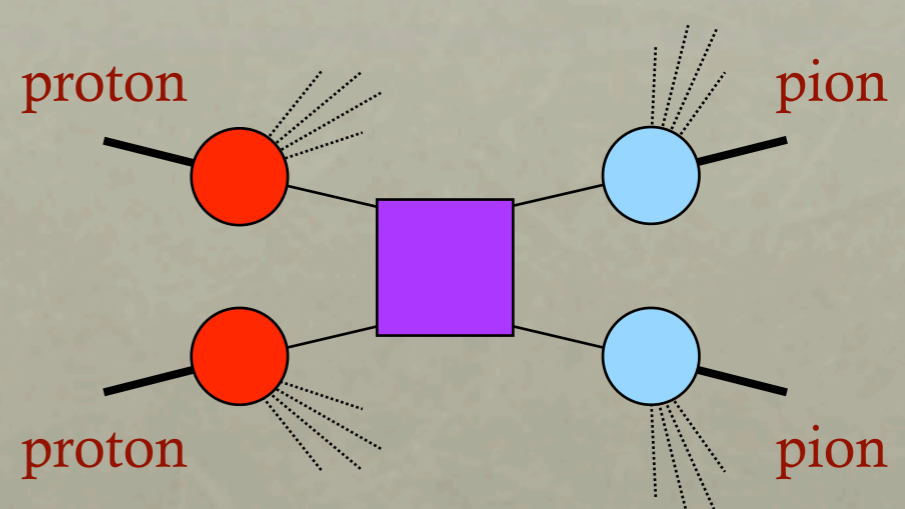
SIDIS



Drell-Yan

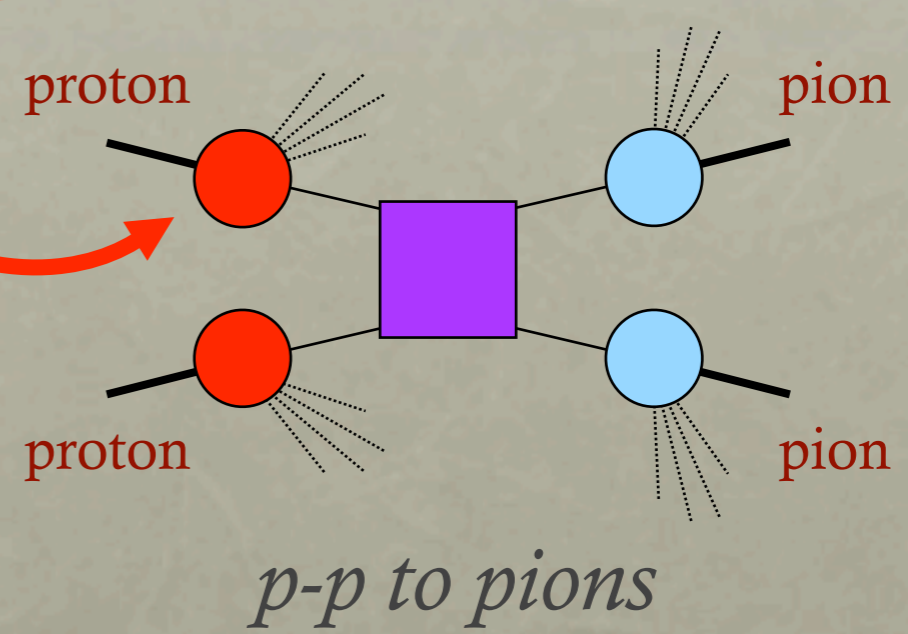
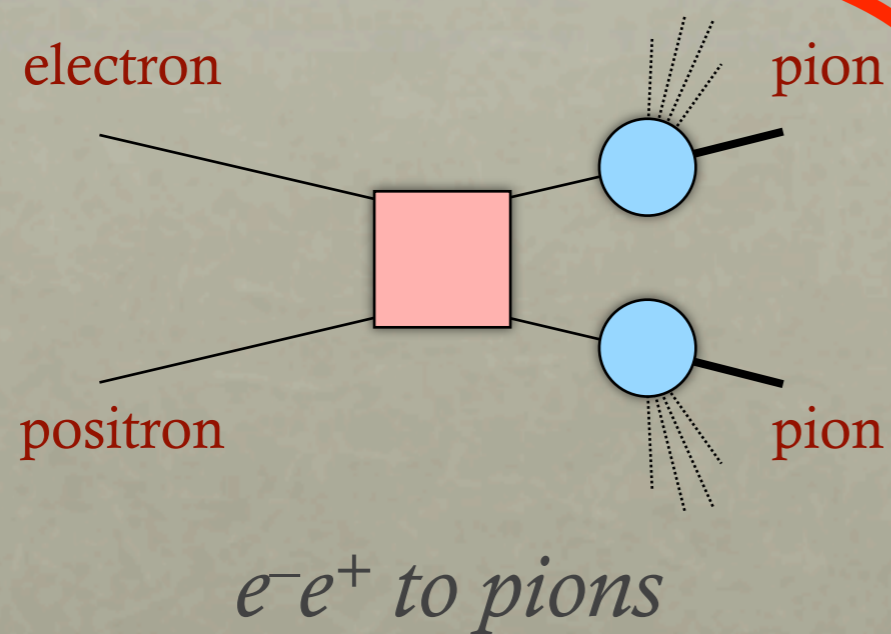
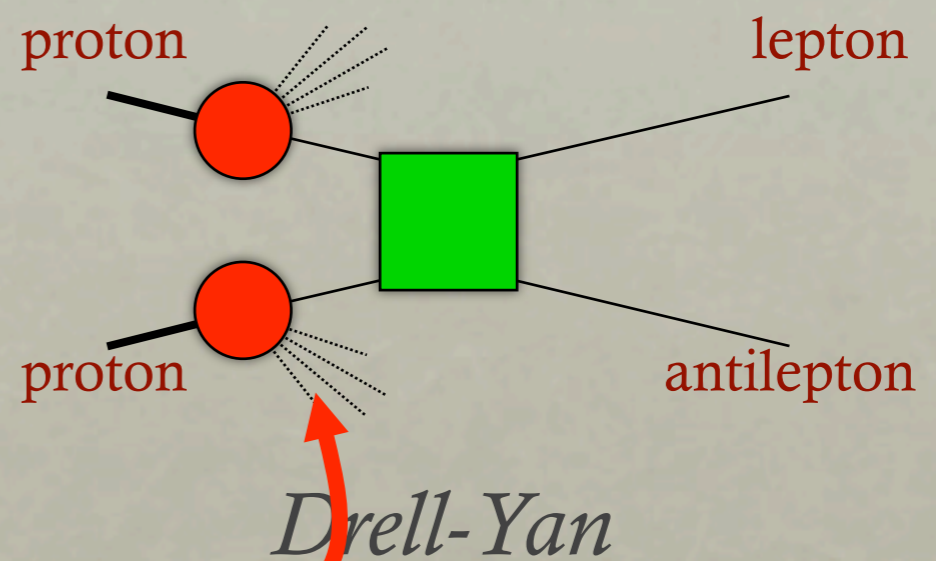
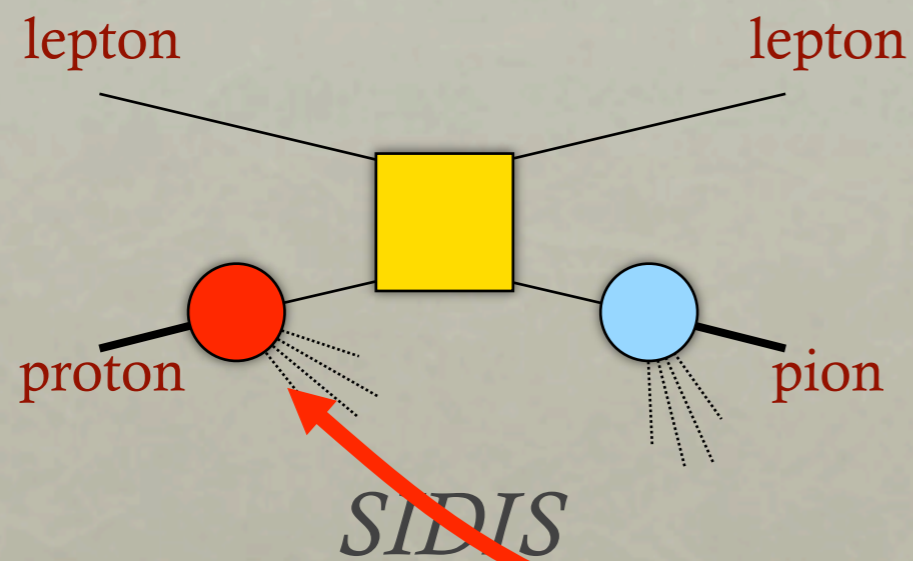


e^-e^+ to pions

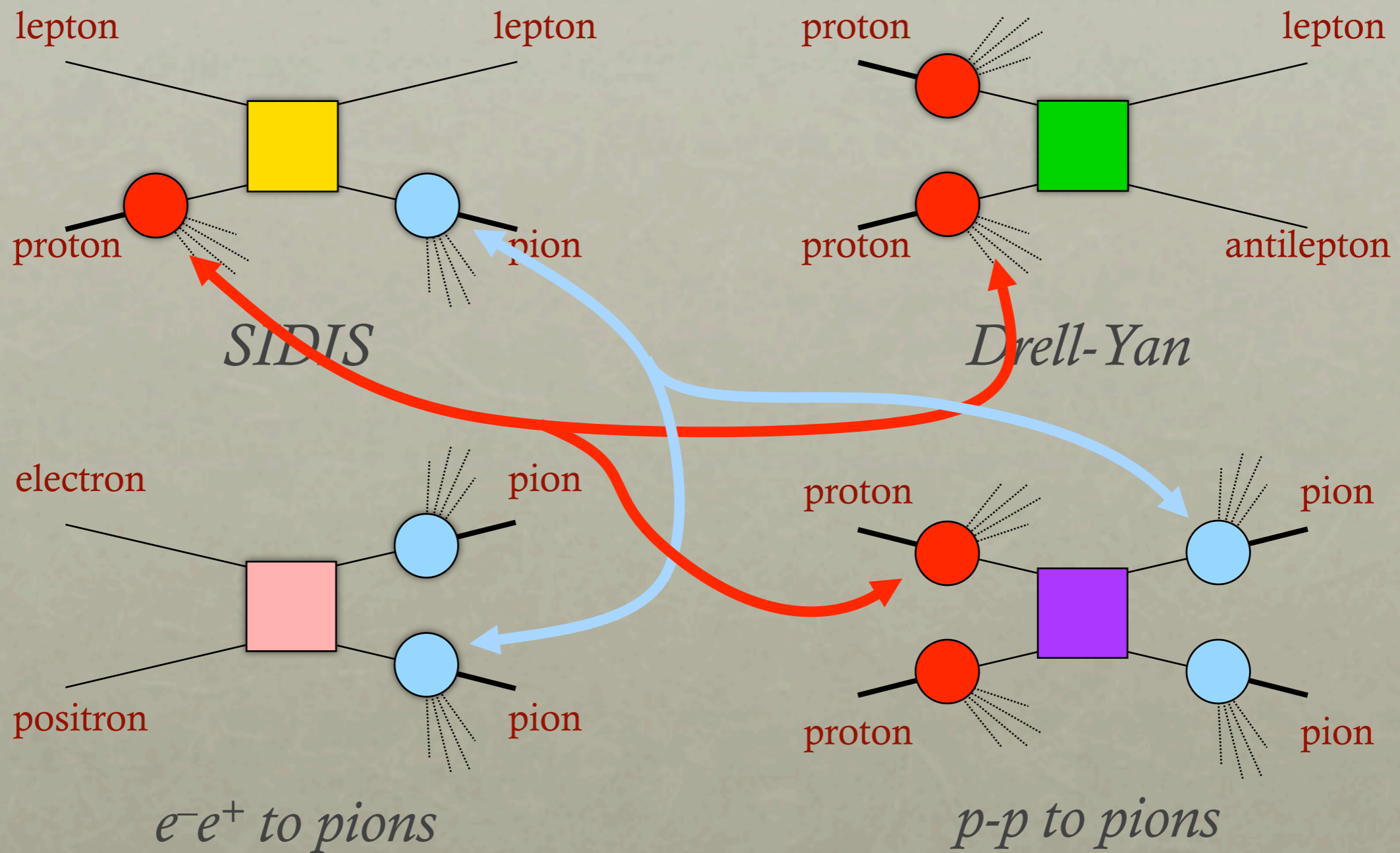


$p-p$ to pions

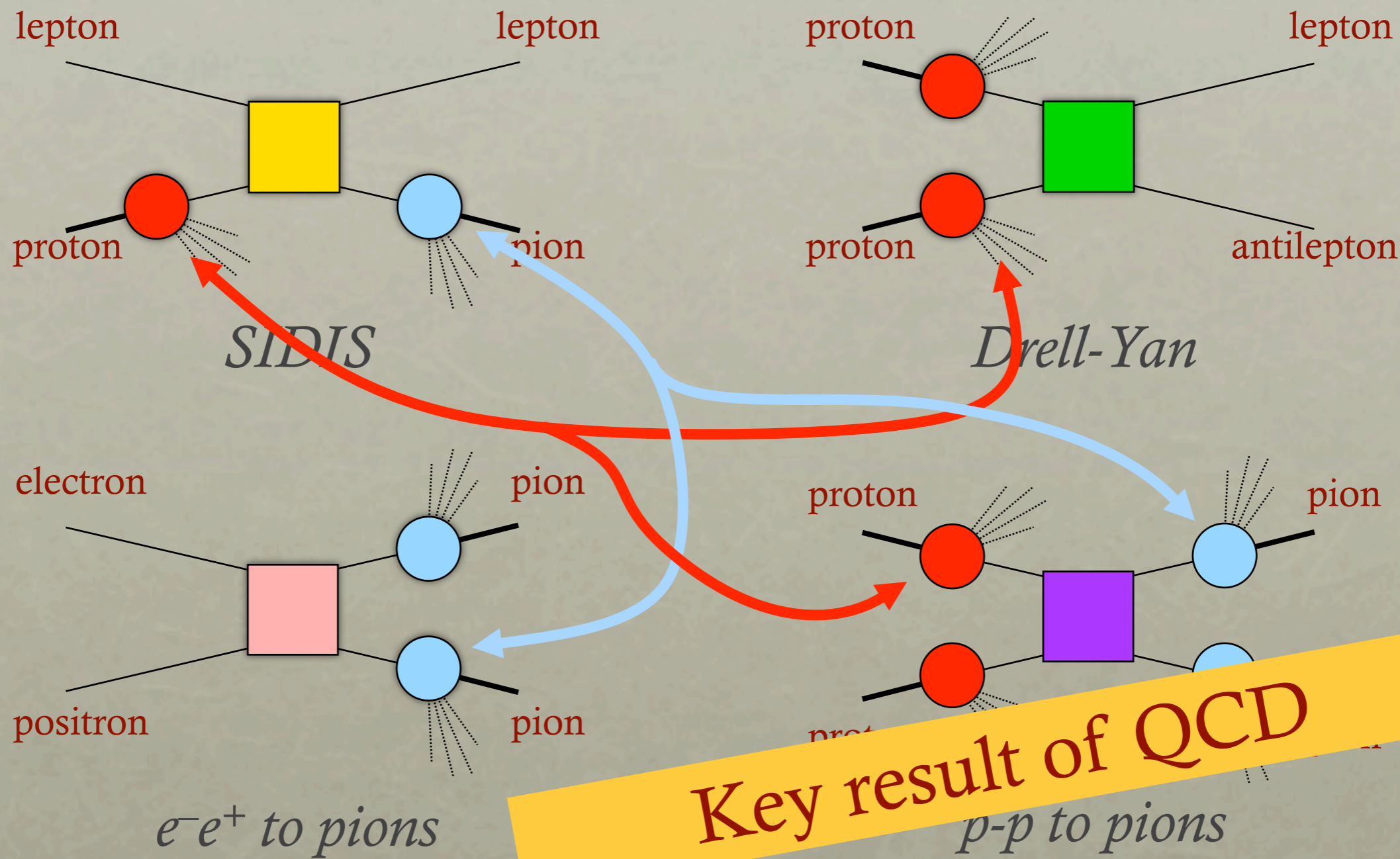
Universality



Universality



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Parton distribution functions

Parton distribution functions (PDFs) are probability densities to find a parton with a given longitudinal momentum and a given spin

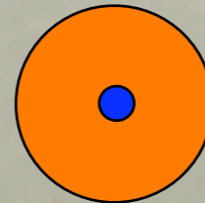
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*Photon moves into the screen/
proton moves out of the screen*

$$f_1^q(x) = q(x) = \text{img}$$



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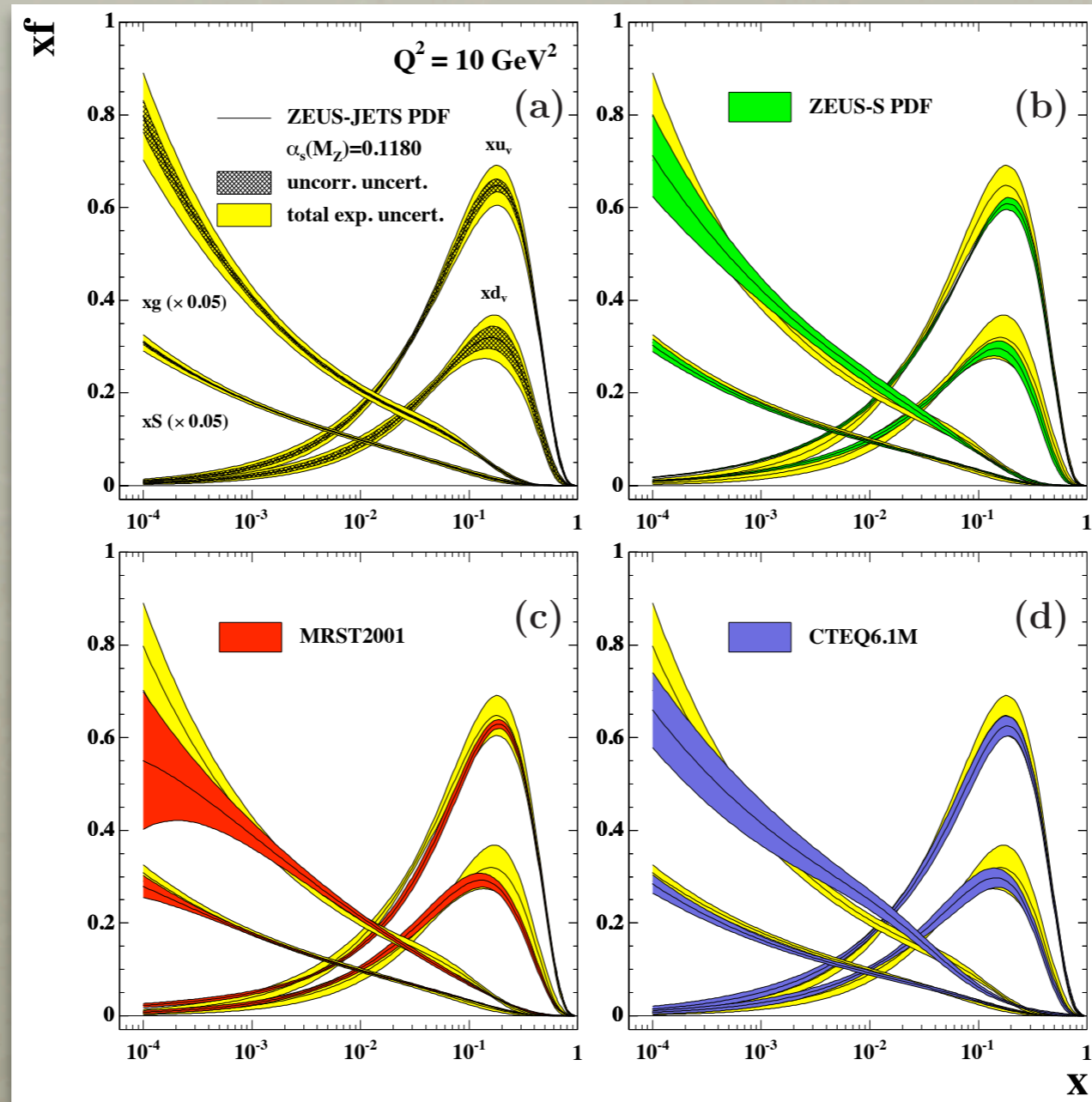
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PDFs from global fits

$x q(x)$

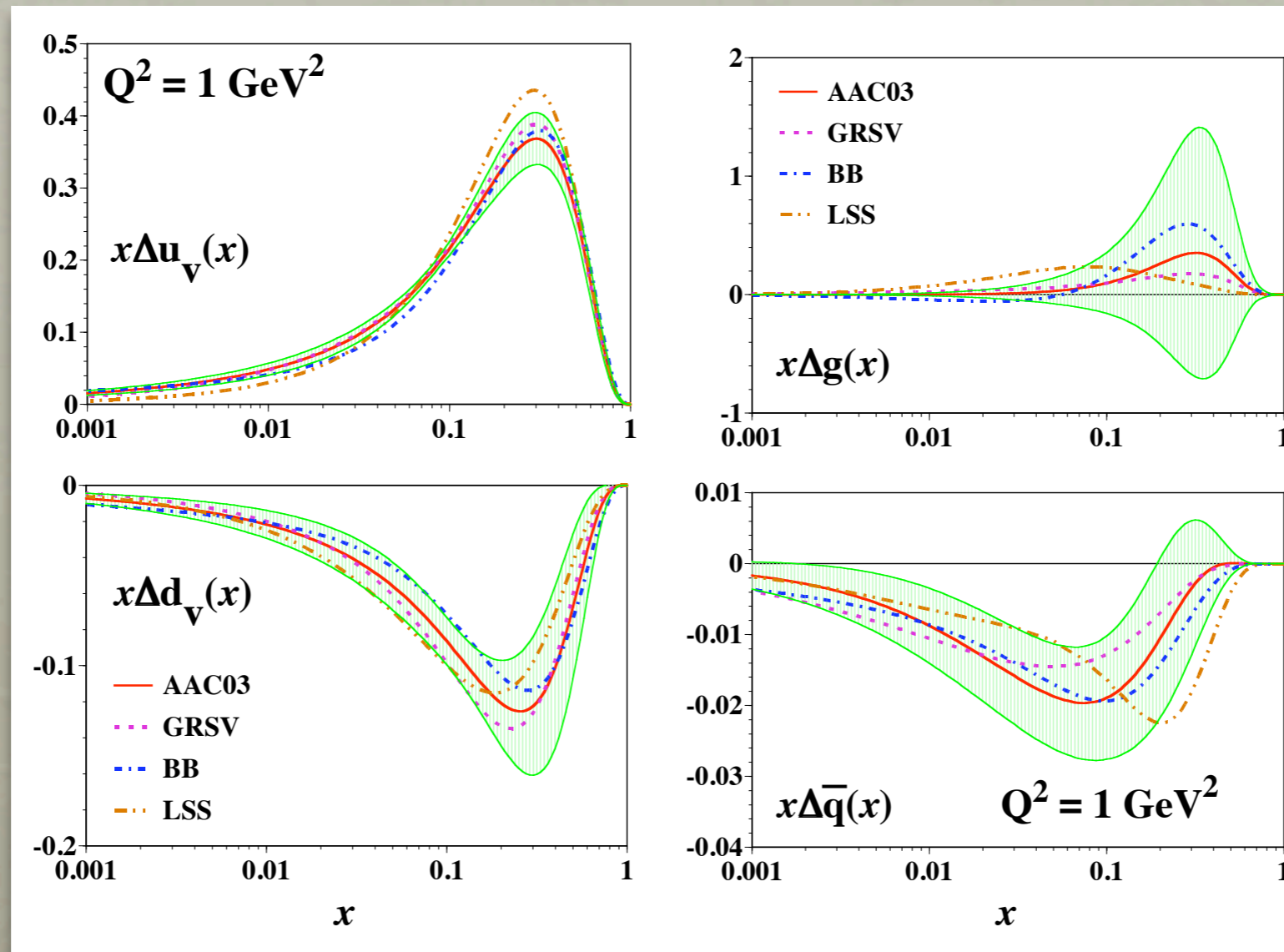


ZEUS Coll, EPJ C42 (05)

Helicity PDFs from global fits

see talk by S. Kuhn this morning

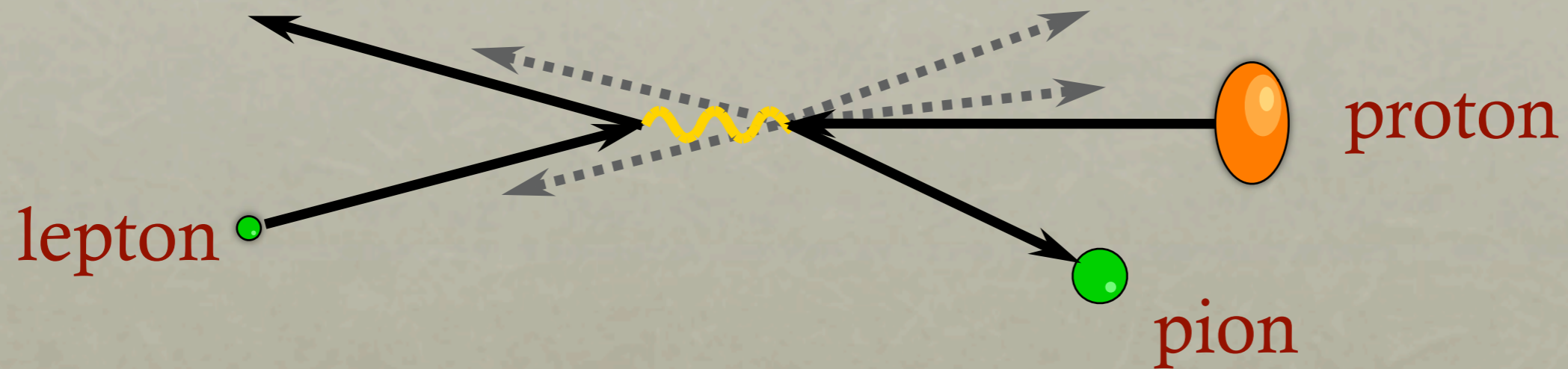
$$x \Delta q(x)$$



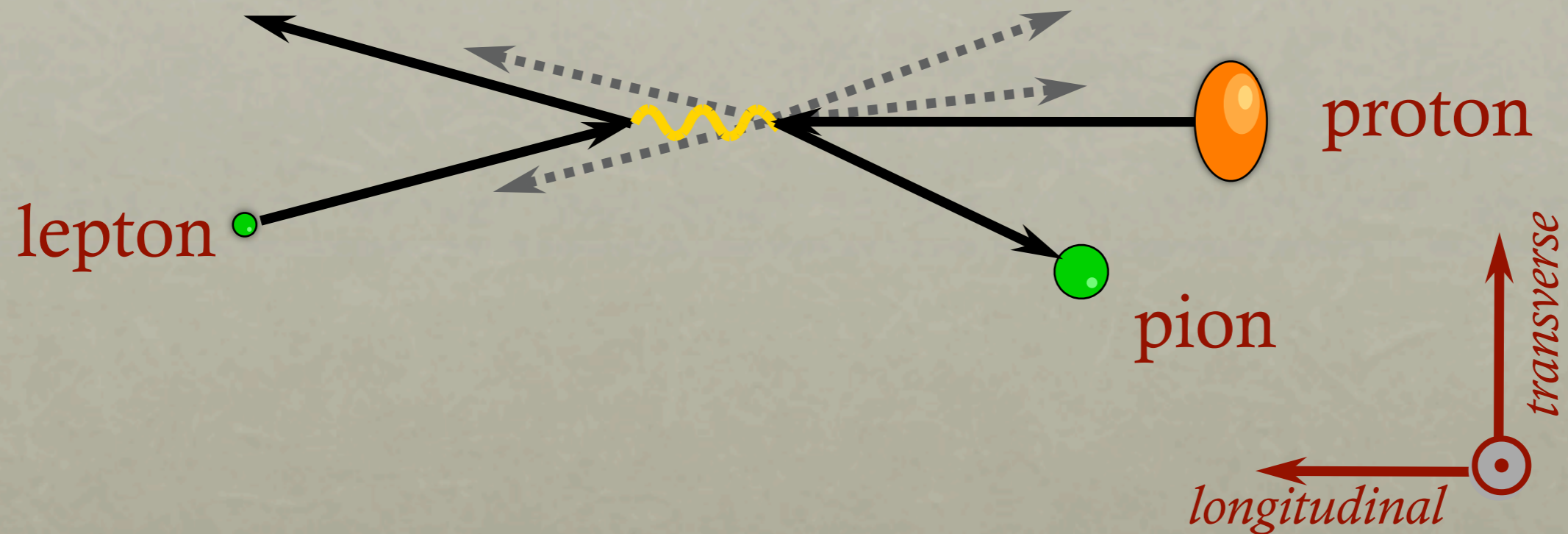
AAC, Hirai et al. PRD69 (04)

Transverse parton distribution functions

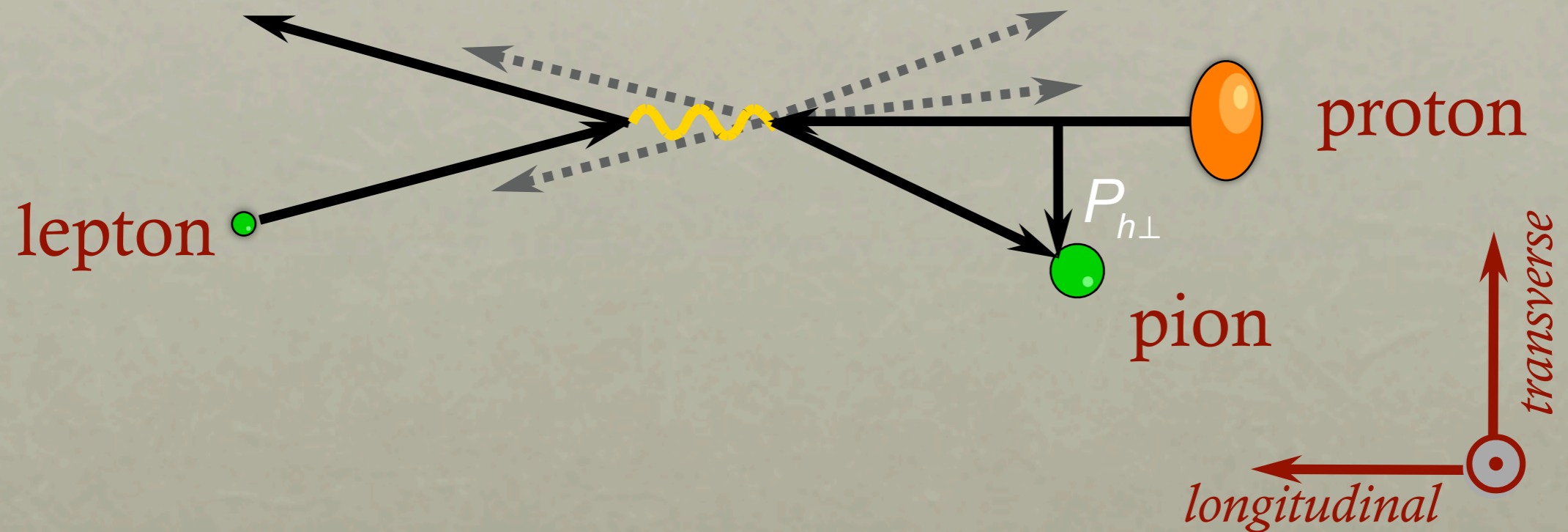
Transverse vs. longitudinal



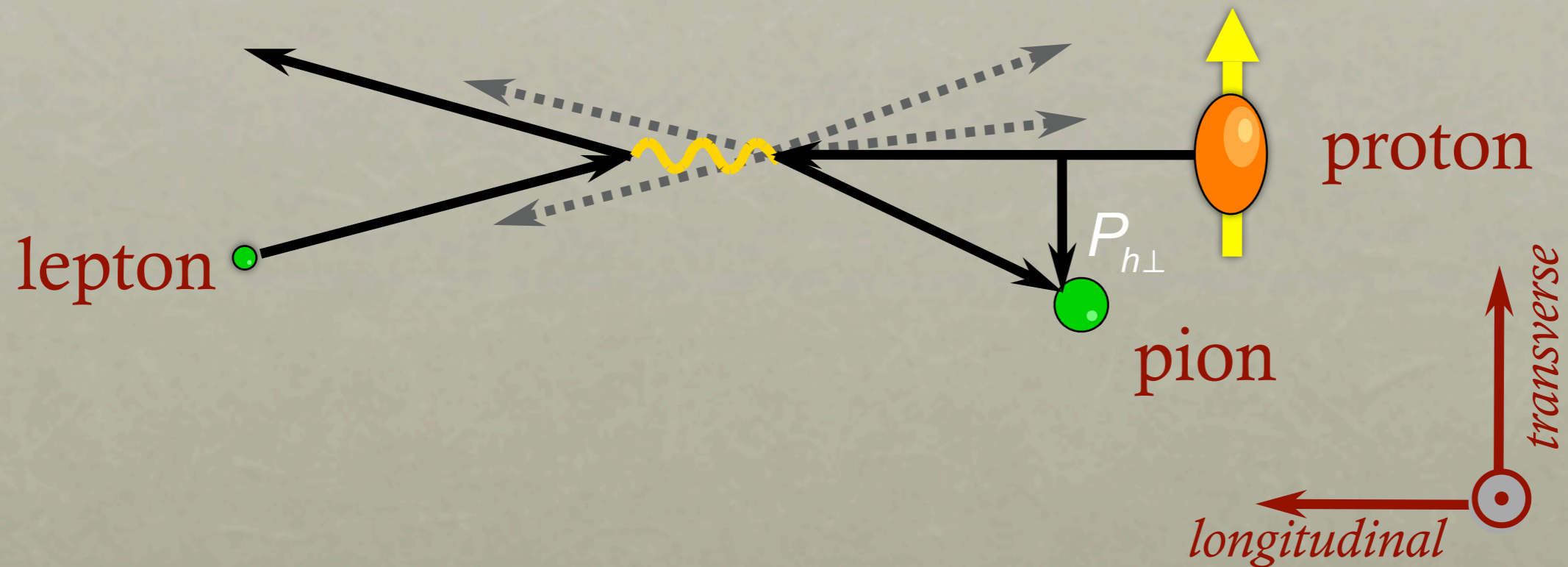
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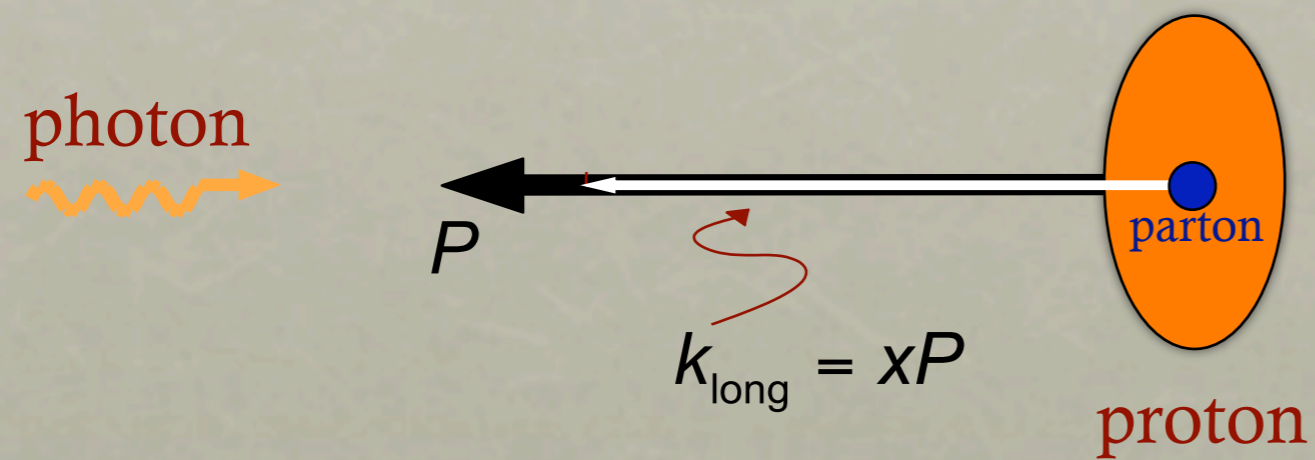
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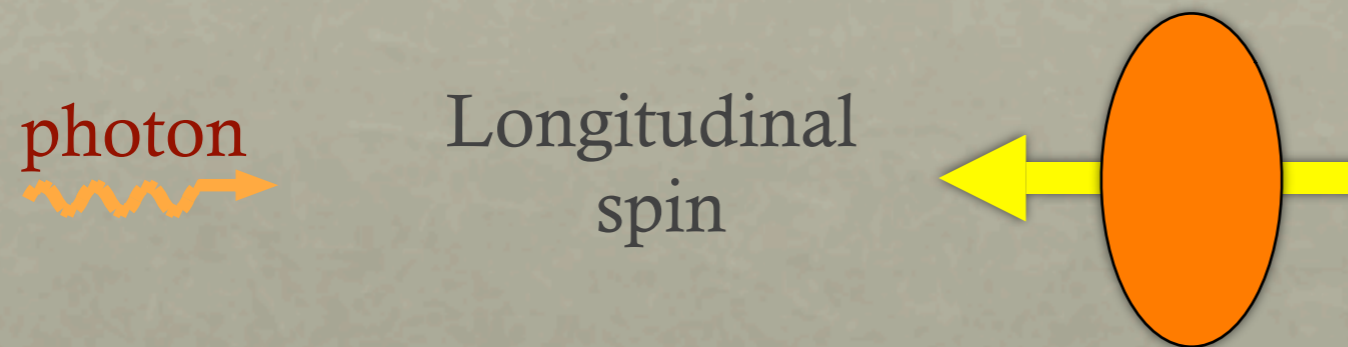
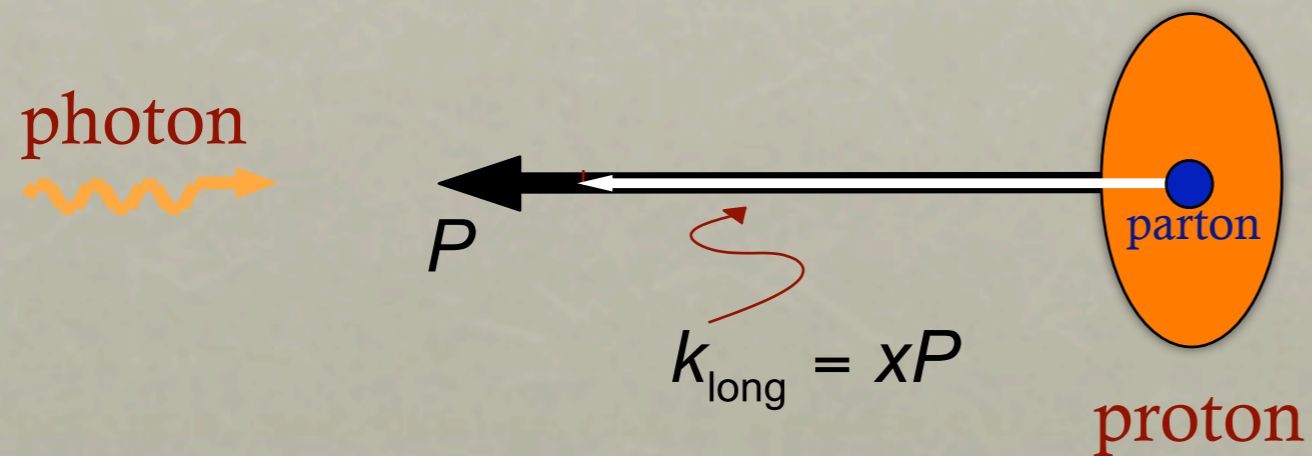
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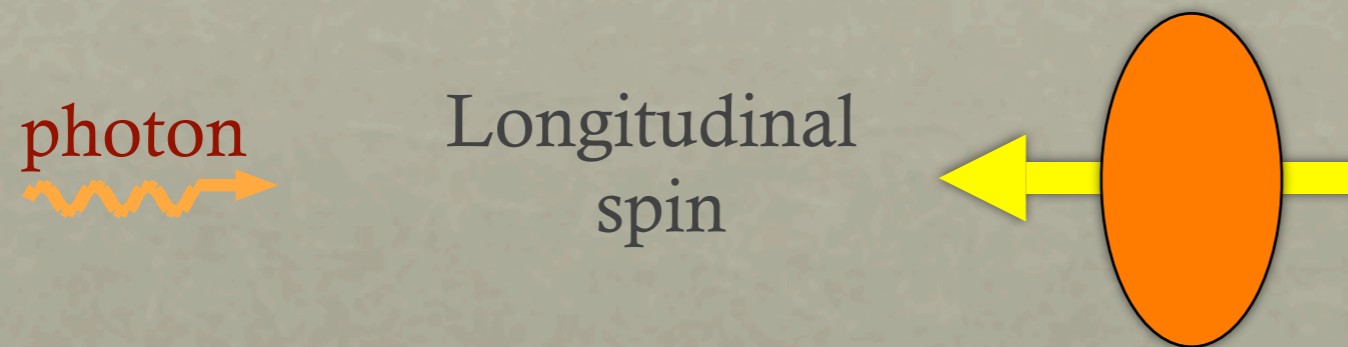
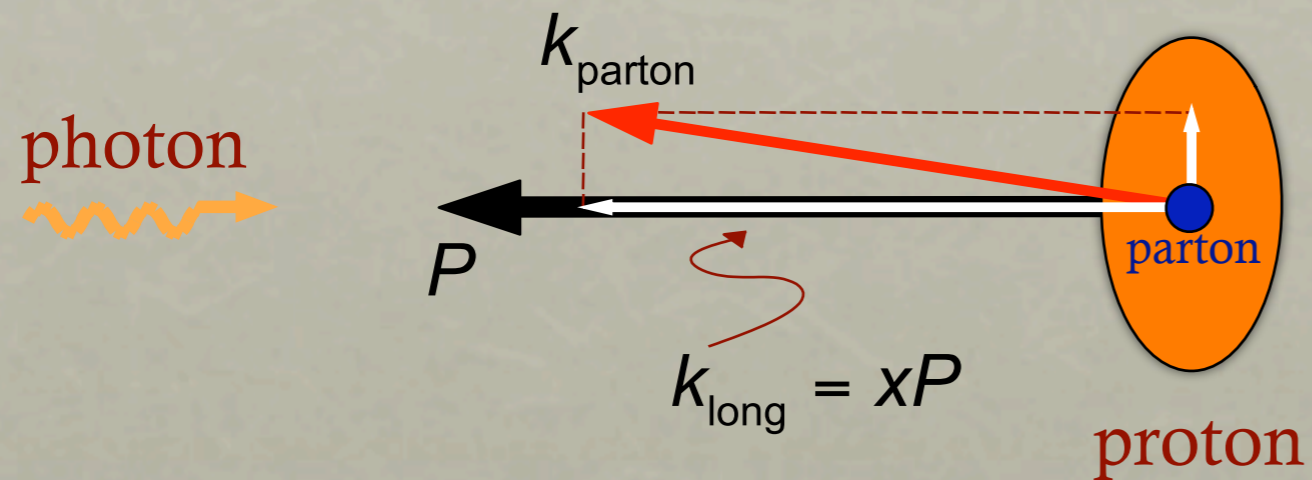
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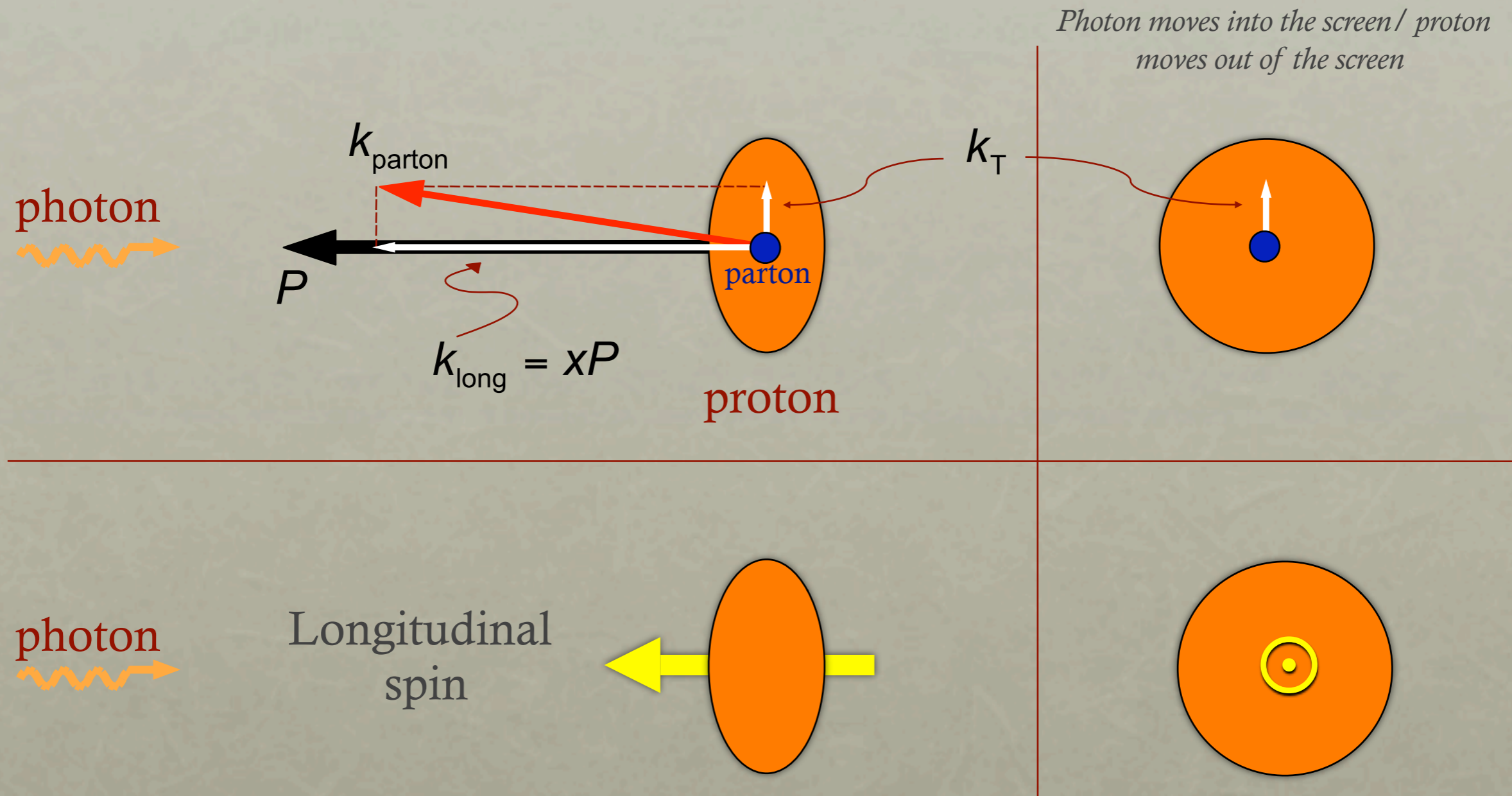
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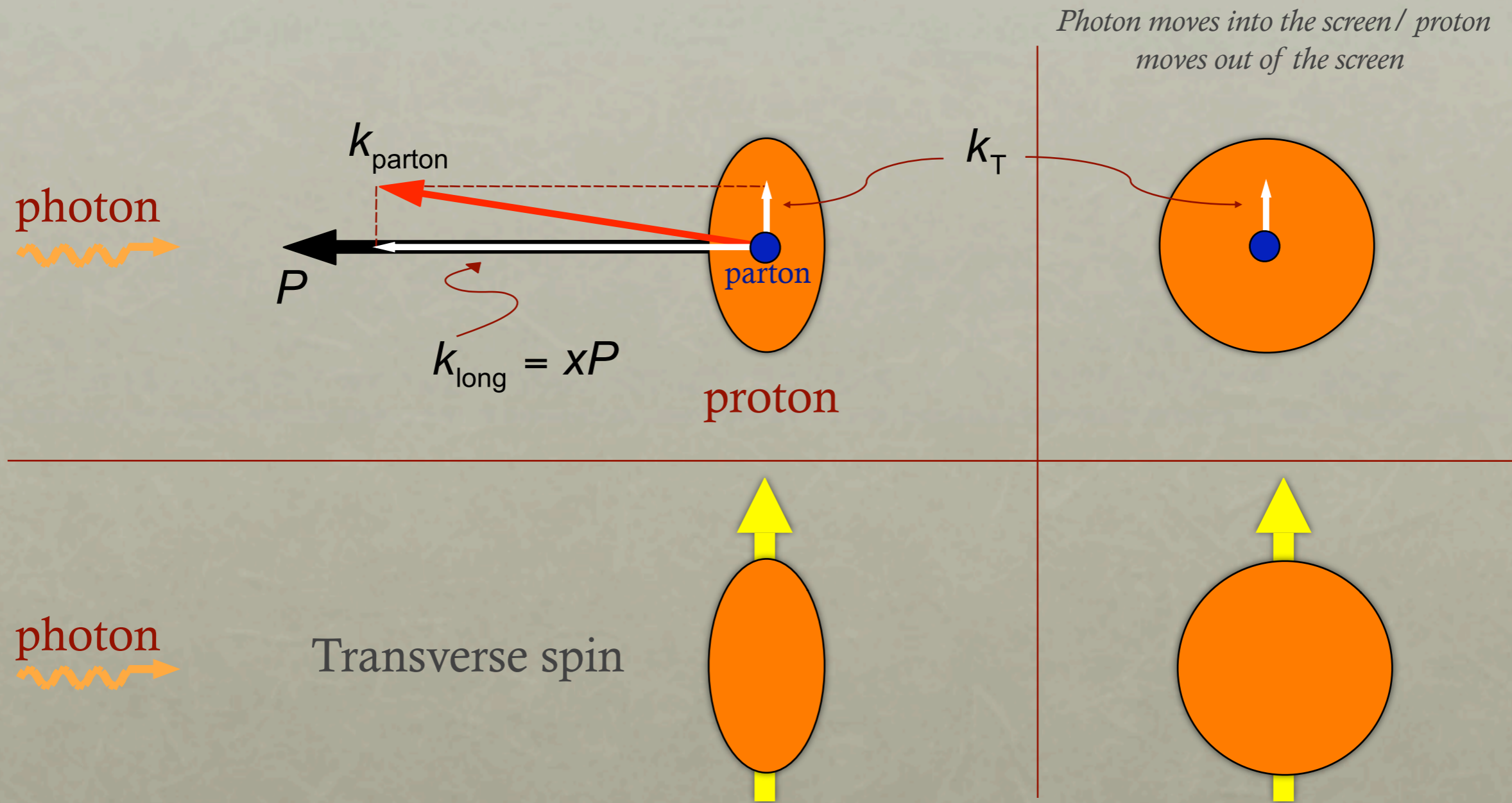
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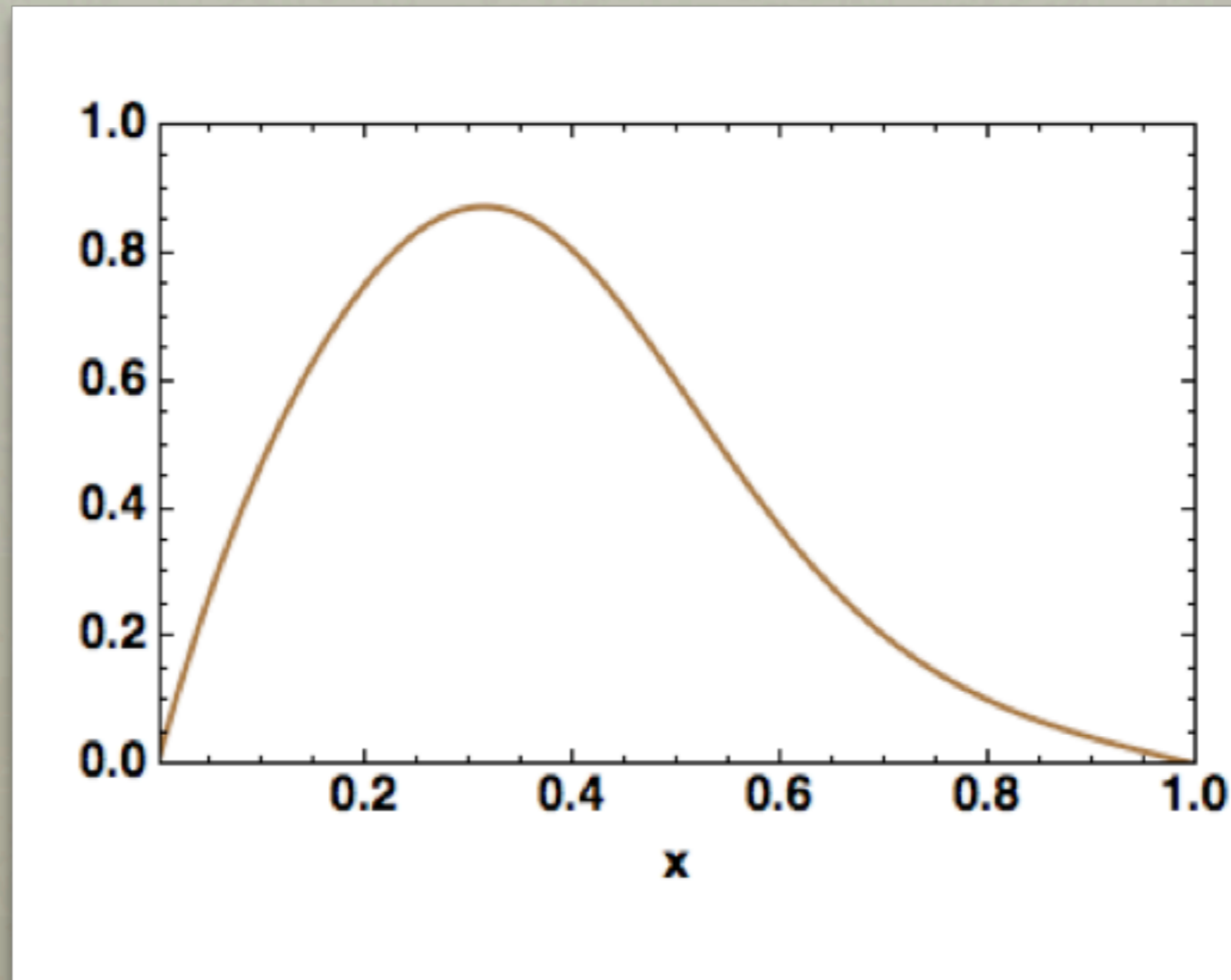
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$$h_1^q(\mathbf{x}) = \Delta_T q(\mathbf{x}) = \text{orange circle with blue dot and upward arrow} - \text{orange circle with blue dot and downward arrow}$$

Transverse momentum distributions

up quarks

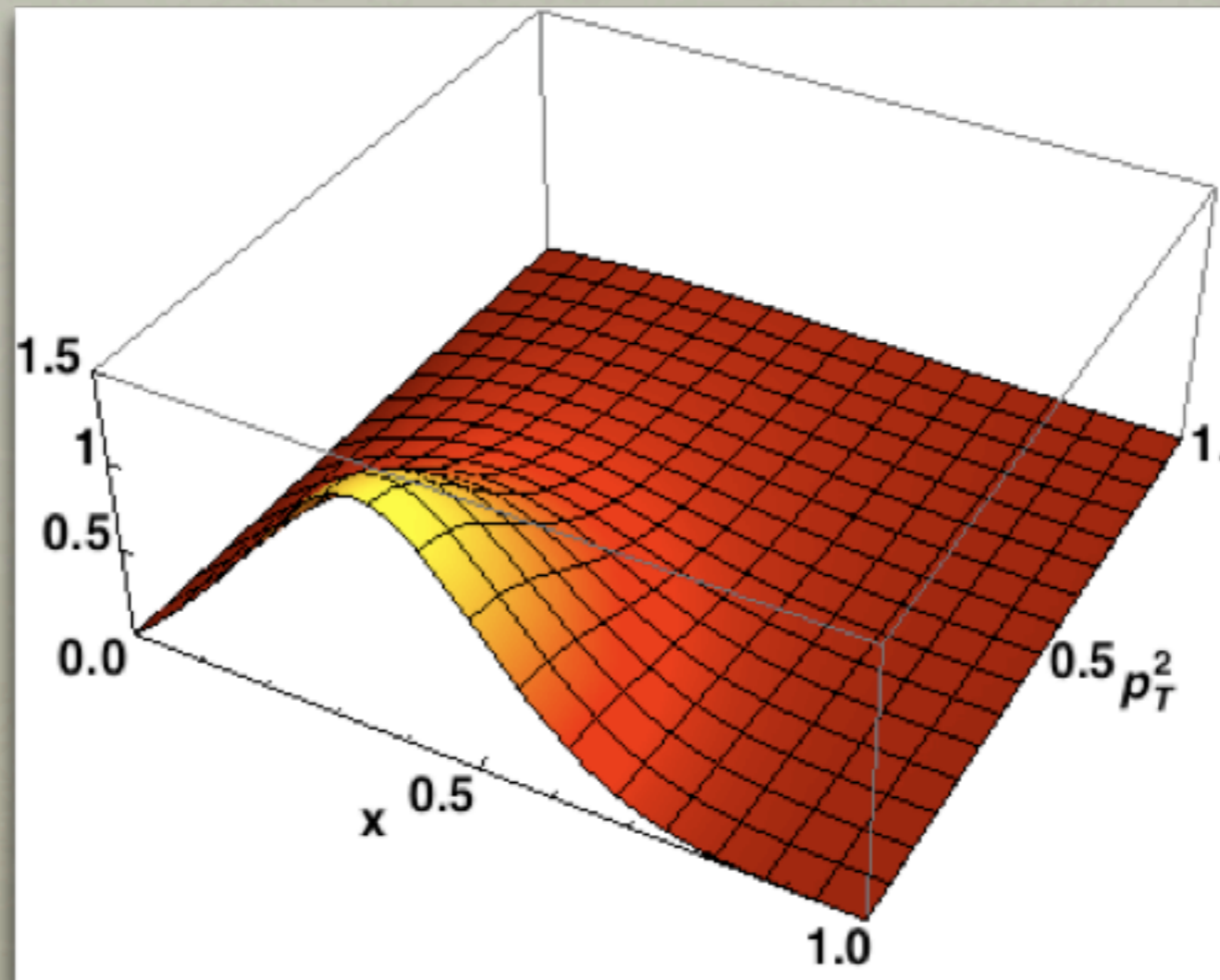
$x q(x)$



Transverse momentum distributions

up quarks

$$x q(x, p_T^2)$$



Orbital angular momentum

Jaffe, Manohar, NPB 337 (90)

$$\frac{1}{2} = \frac{1}{2} \int dx g_1^{q+\bar{q}}(x) + L_z^{q+\bar{q}} + \int dx g_1^g(x) + L_z^g$$

Ji, PRL 78 (97)

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Gauge invariant

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Ji, PRL 78 (97)

Transversity

Transversity vs Helicity

- Different due to relativistic effects
- Different integral (tensor vs axial charge)

$$\delta\Sigma = 0.56$$

$$\Delta\Sigma = 0.18$$

- Different evolution (no gluons vs gluons)

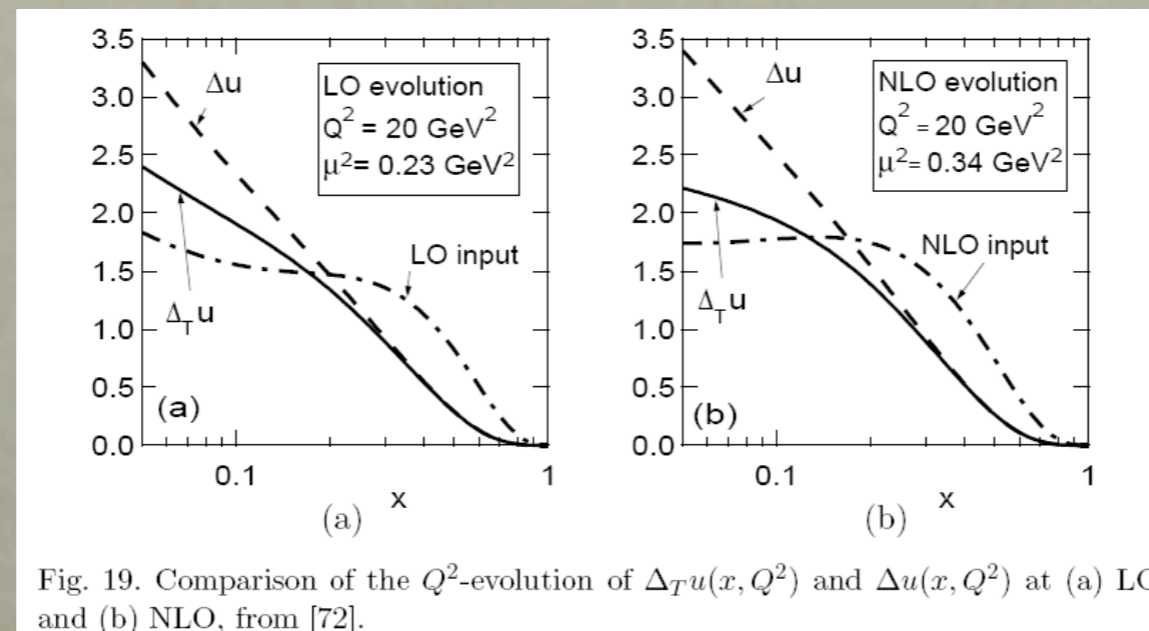


Fig. 19. Comparison of the Q^2 -evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO, from [72].

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Aoki et al., PRD 56 (97)

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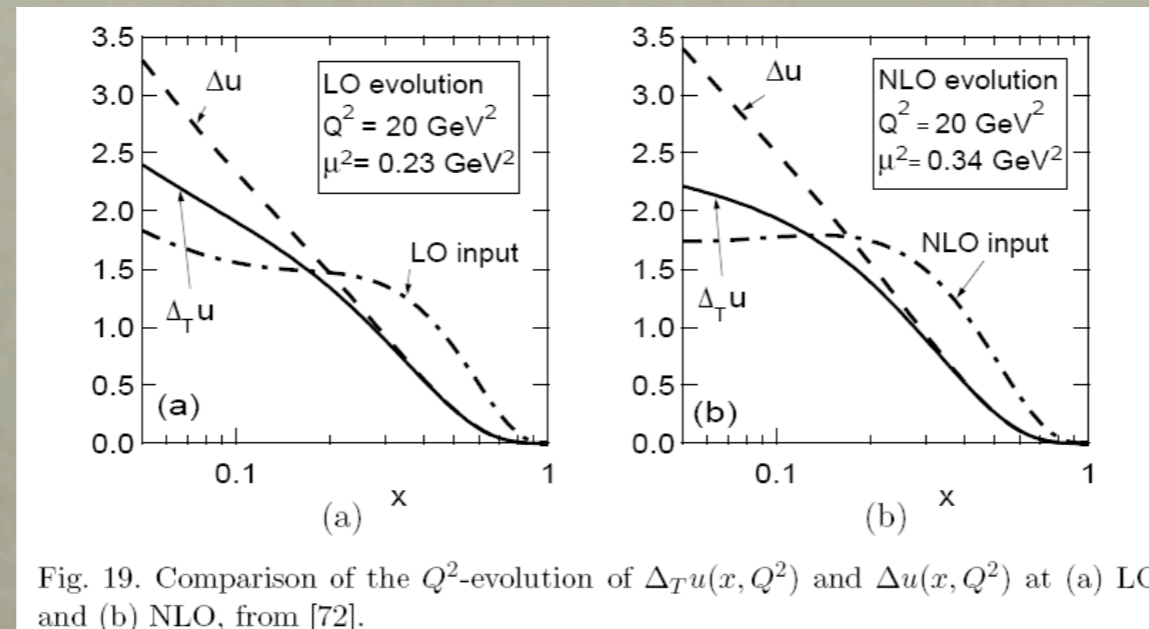
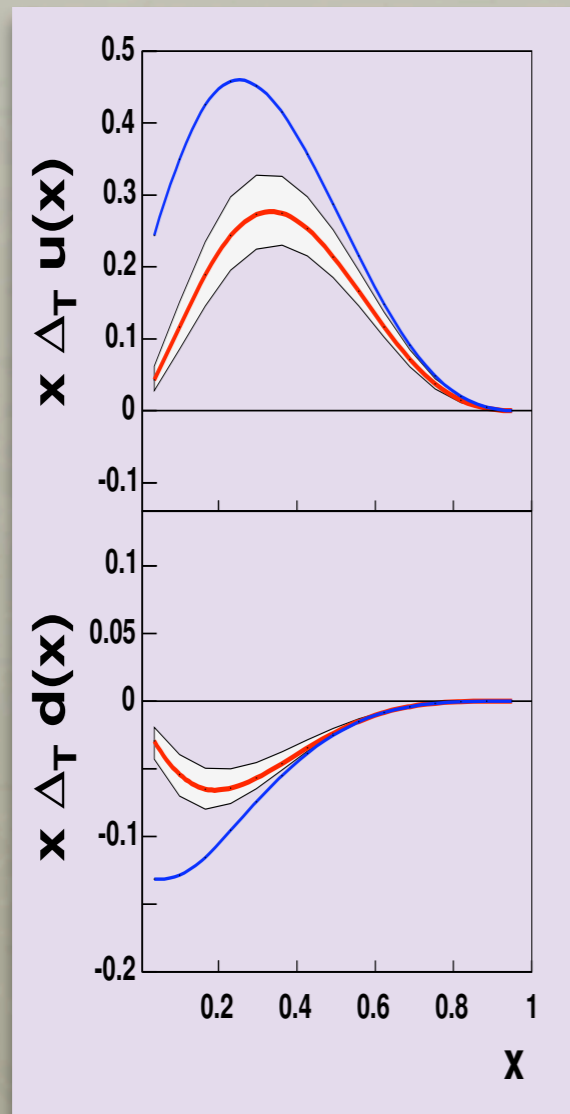


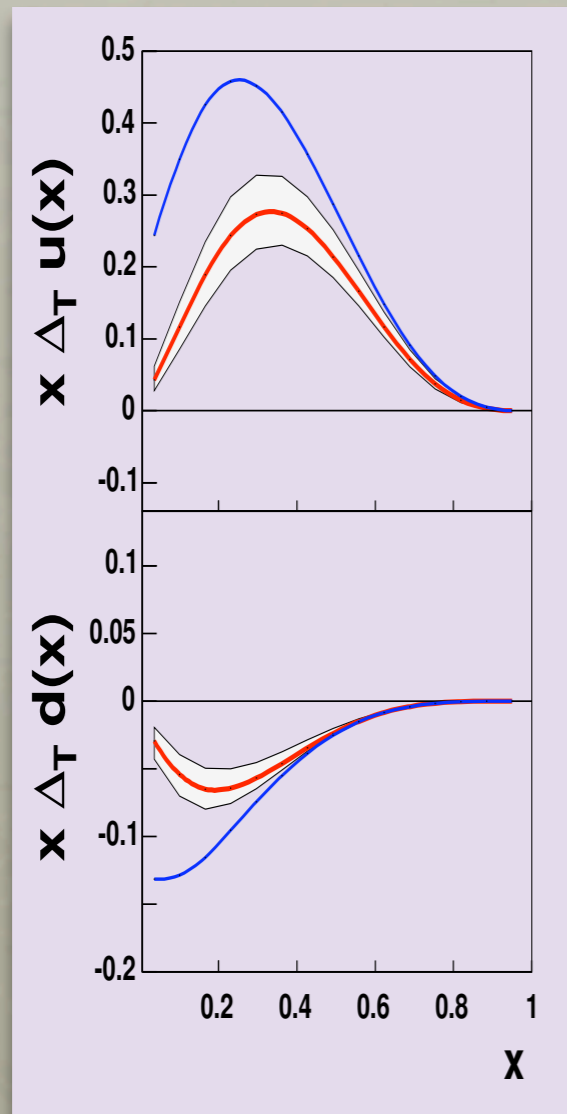
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Extraction of transversity



A. Prokudin, talk at DIS08 (Anselmino et al, 0807.0173)

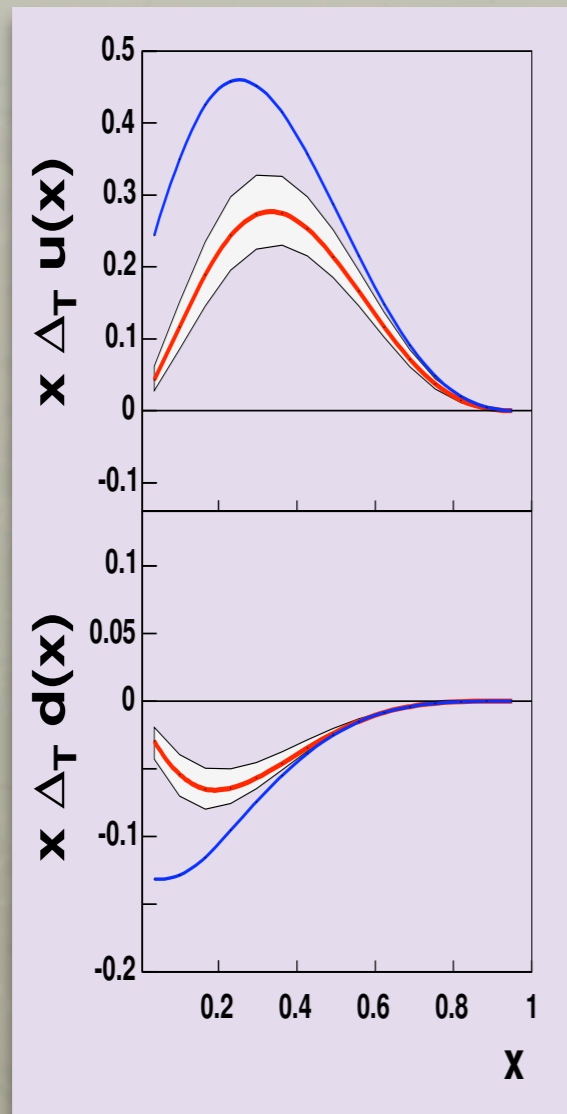
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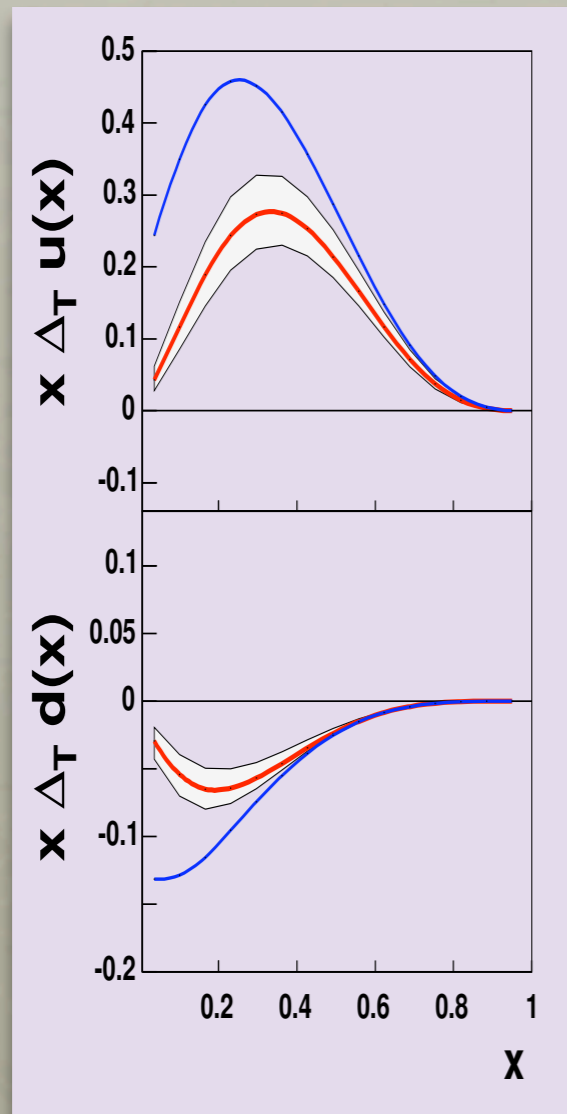
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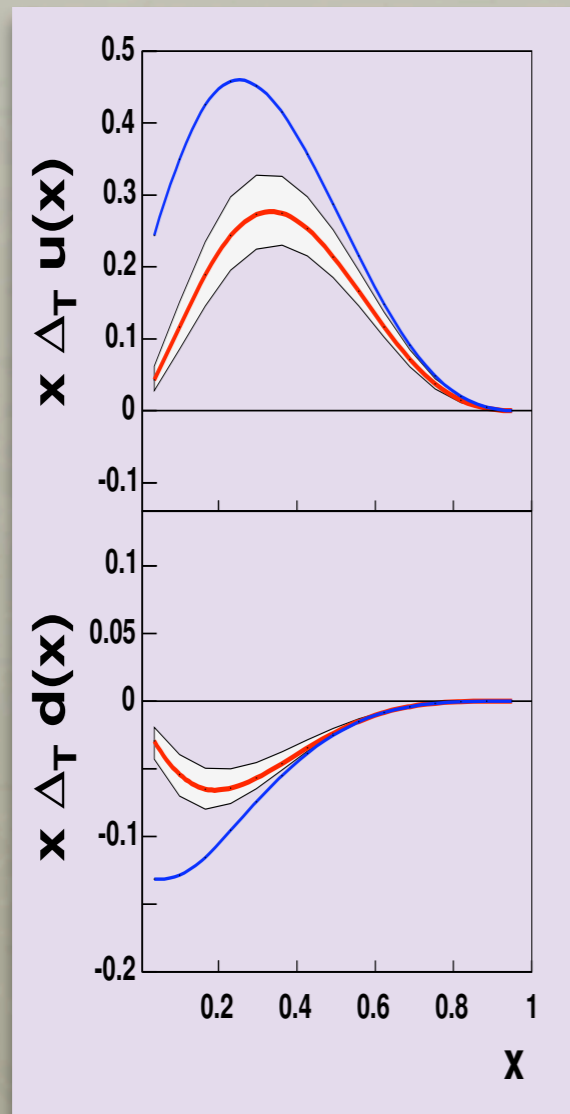
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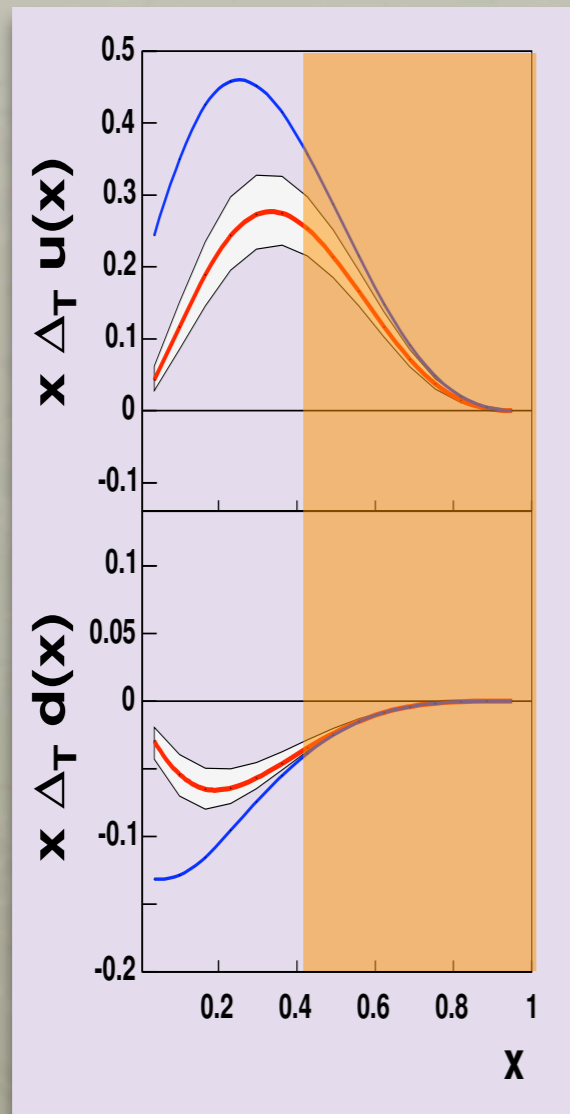
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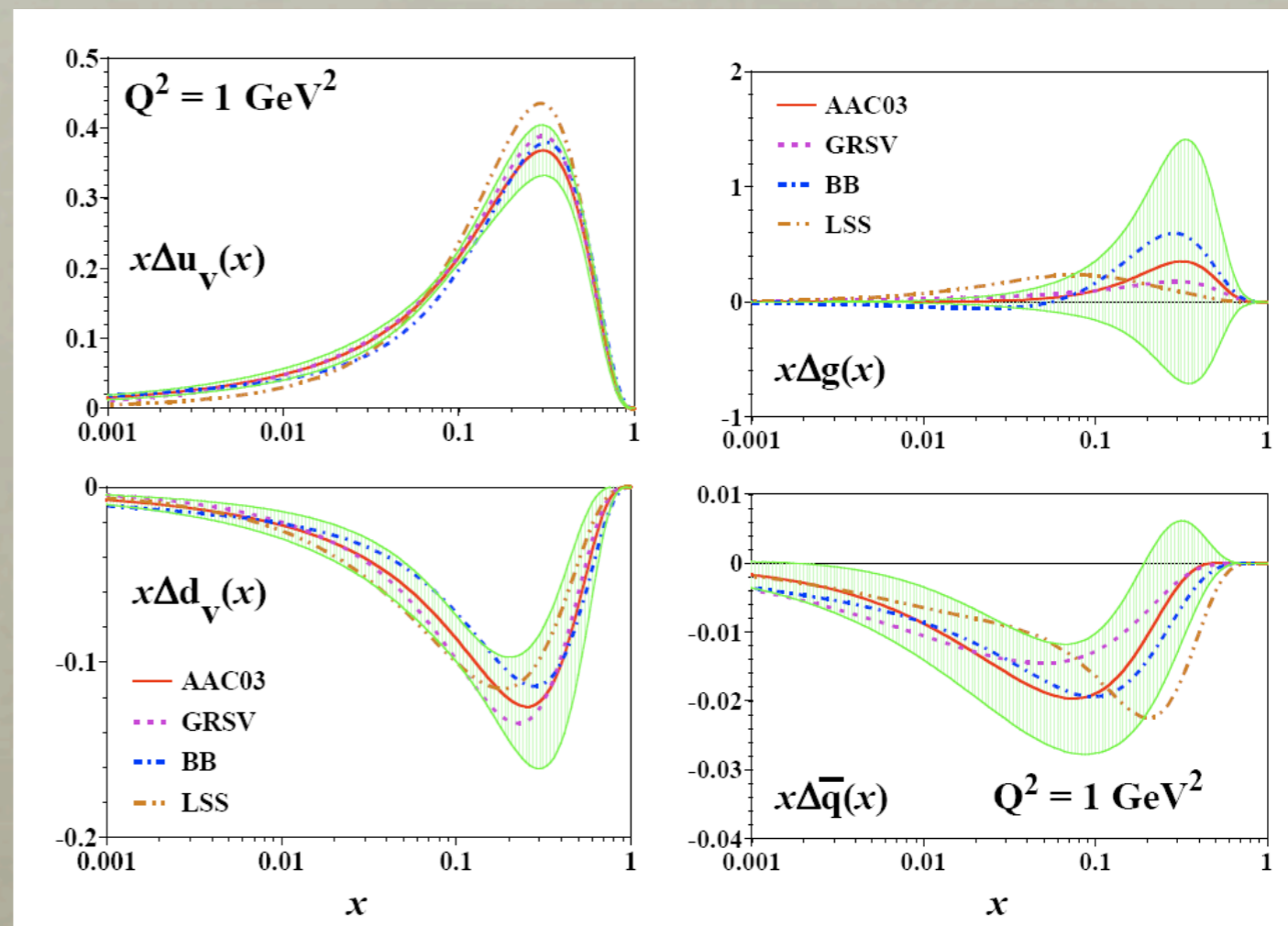
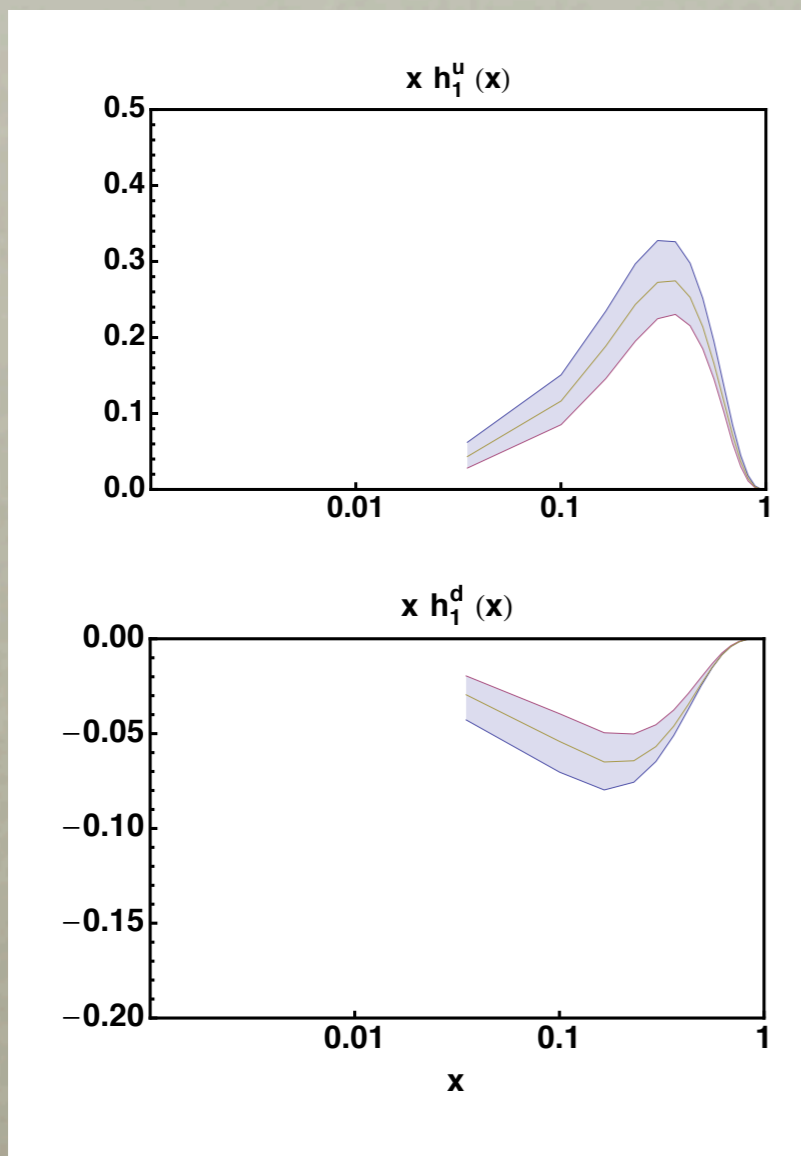
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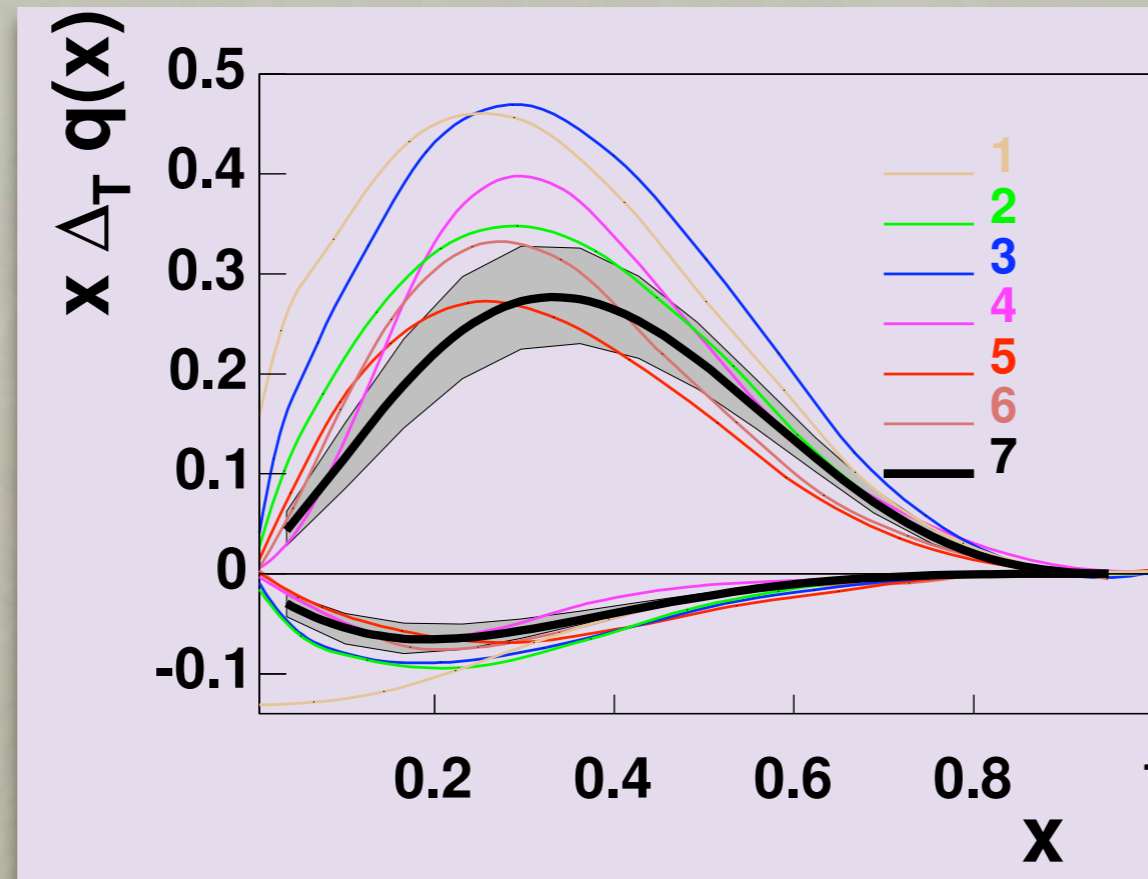
A. Prokudin, talk at DIS08 (Anselmino et al, 0807.0173)



Anselmino et al, 0807.0173

AAC, Hirai et al. PRD 69 (04)

Comparison with models



[1] Soffer et al. PRD 65 (02)

[4] Wakamatsu, PLB 509 (01)

[2] Korotkov et al. EPJC 18 (01)

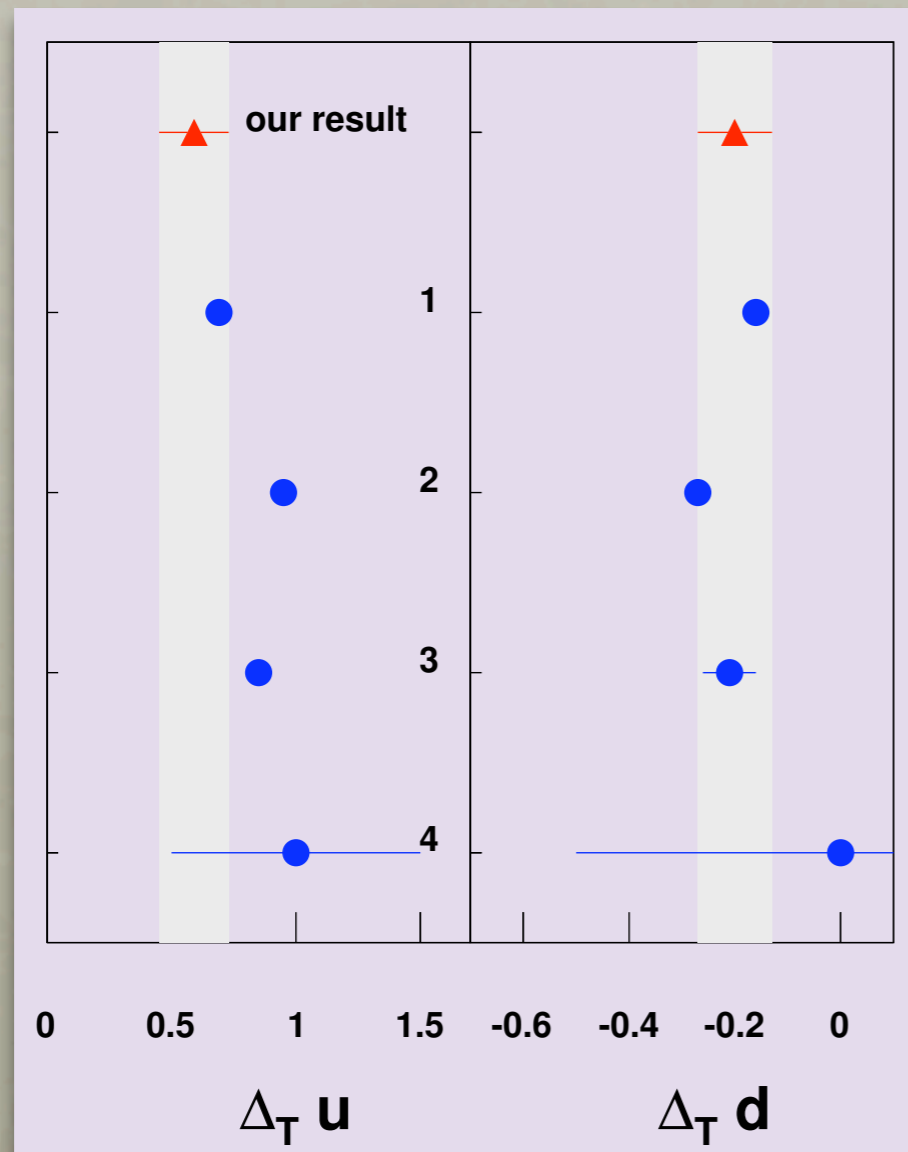
[5] Pasquini et al., PRD 72 (05)

[3] Schweitzer et al., PRD 64 (01)

[6] Cloet, Bentz, Thomas, PLB 659 (08)

Nucleon tensor charges

Integrals over x of transversity



[our result] Anselmino et al. DIS 08

[1] Diquark spectator model,
Cloet, Bentz, Thomas, PLB 659 (08)

[2] Chiral quark soliton model,
Wakamatsu, PLB 653 (07)

[3] Lattice QCD,
Goekeler et al. PLB 627 (05)

[4] QCD sum rules,
He, Ji, PRD 52 (95)

Transverse momentum distributions (TMDs)

Relation to GPDs

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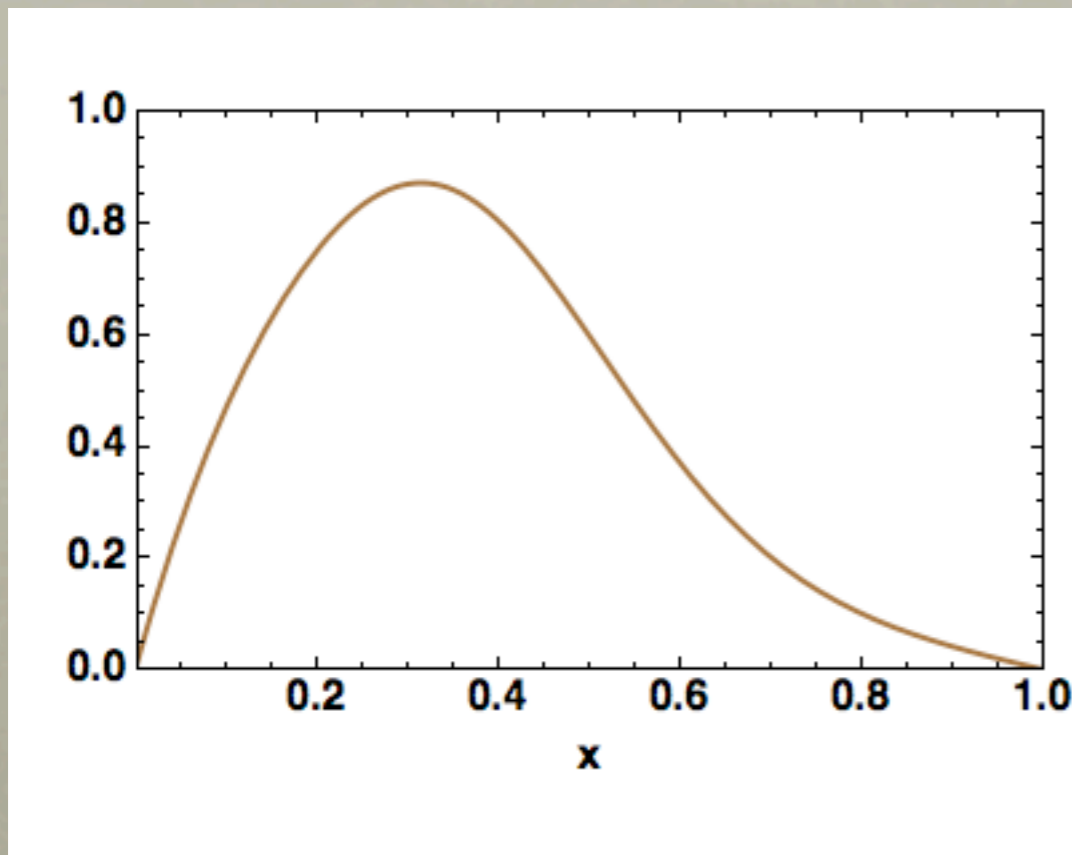
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*X. Ji, PRL 91 (03), Meissner et al. arXiv:0805.3165
for even more dim. (8), see Collins, Rogers, Stasto, PRD77 (08)*

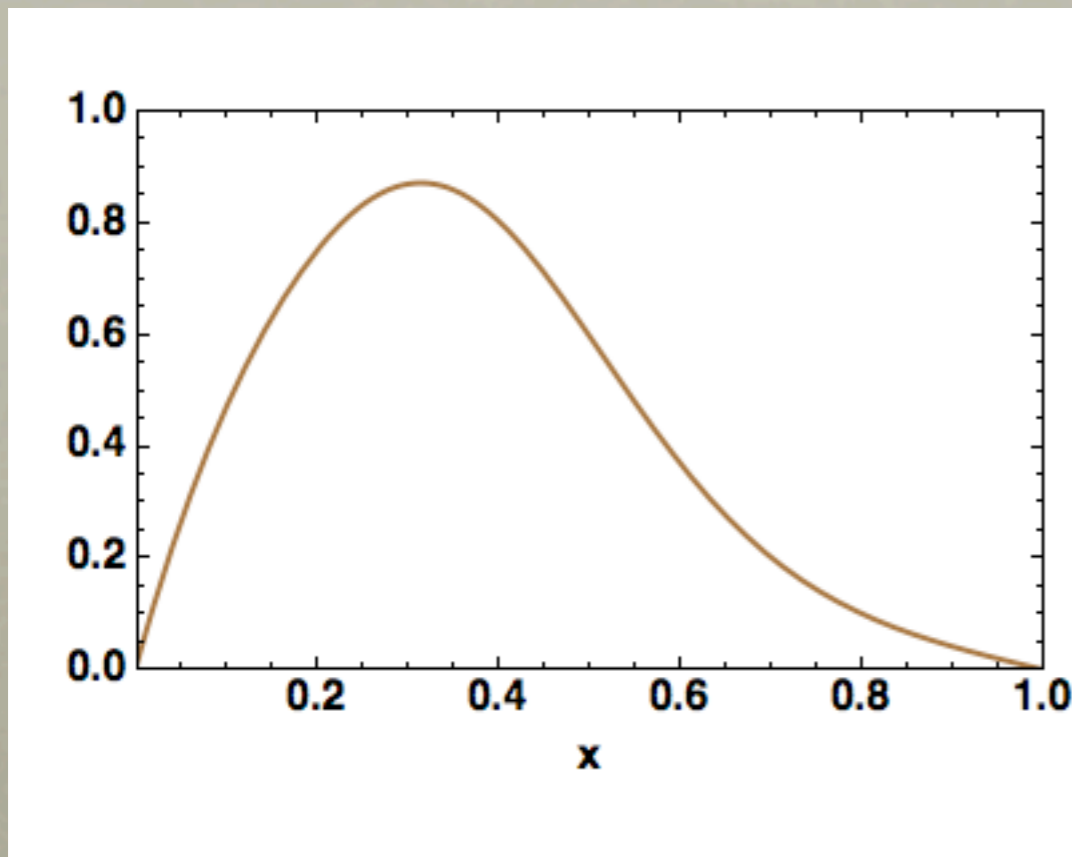
Transverse momentum distributions

$$x f_1^u(x)$$

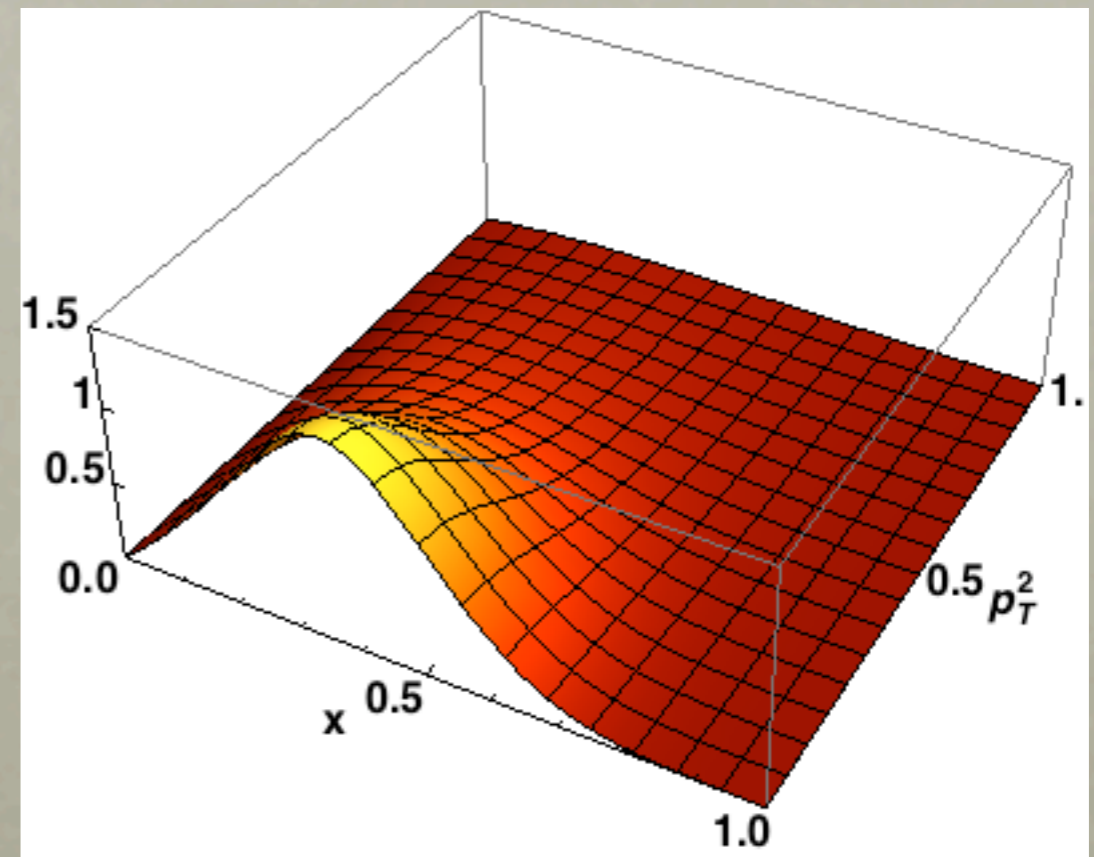


Transverse momentum distributions

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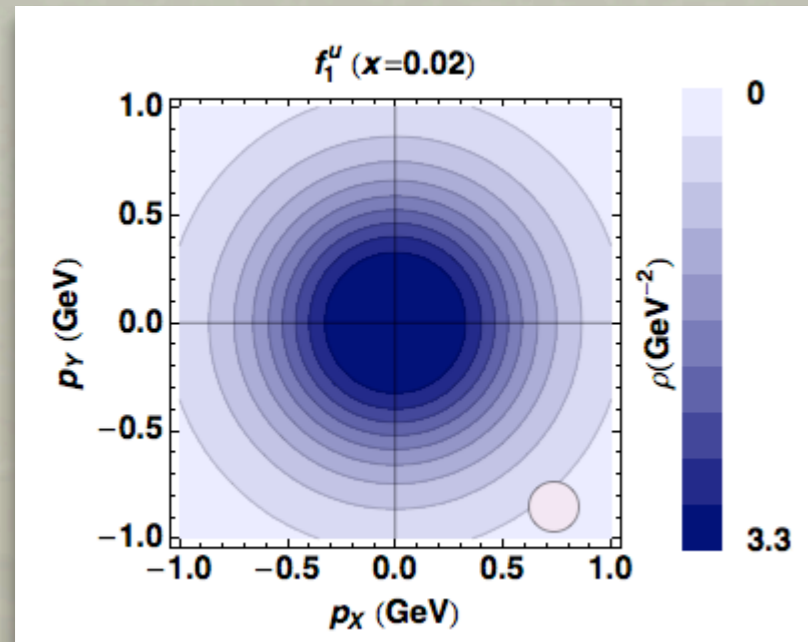


$$x f_1^u(x, p_T^2)$$

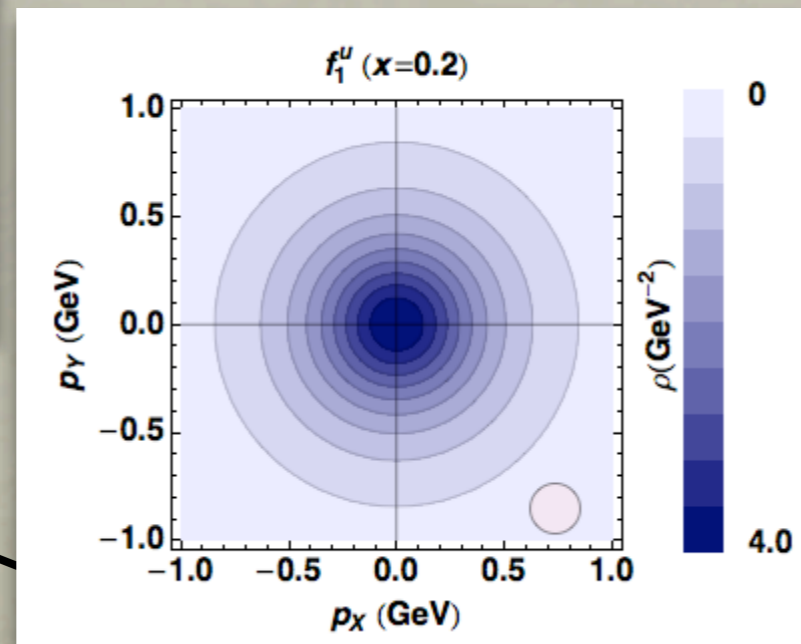


*A.B., F. Conti, M. Radici, arXiv:0807.0323
see also talk by B. Pasquini*

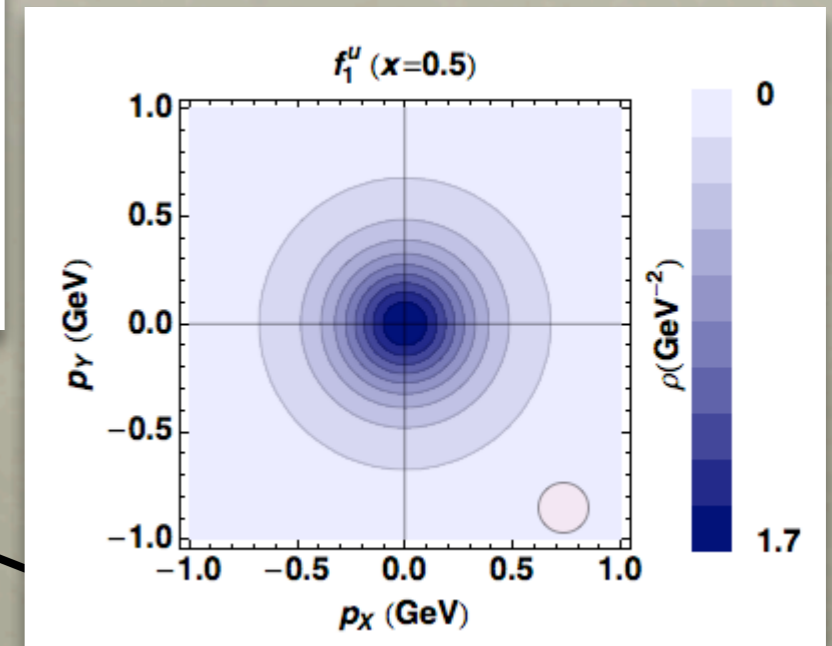
Nucleon tomography in momentum space



0.02



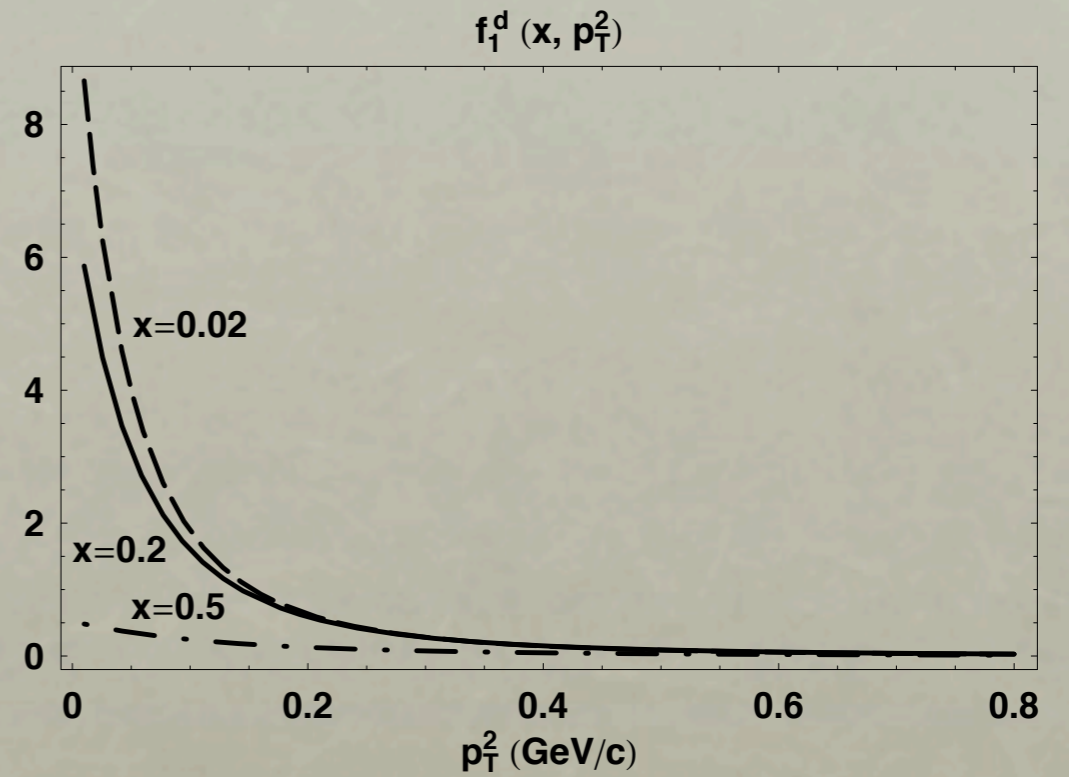
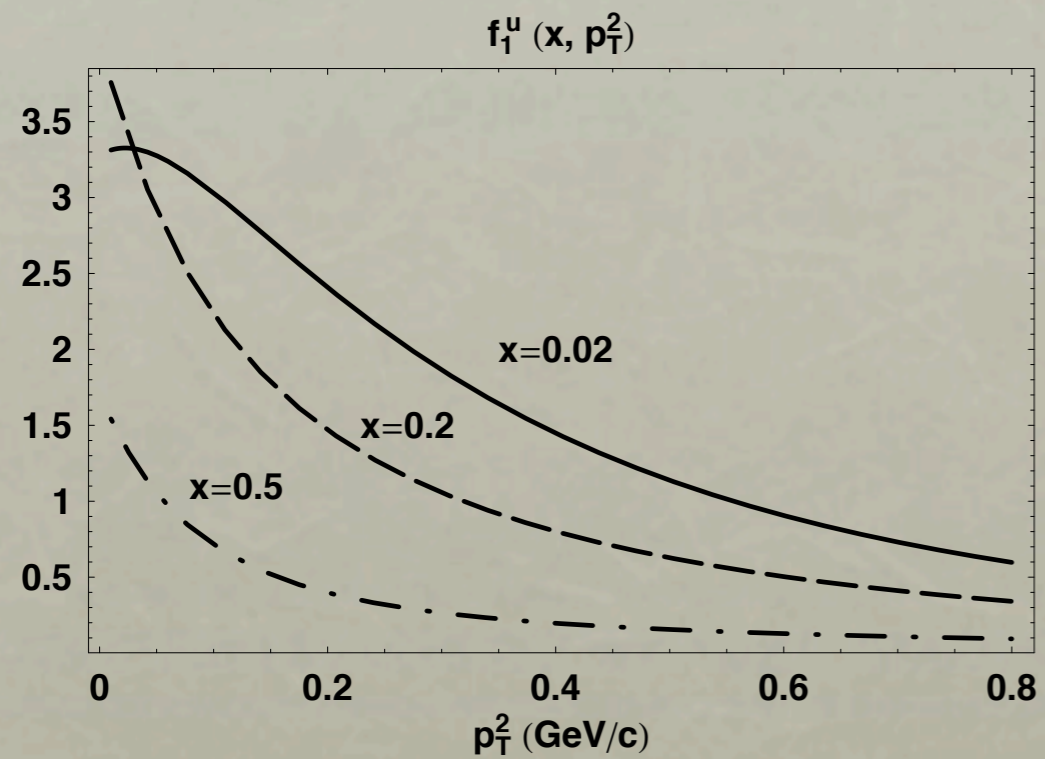
0.2



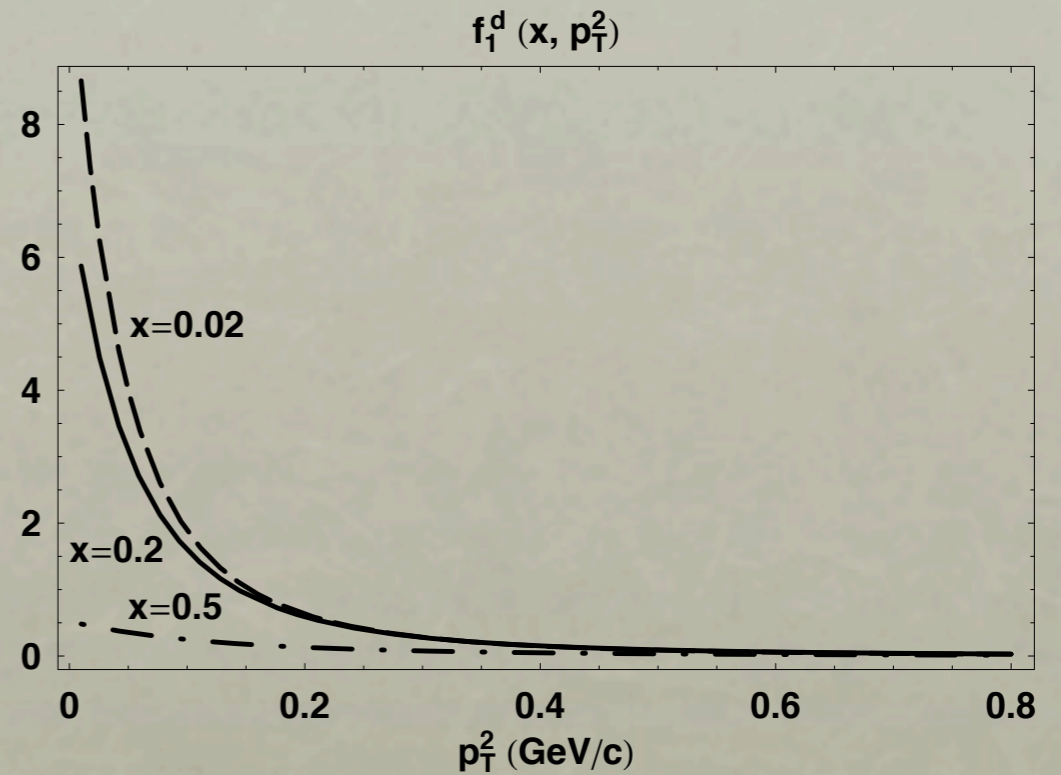
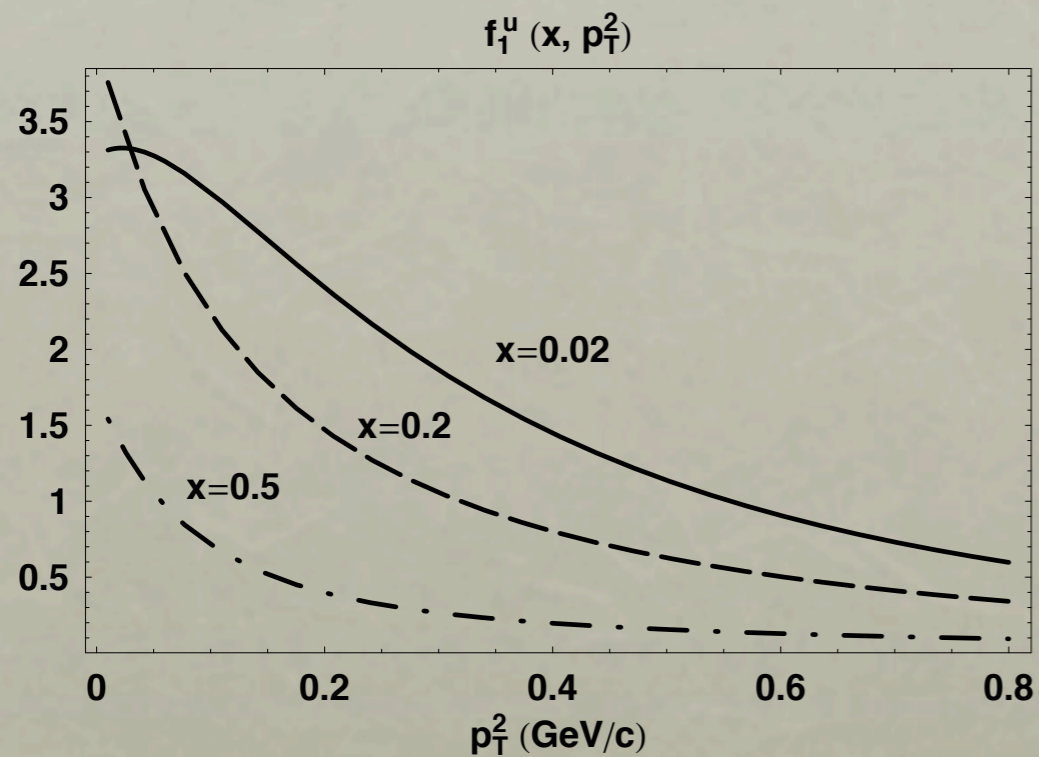
0.5

x

Nontrivial features

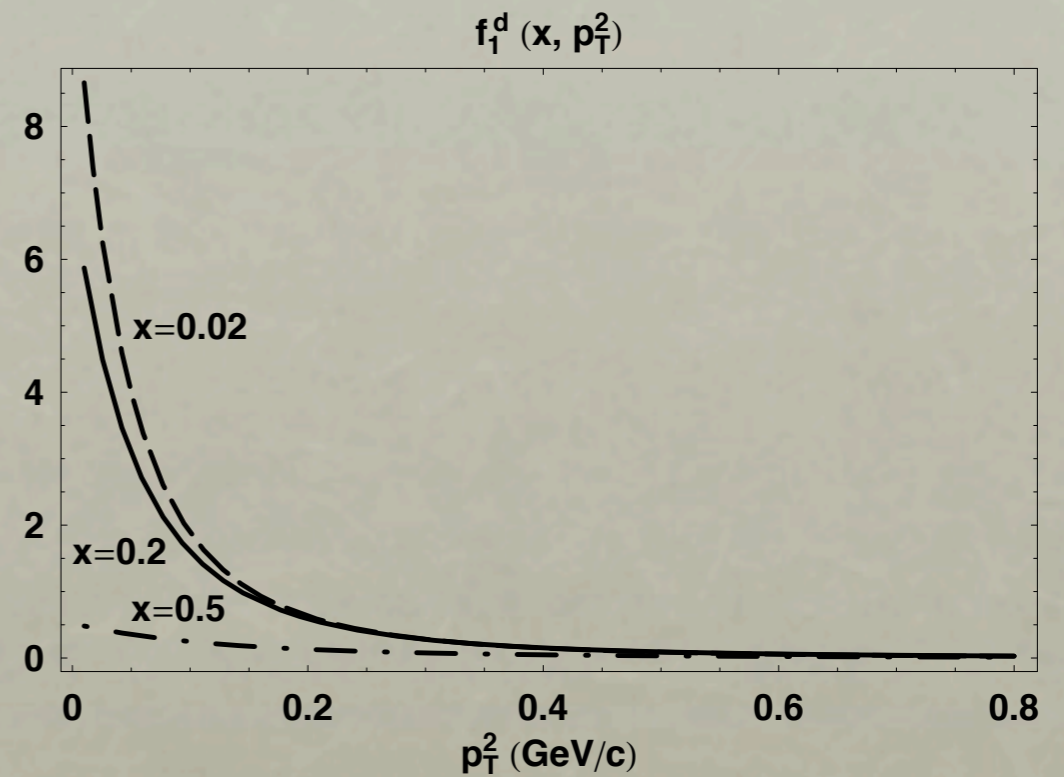
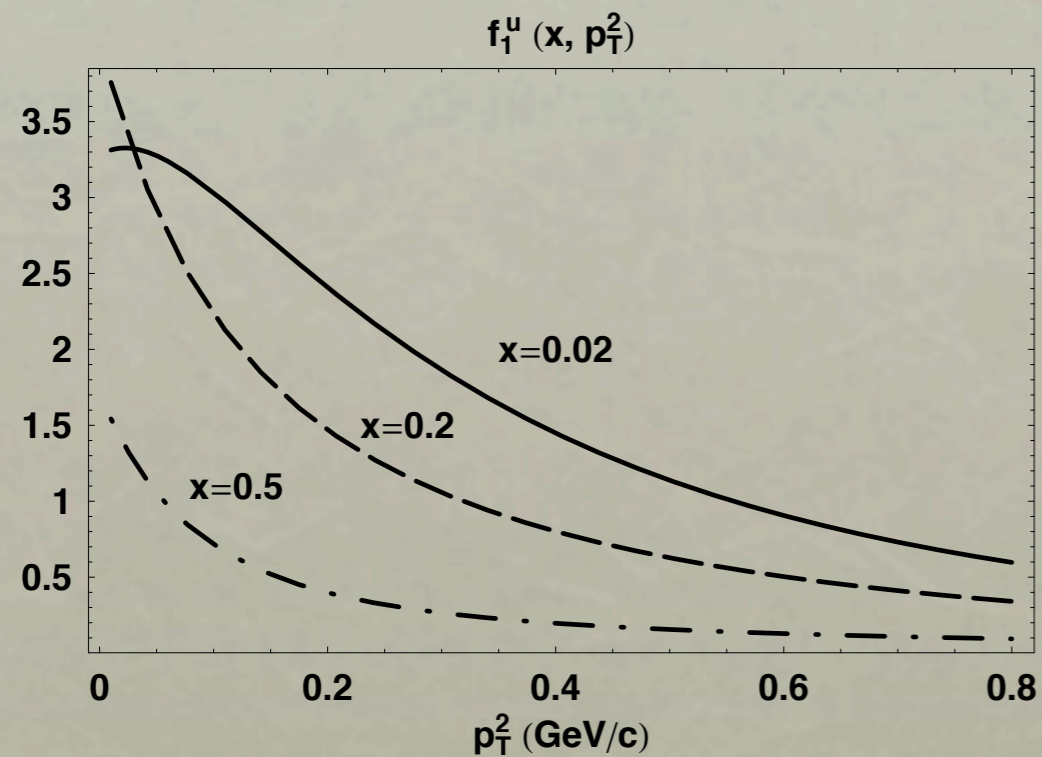


Nontrivial features



Simple model calculations suggests

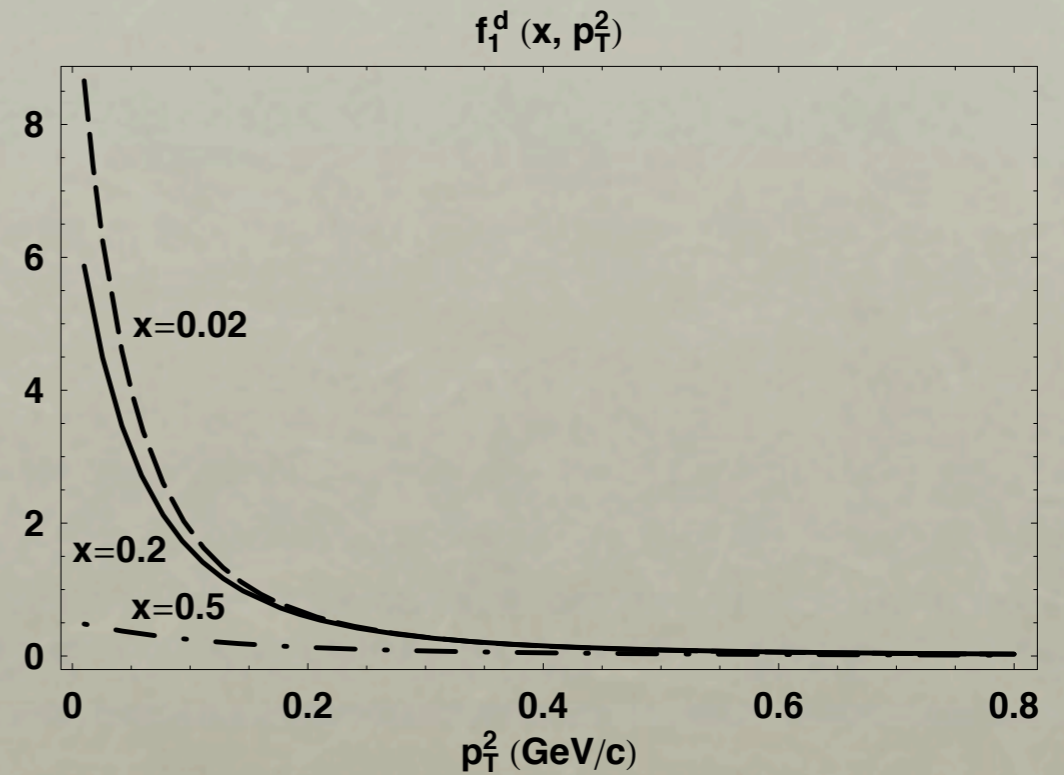
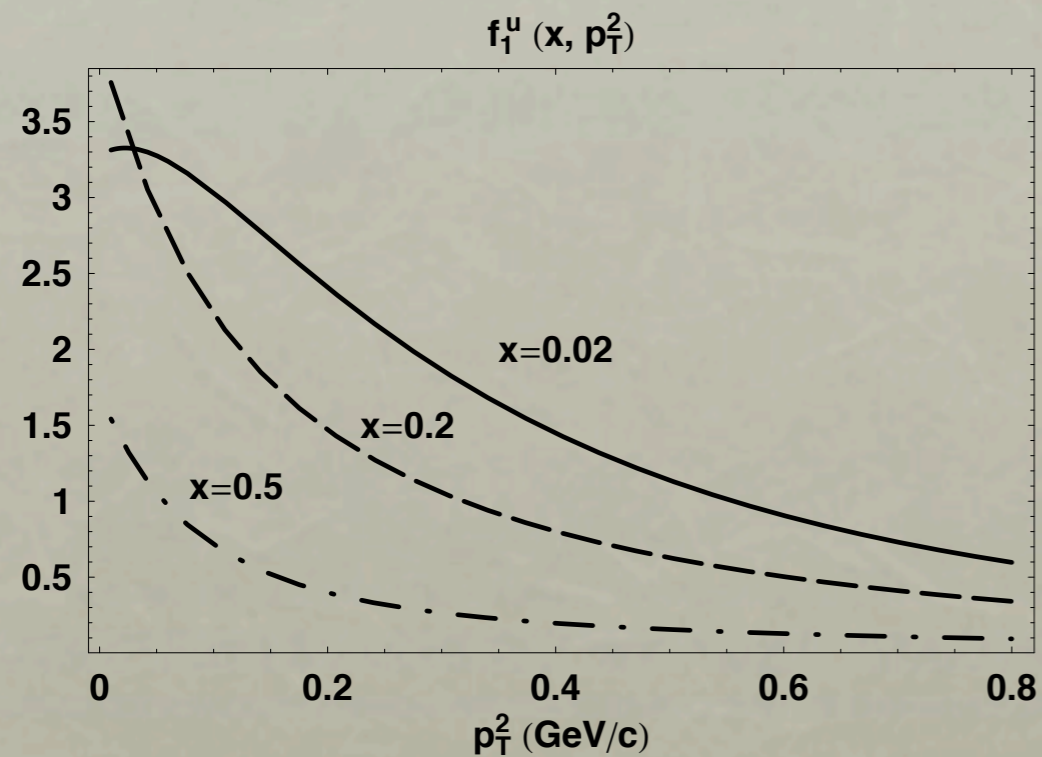
Nontrivial features



Simple model calculations suggests

- x -dependence

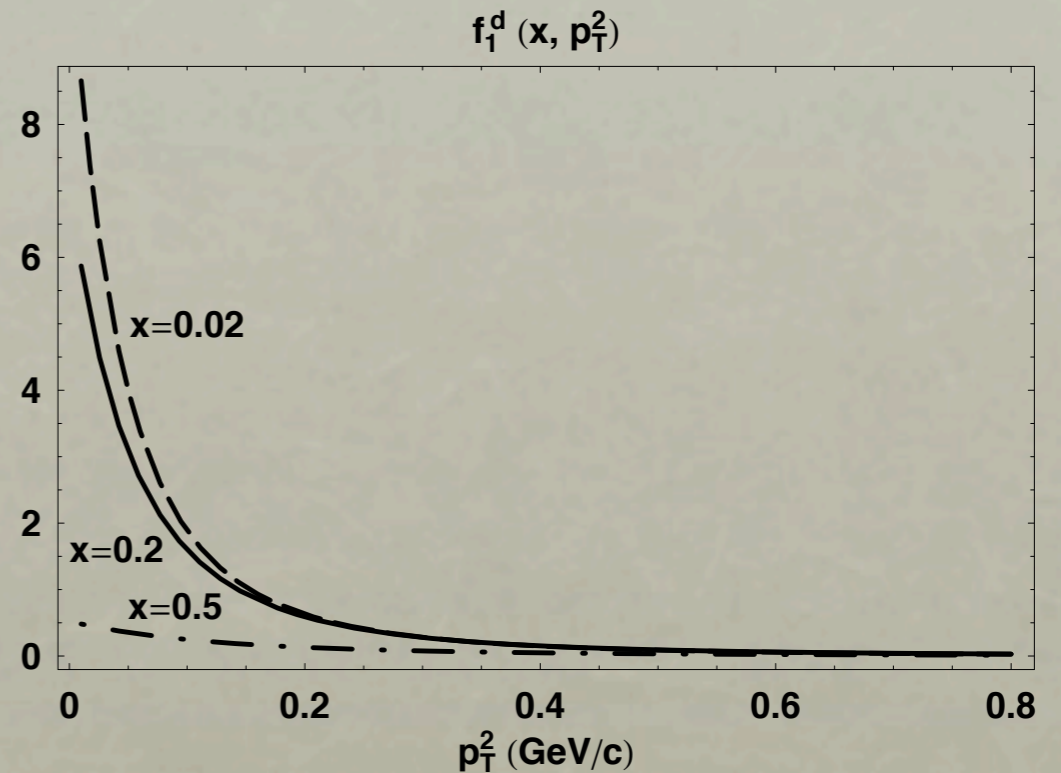
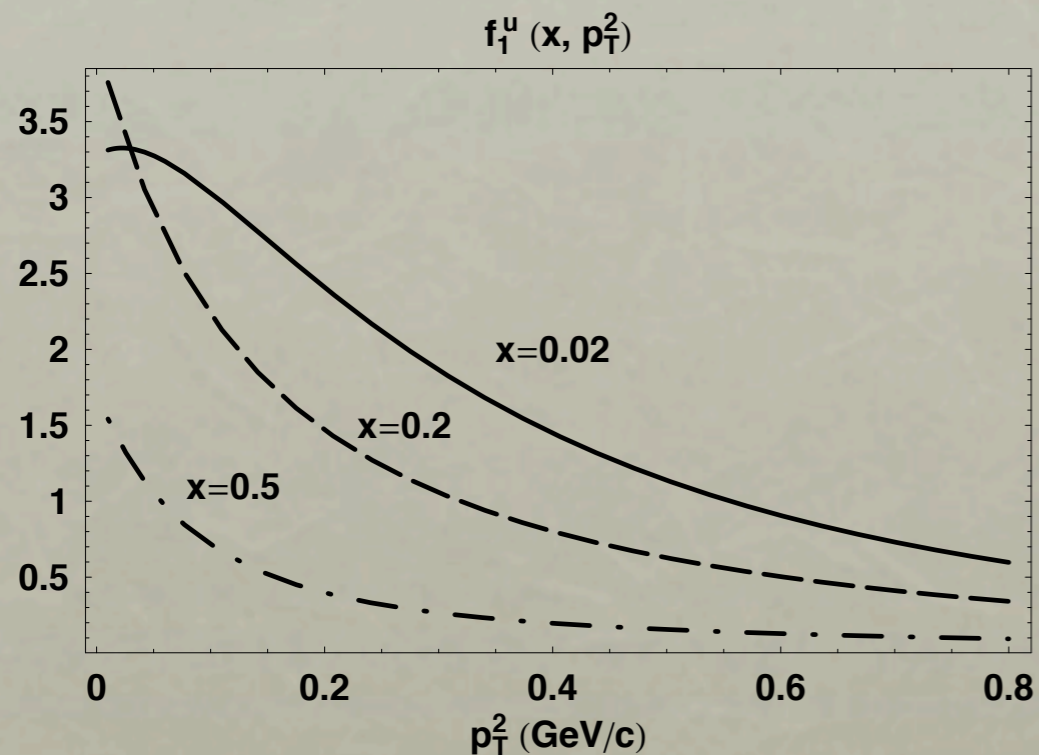
Nontrivial features



Simple model calculations suggests

- x -dependence
- flavor dependence

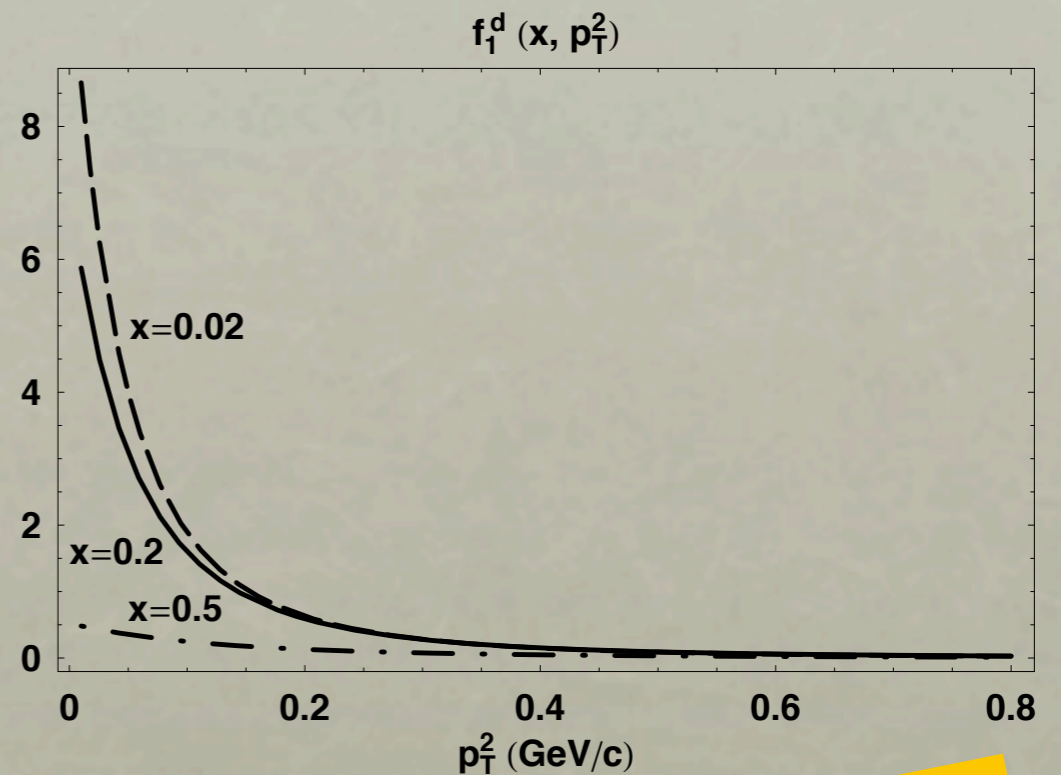
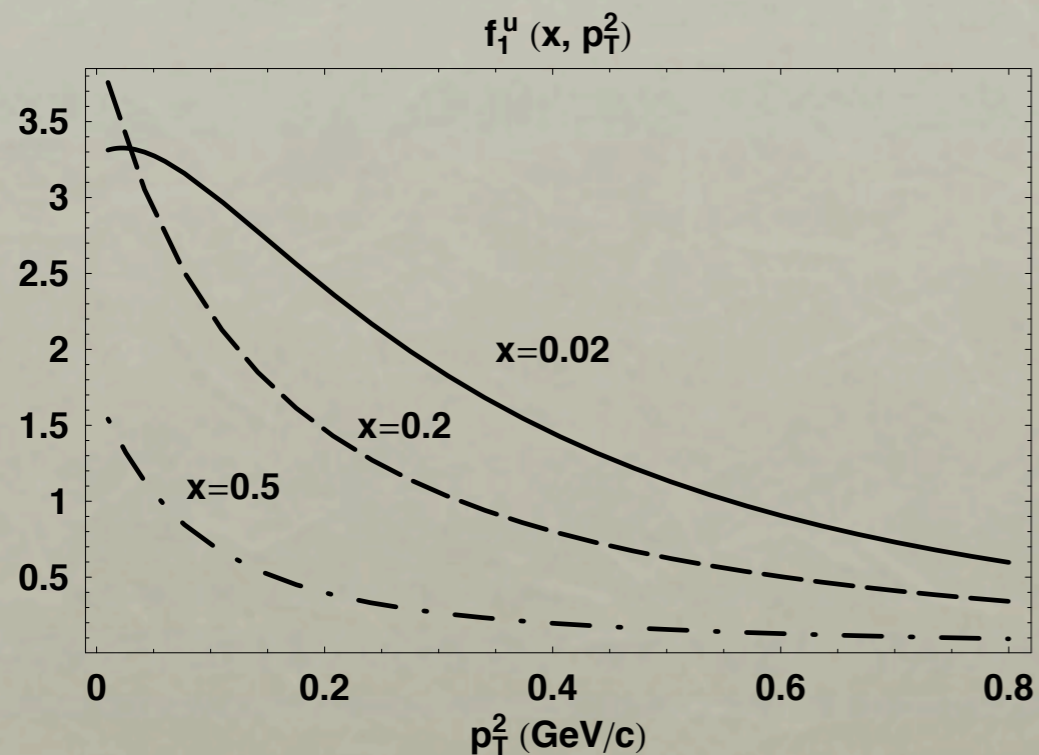
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Fundamental information on the nucleon structure almost as important as standard collinear PDFs

Experimental results

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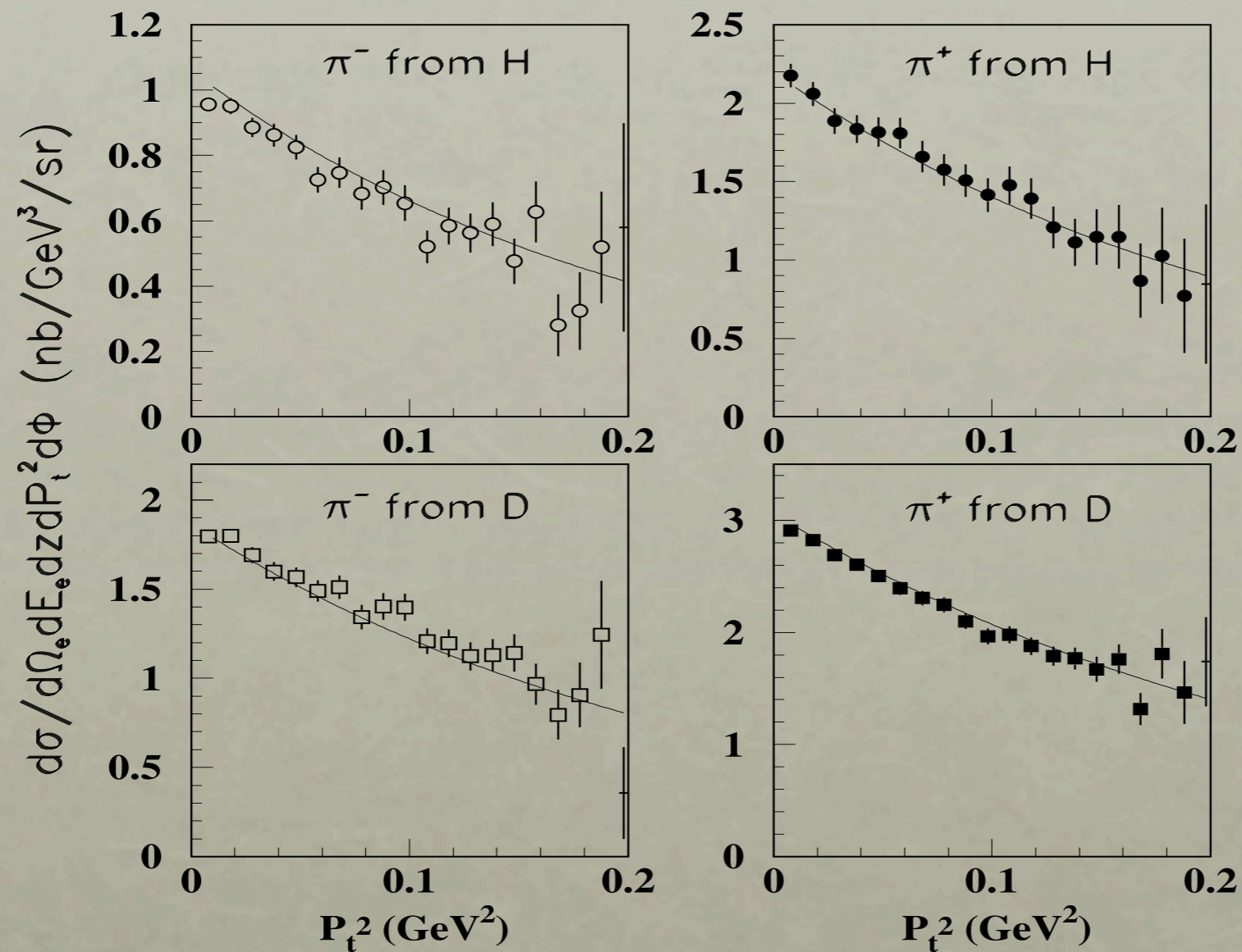
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- Interesting analysis done at JLab Hall C: down quarks have higher transverse momentum than up quarks

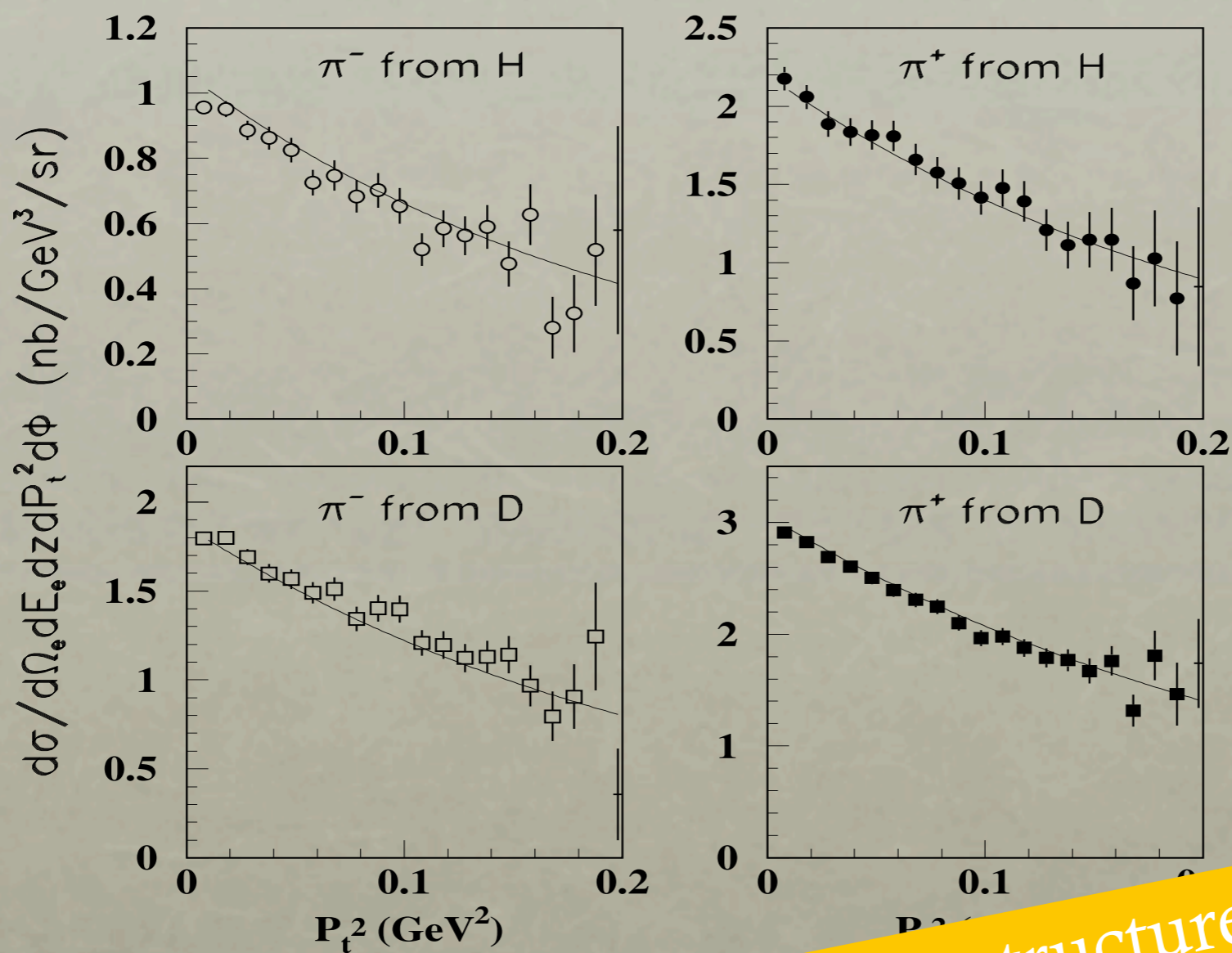
Mkrtchyan et al., arXiv: 0709.3020

SIDIS data with hadron identification



JLab Hall C, Mkrtchyan et al., PLB665 (08)

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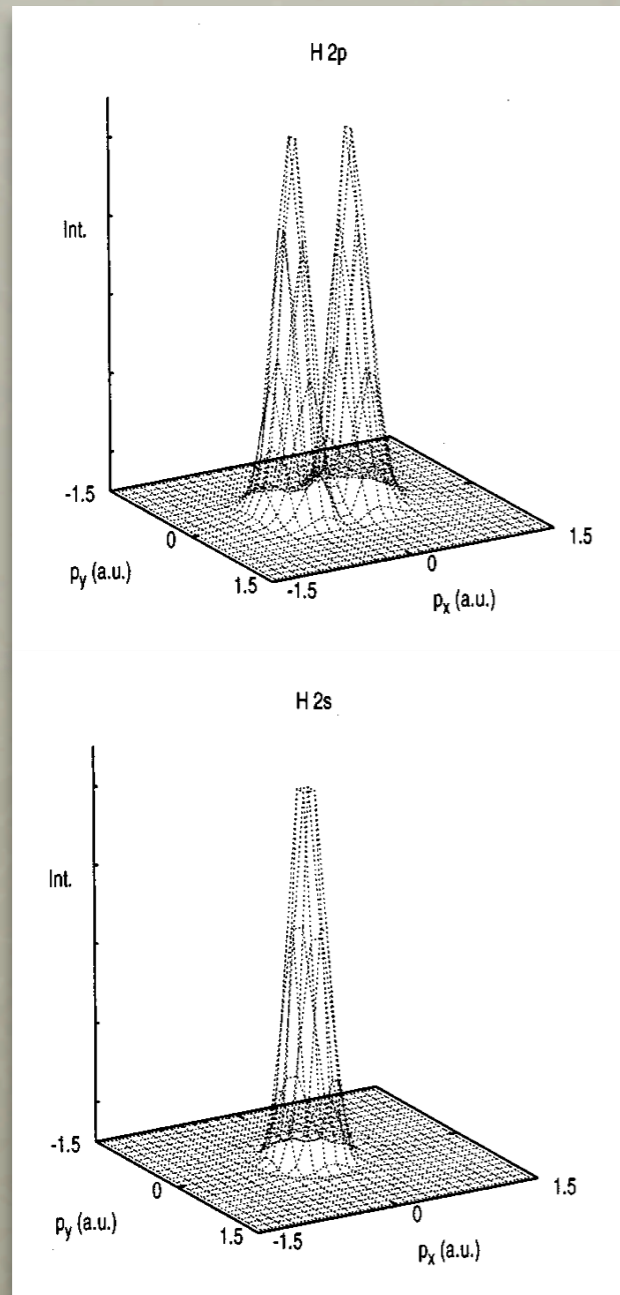


Essential to study flavor structure

JLab Hall C, Mkrtchyan et al., PLB665 (08)

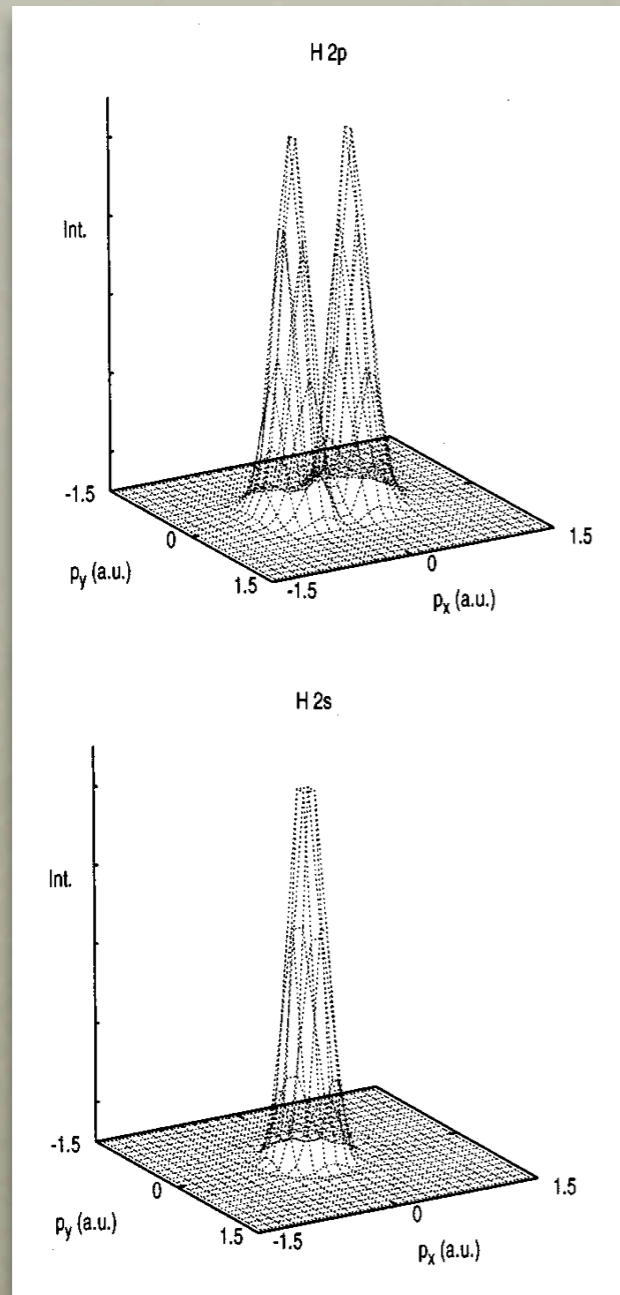
Orbital angular momentum

Hydrogen atom wavefunctions
in momentum space



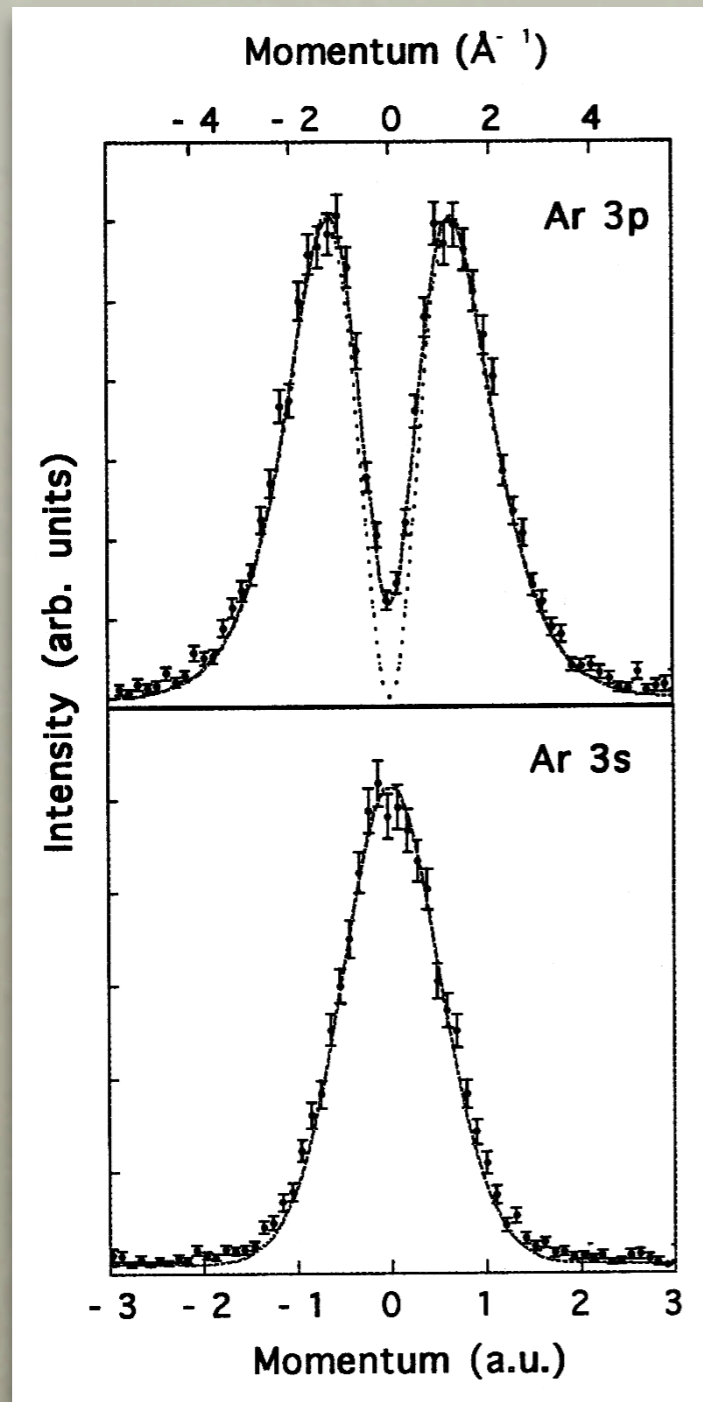
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- In atomic physics, wavefunctions with orbital angular momentum have distinct shapes

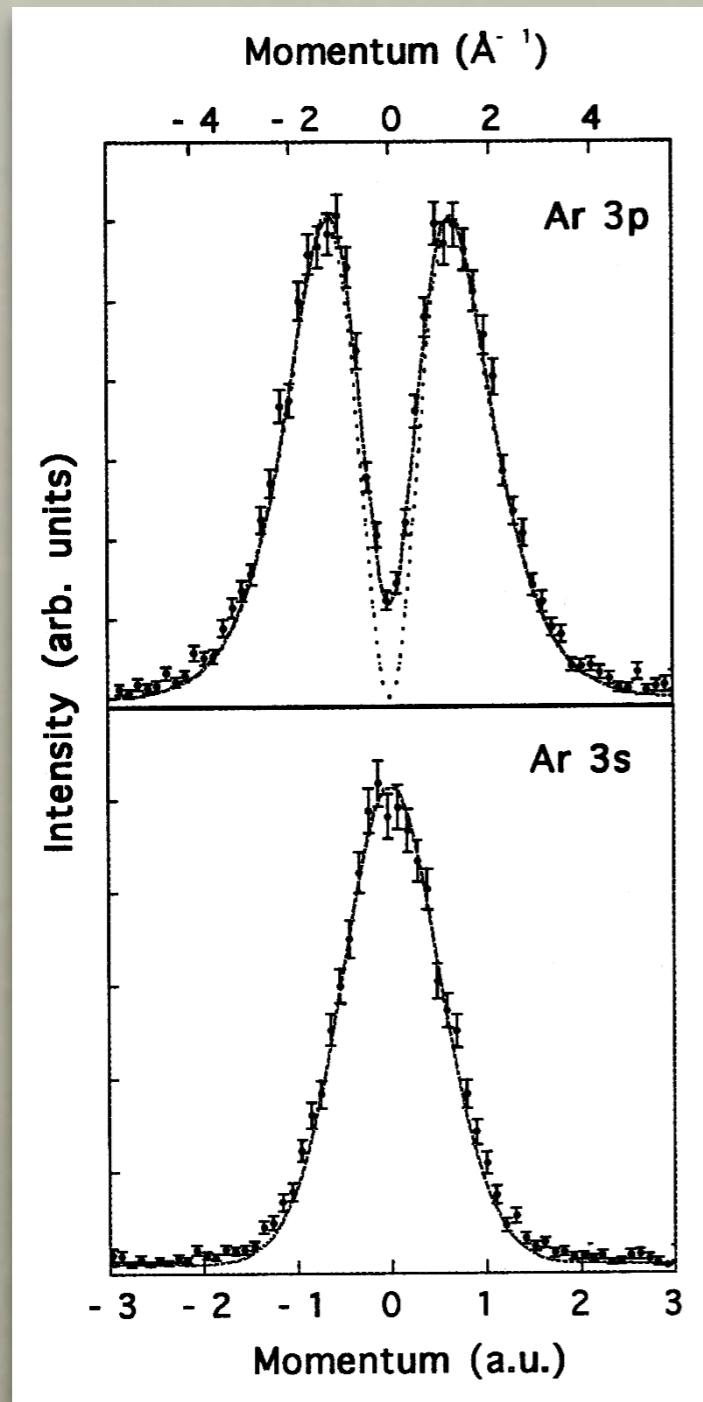
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$$f_1(x, p_T^2) = |\psi_{s\text{-wave}}|^2 + |\psi_{p\text{-wave}}|^2 + \dots$$

Orbital angular momentum

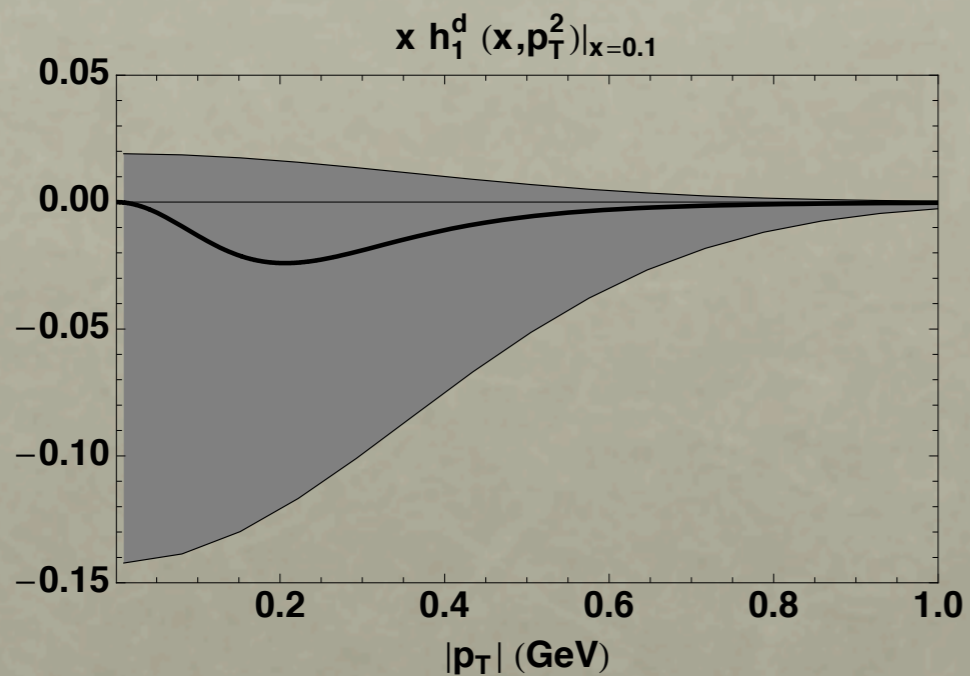
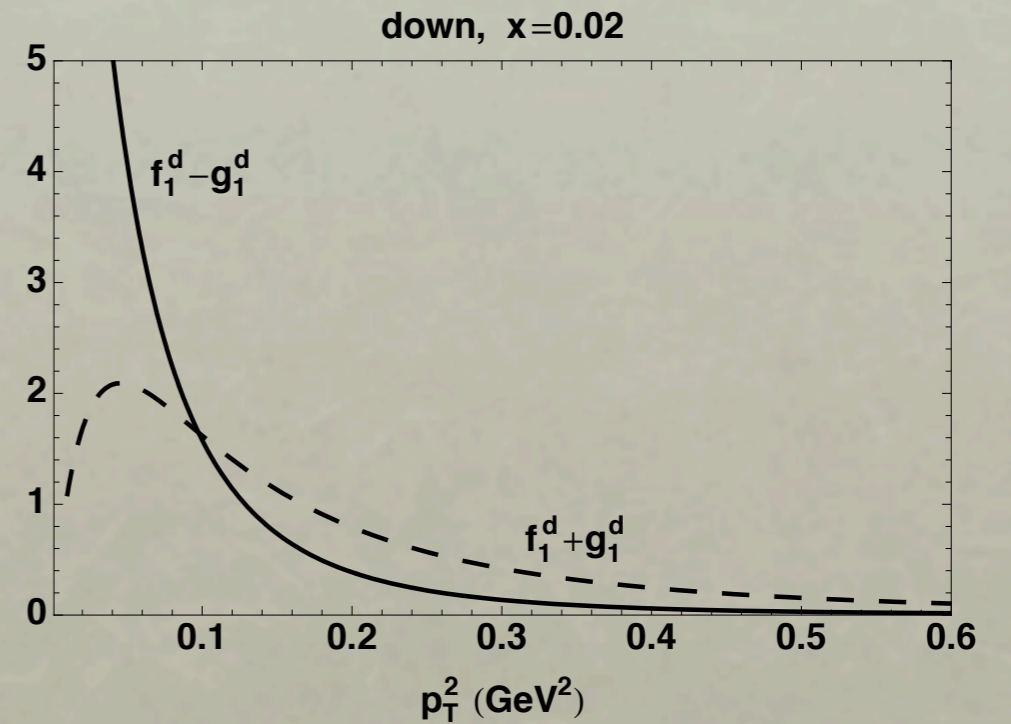
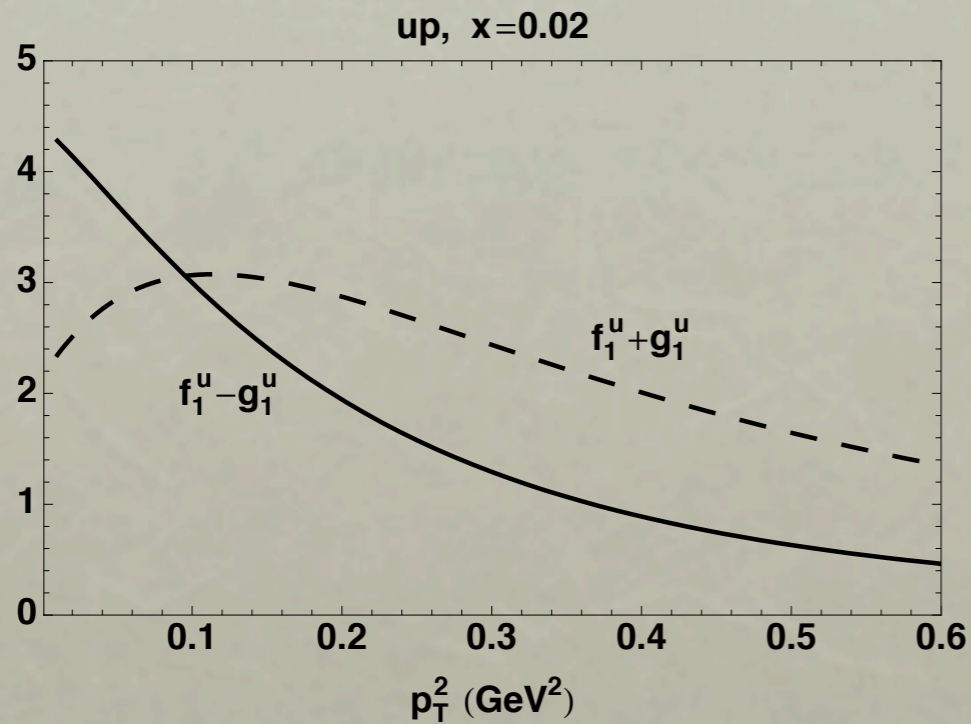


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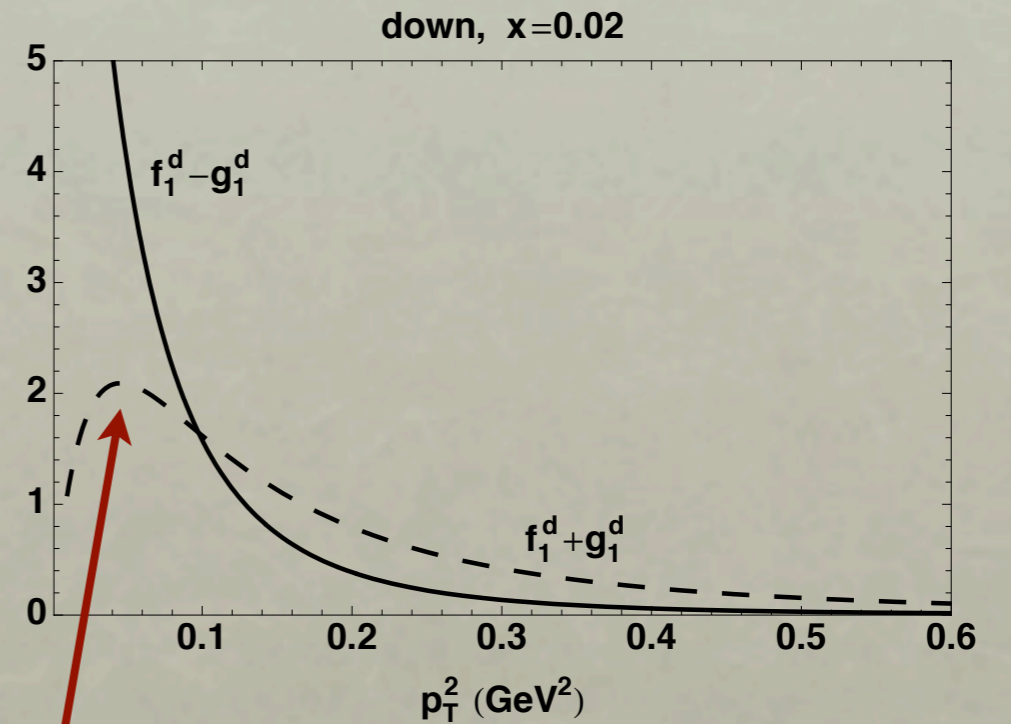
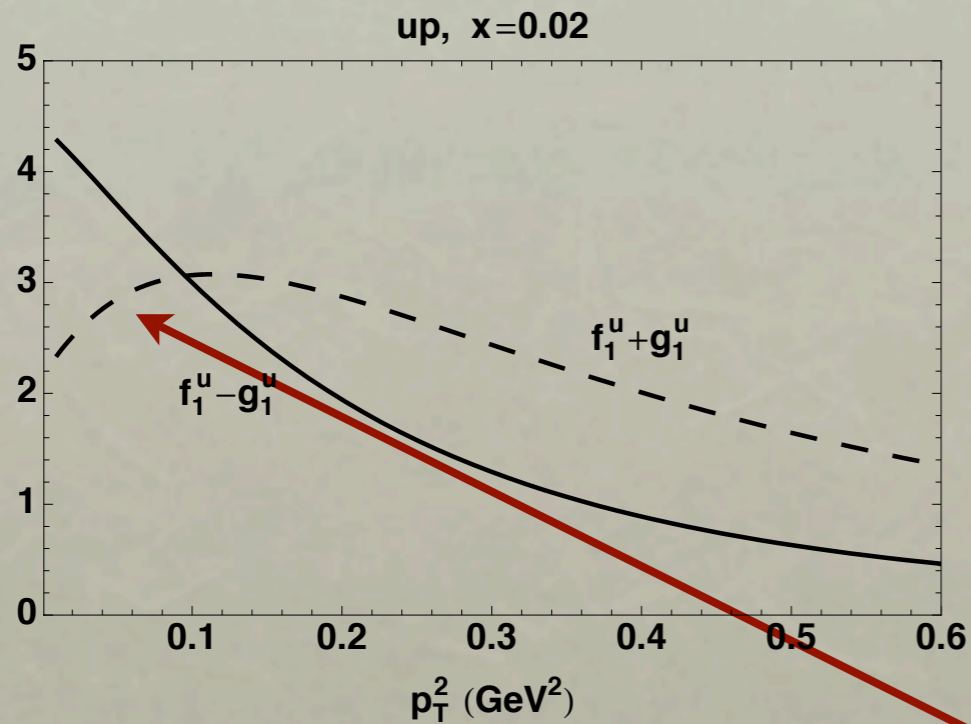
$$\text{At low } p_T \quad |\psi_{p\text{-wave}}|^2 \sim p_T^2$$

TMDs and orbital angular mom.

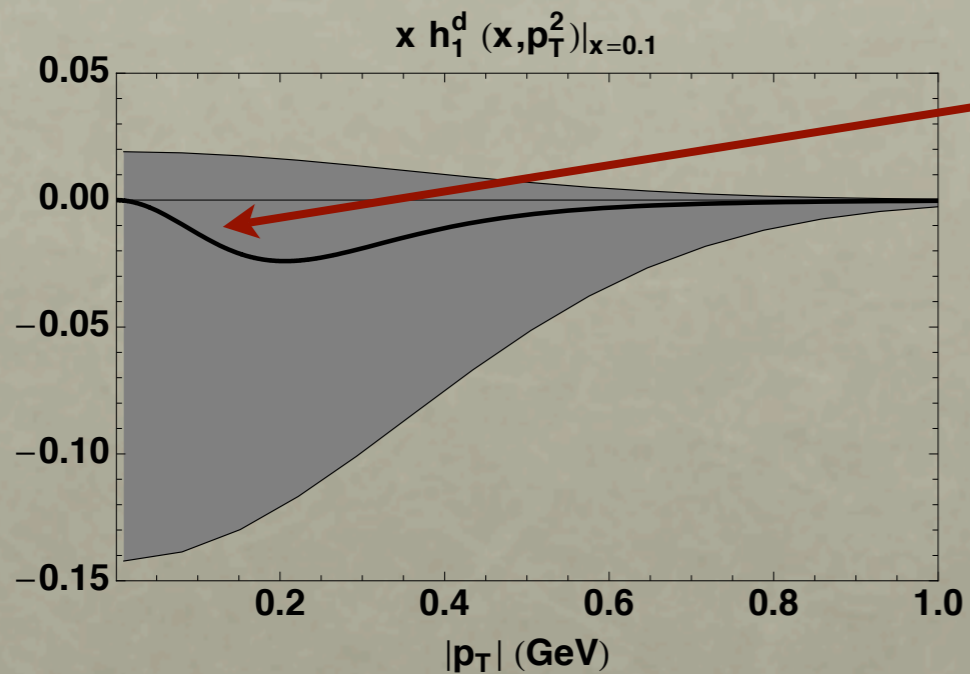


A.B., F. Conti, M. Radici, in preparation

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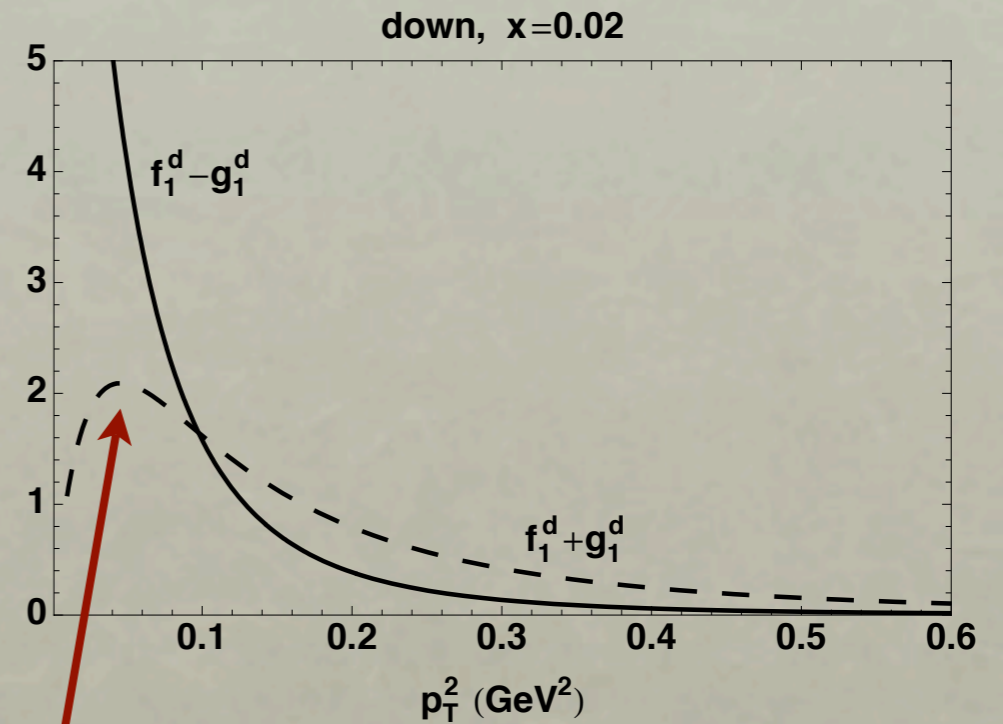
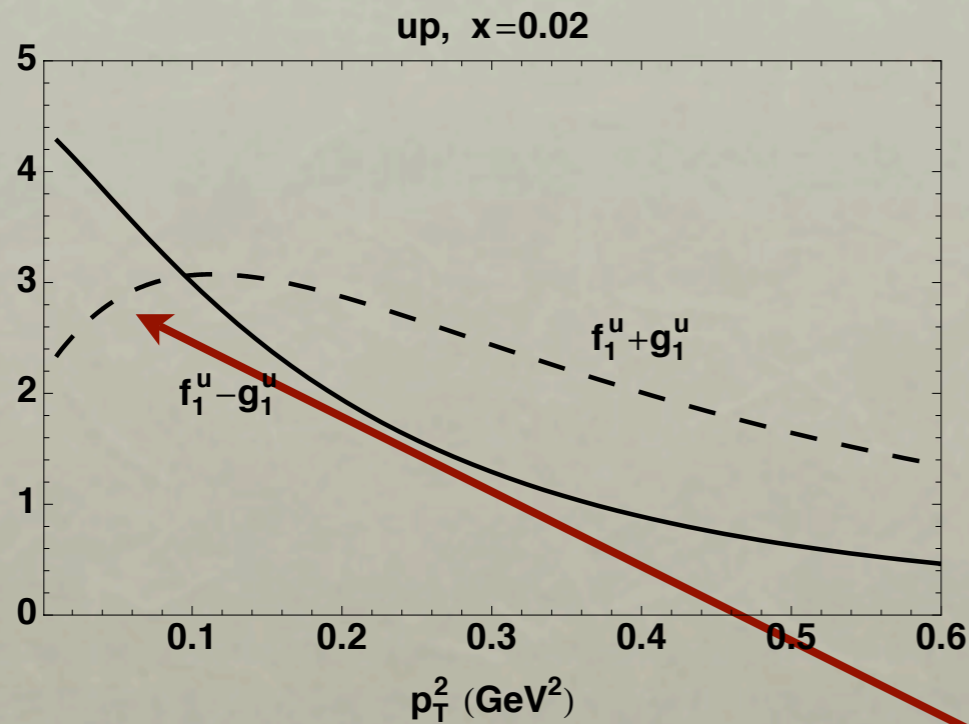


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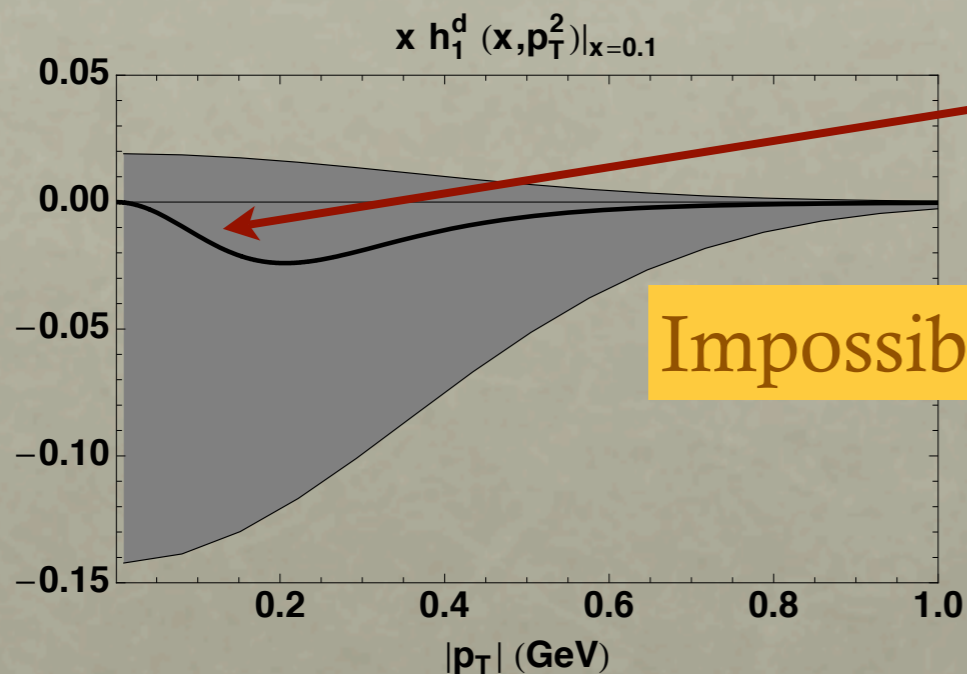


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Signs of orbital ang. mom.



Impossible to reproduce using simple Gaussians

A.B., F. Conti, M. Radici, in preparation

TMDs and orbital angular mom.

quark pol.

	U	L	T
nucleon pol. U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
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- Some TMDs vanish if there is no quark orbital angular momentum, e.g., Sivers function, g_{1T}, \dots

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Twist-2 TMDs

- Some TMDs vanish if there is no quark orbital angular momentum, e.g., Sivers function, g_{1T}, \dots
- Any quantitative statement about the total orbital angular momentum is model-dependent

Sivers function

Two ingredients

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- Final-state interactions (included in the gauge link)

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Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

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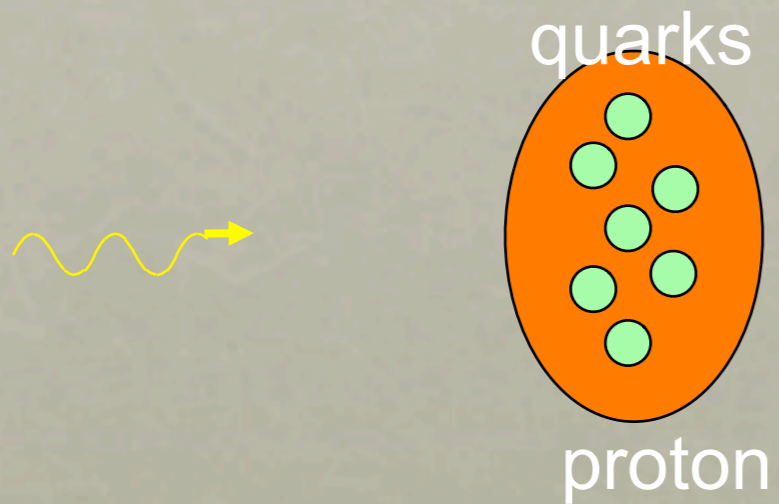
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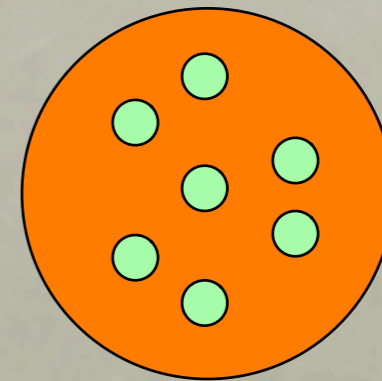
Burkardt, PRD 66 (02); Diehl, EPJ C25 (02); Diehl, Hägler, EPJ C44 (05)

Final-state interactions

Side view



Front view



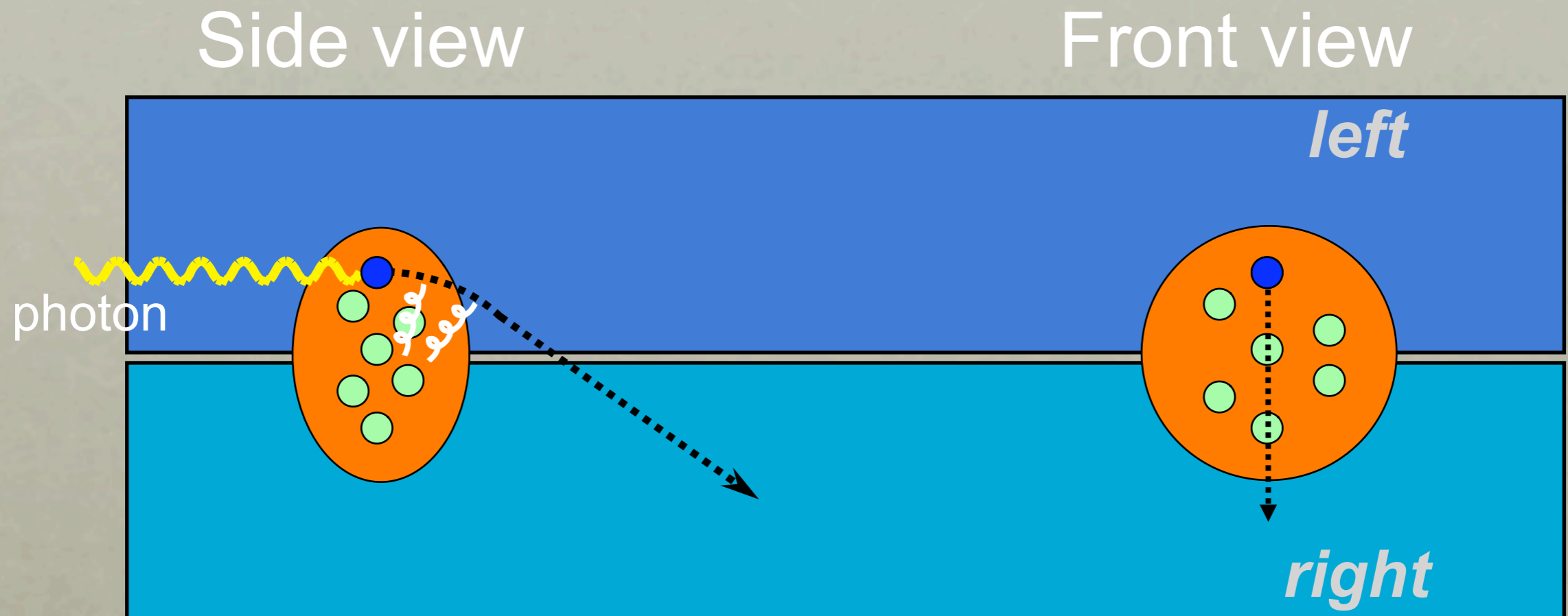
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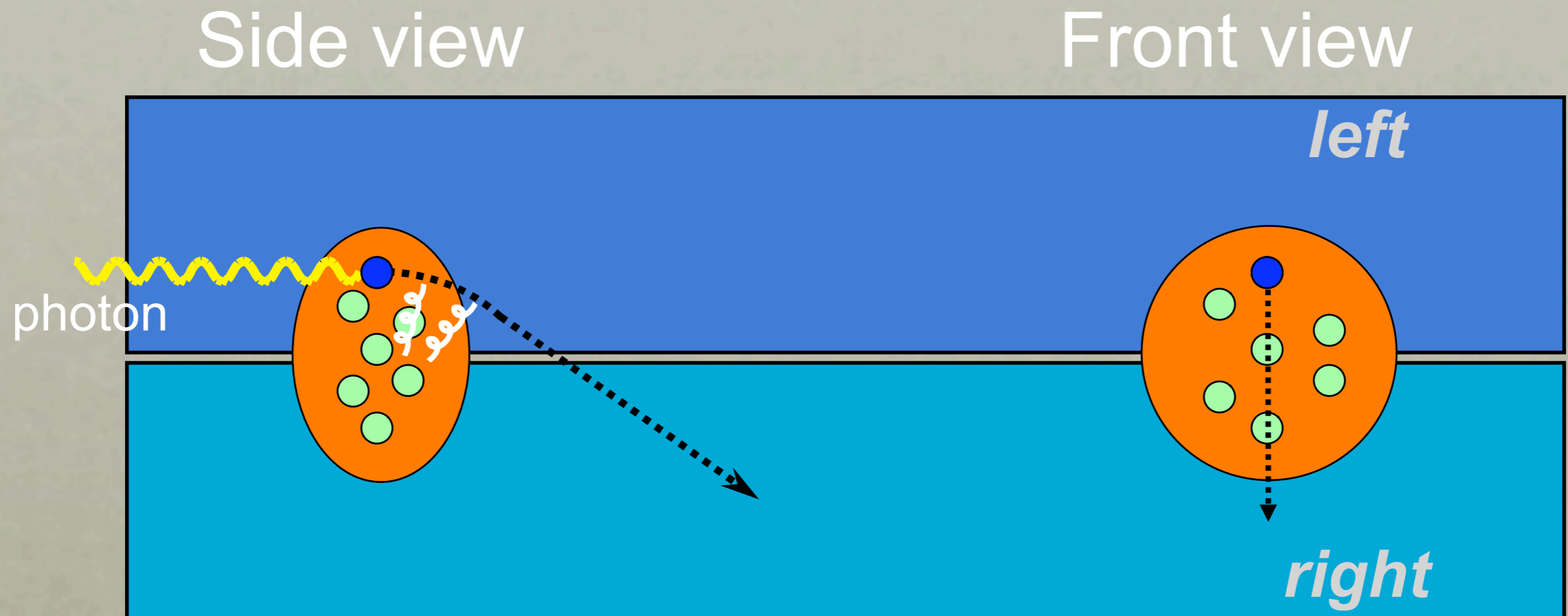
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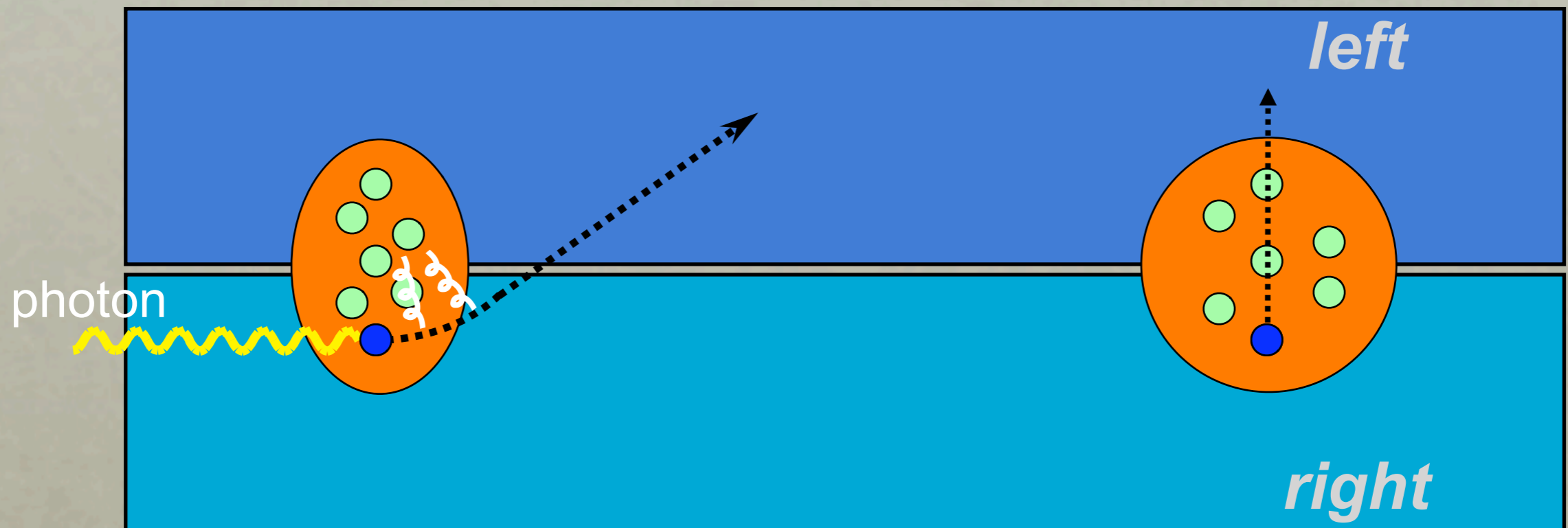


NOTE: QCD tells us that the FSI has to be attractive, since quark and remnants form a color antisymmetric state

Final-state interactions

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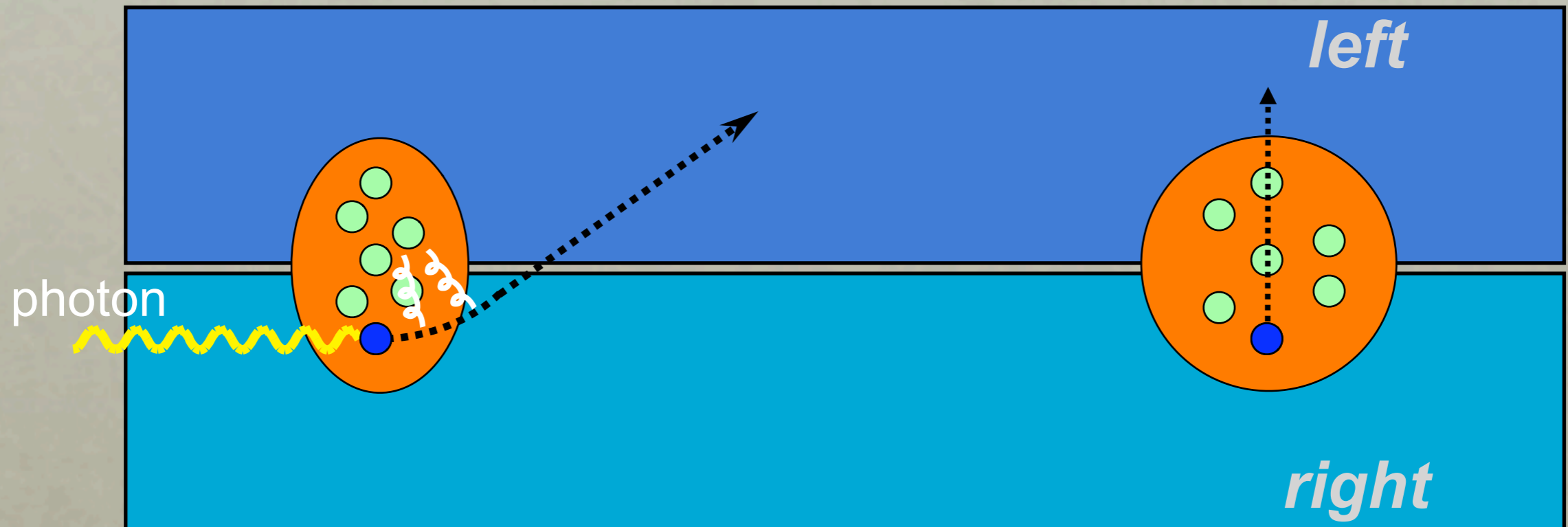
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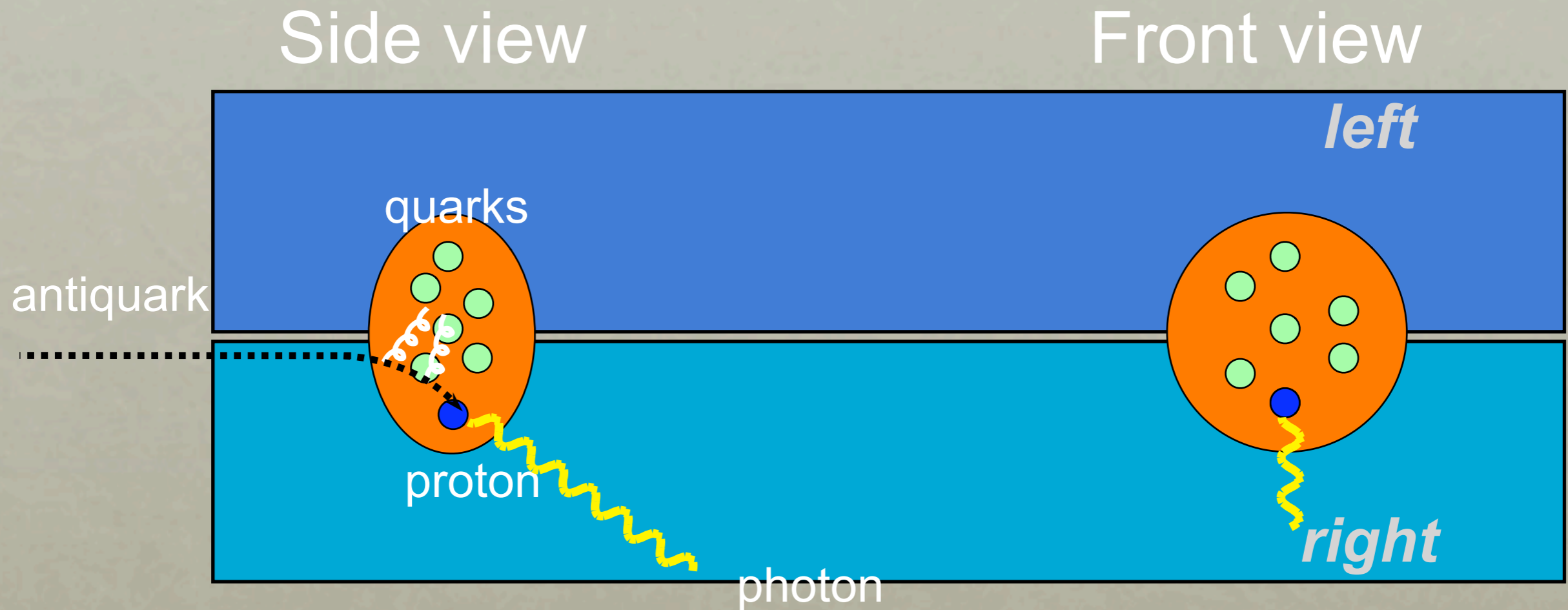
Front view



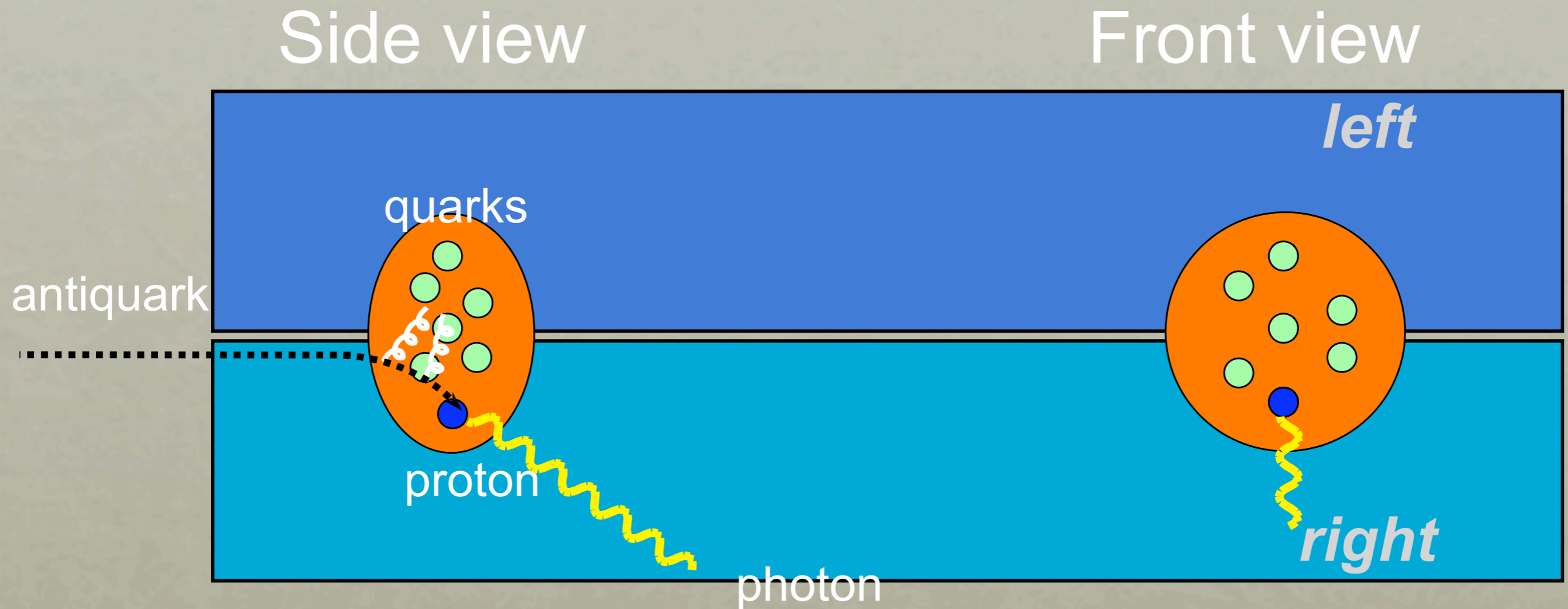
Chromodynamic lensing

Burkardt, PRD 66 (02)

Change of sign in Drell-Yan



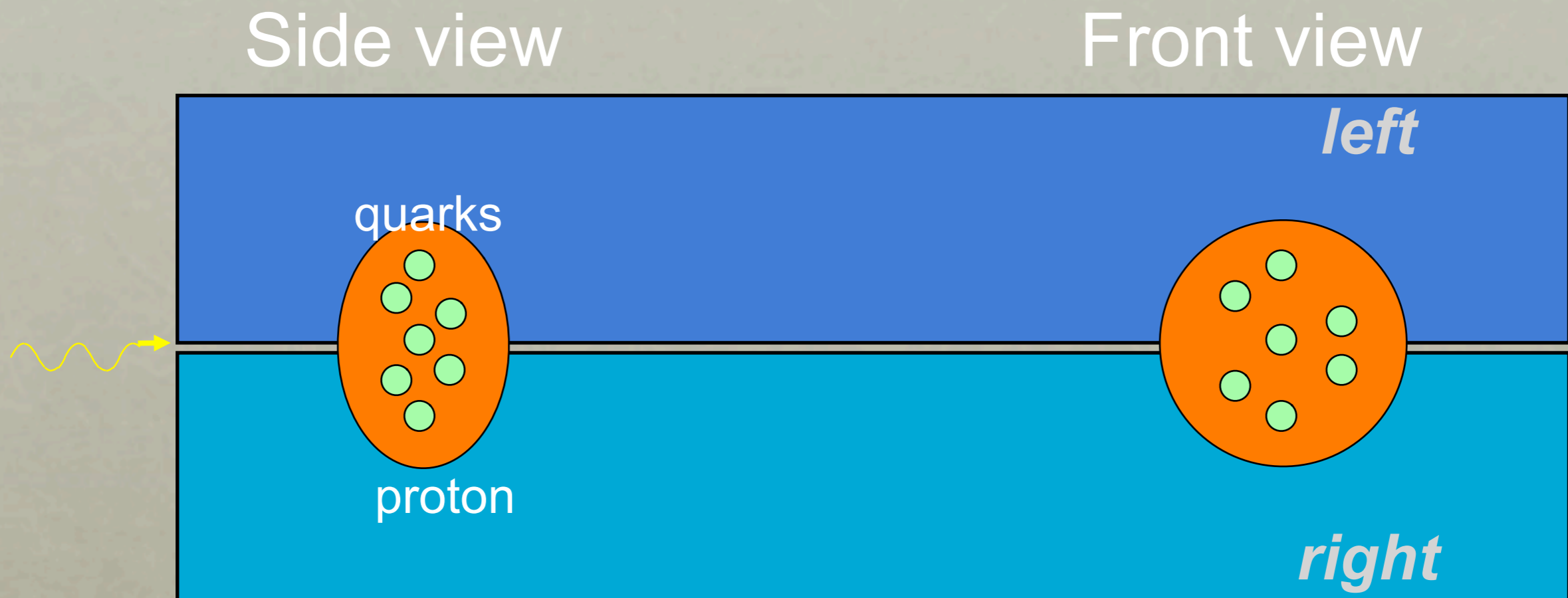
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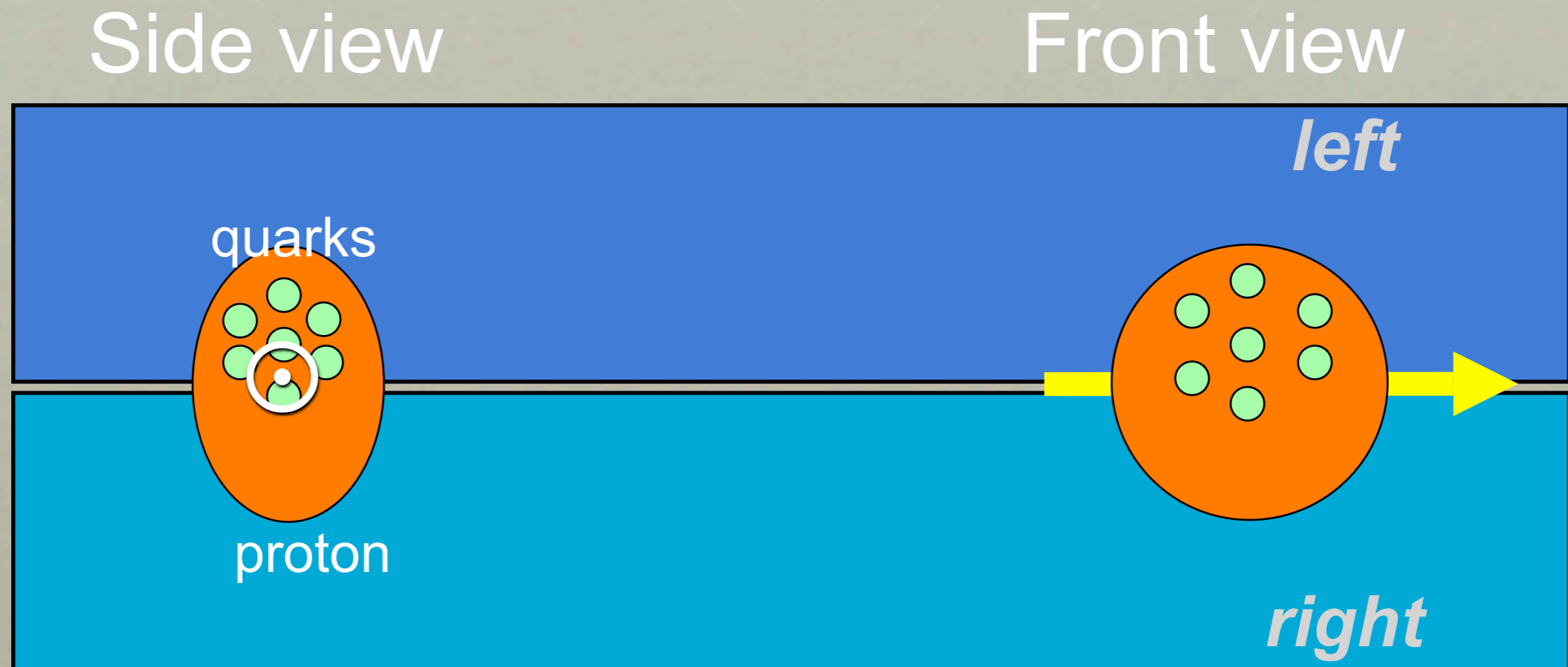
Clear-cut prediction of QCD

Collins, PLB 536 (02)

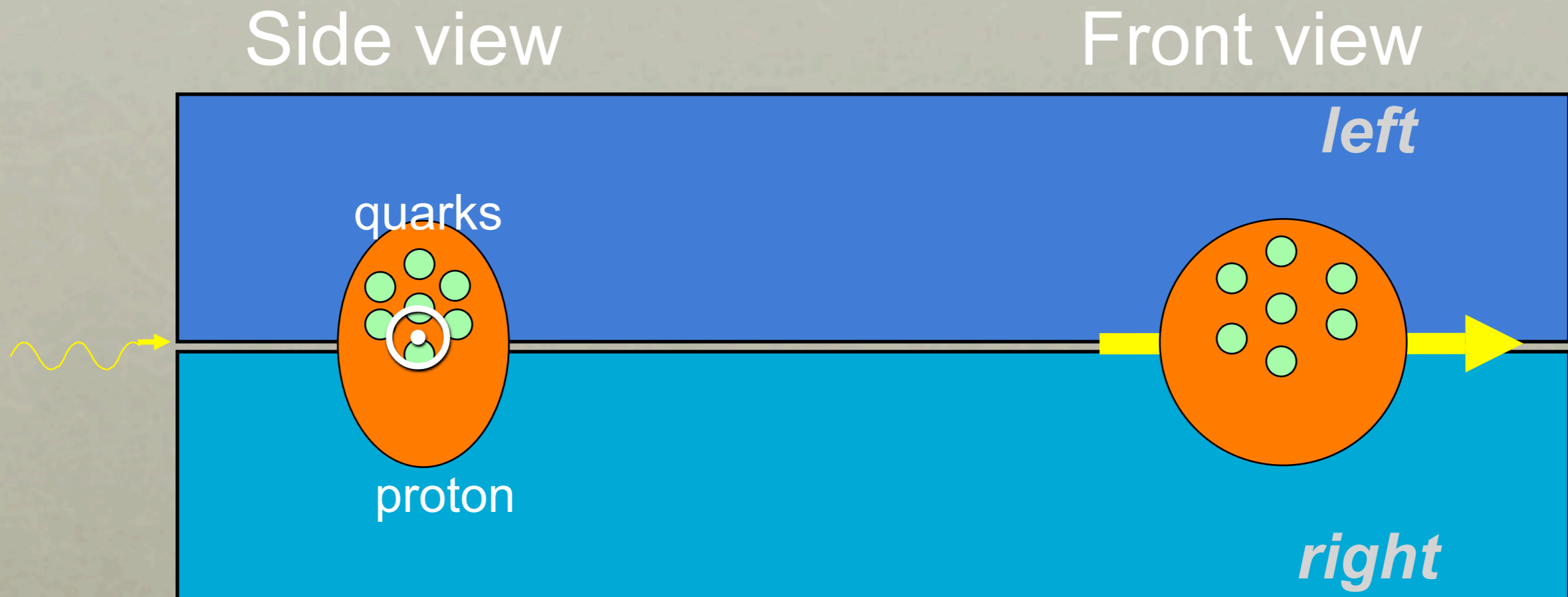
Distortions in transverse space



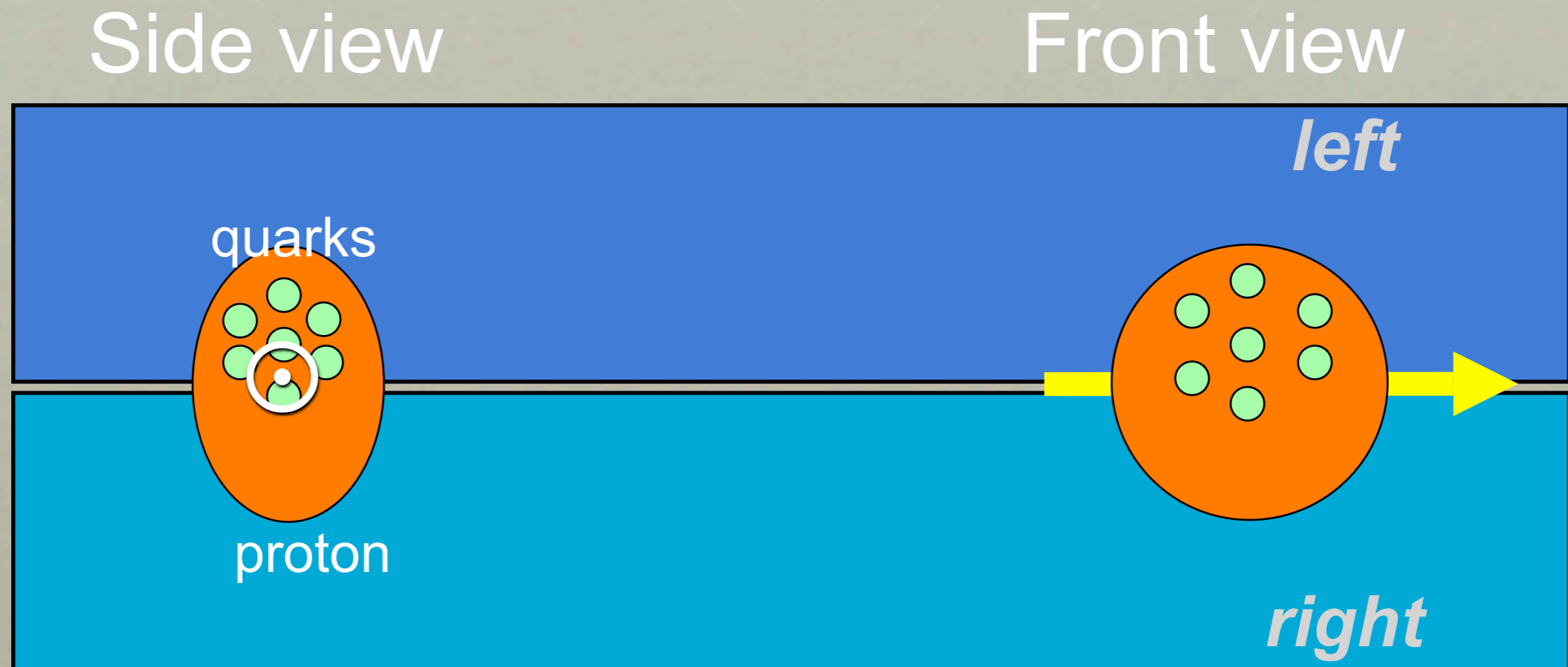
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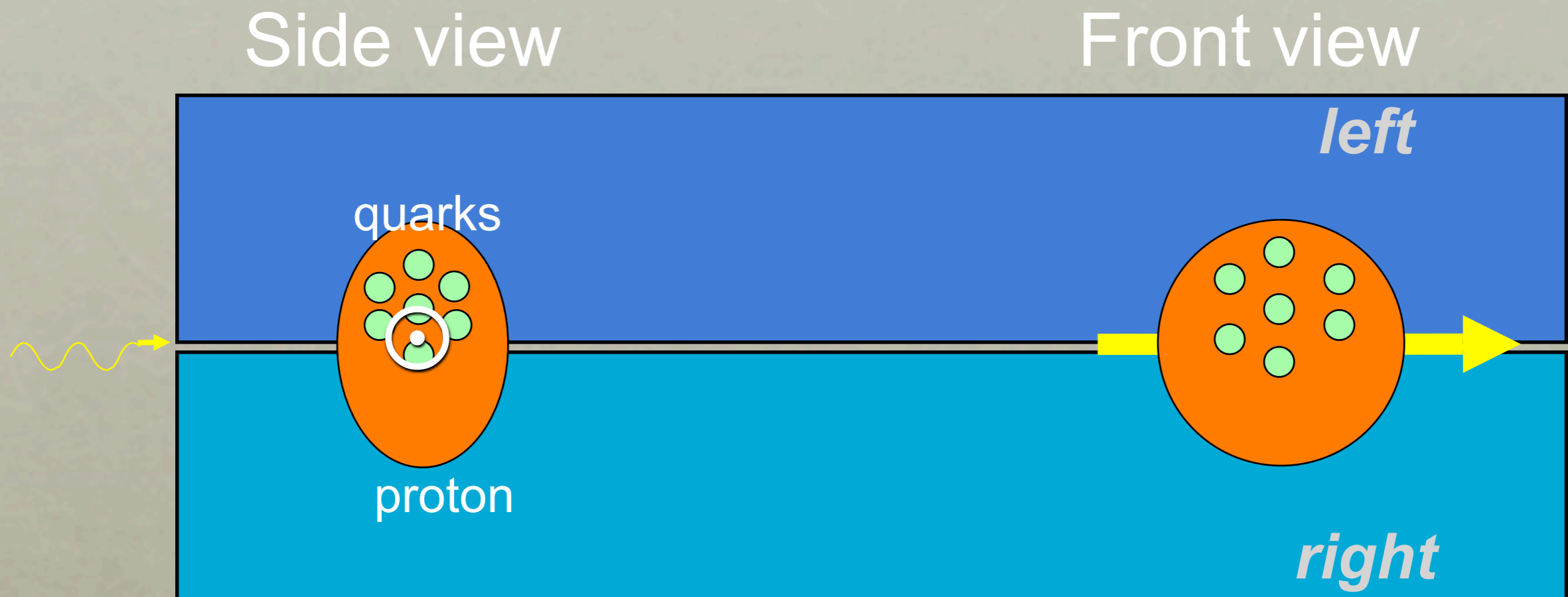
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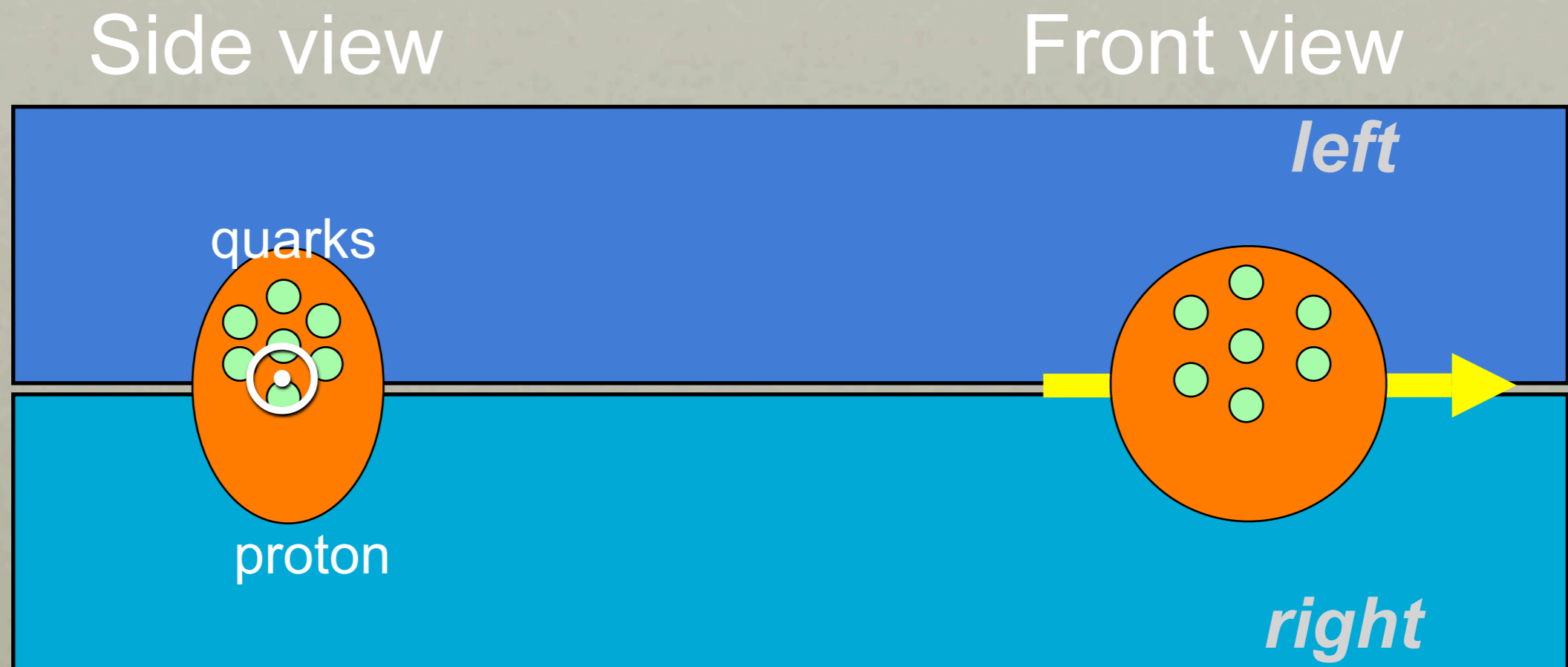


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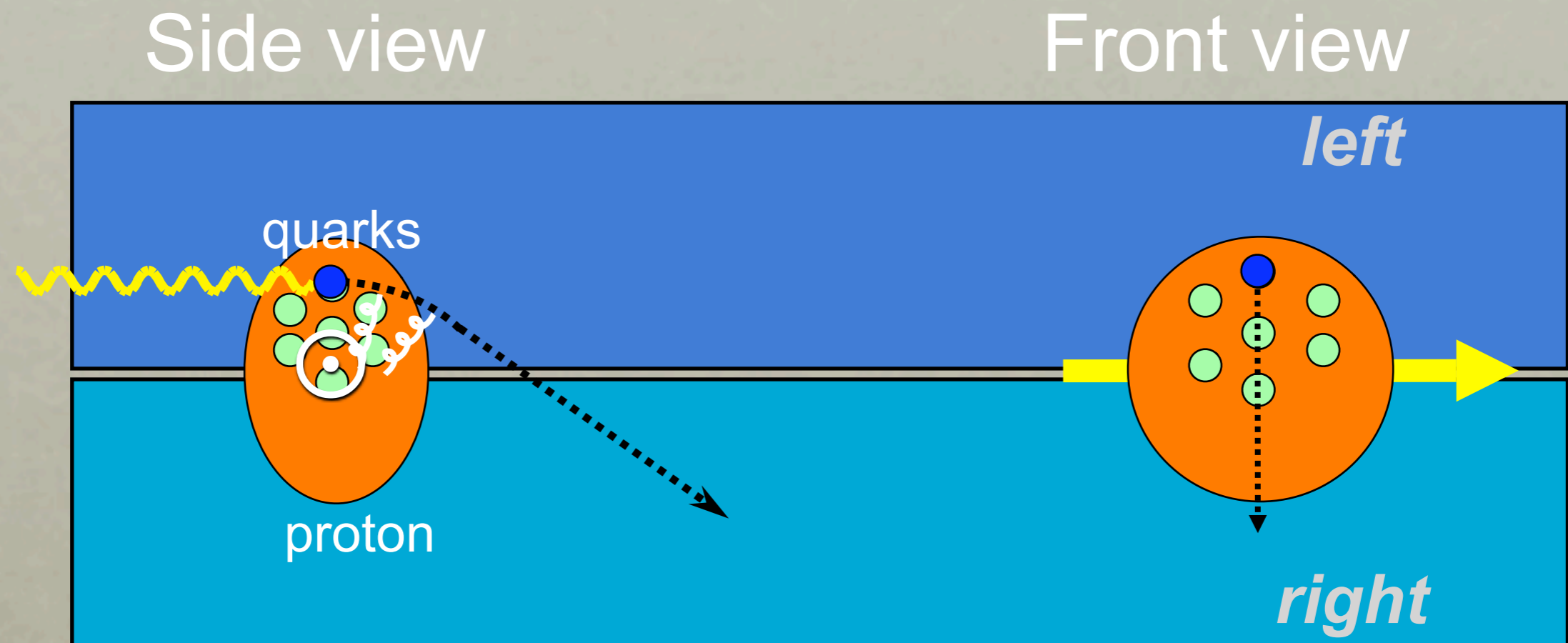
The presence of spin can distort the distribution of quarks in transverse space (orbital angular momentum of quarks is required)

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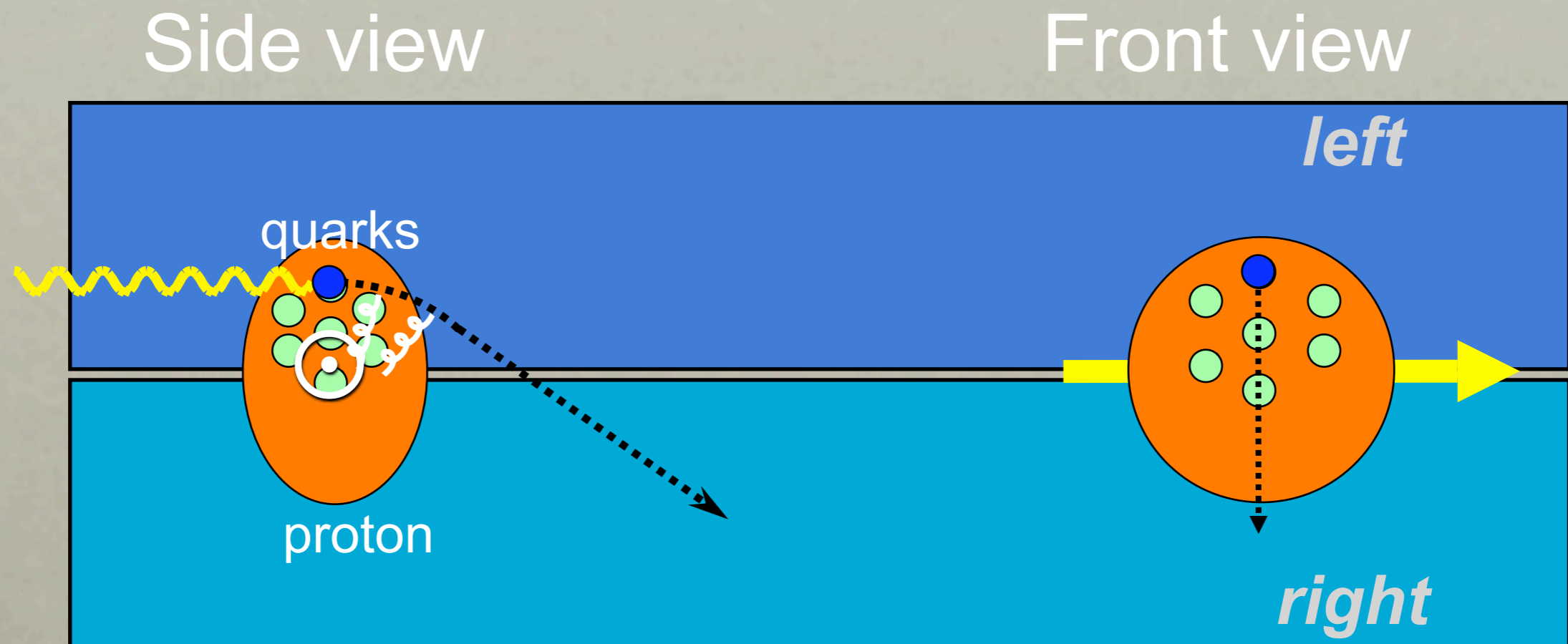
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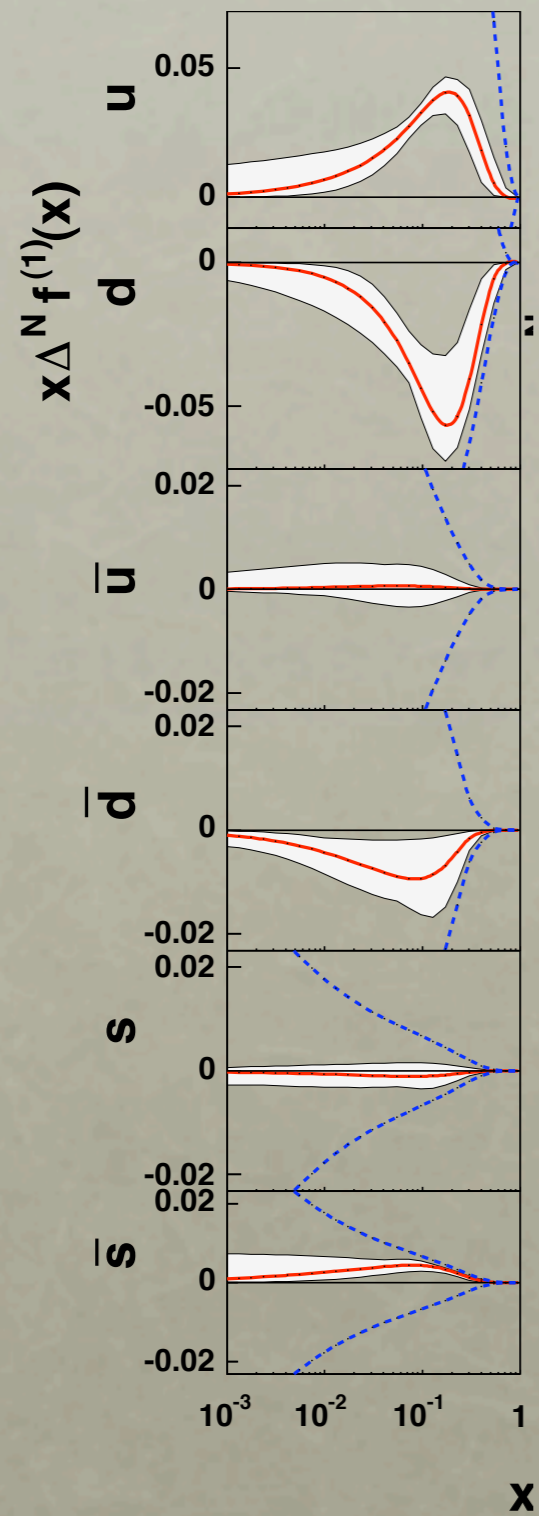
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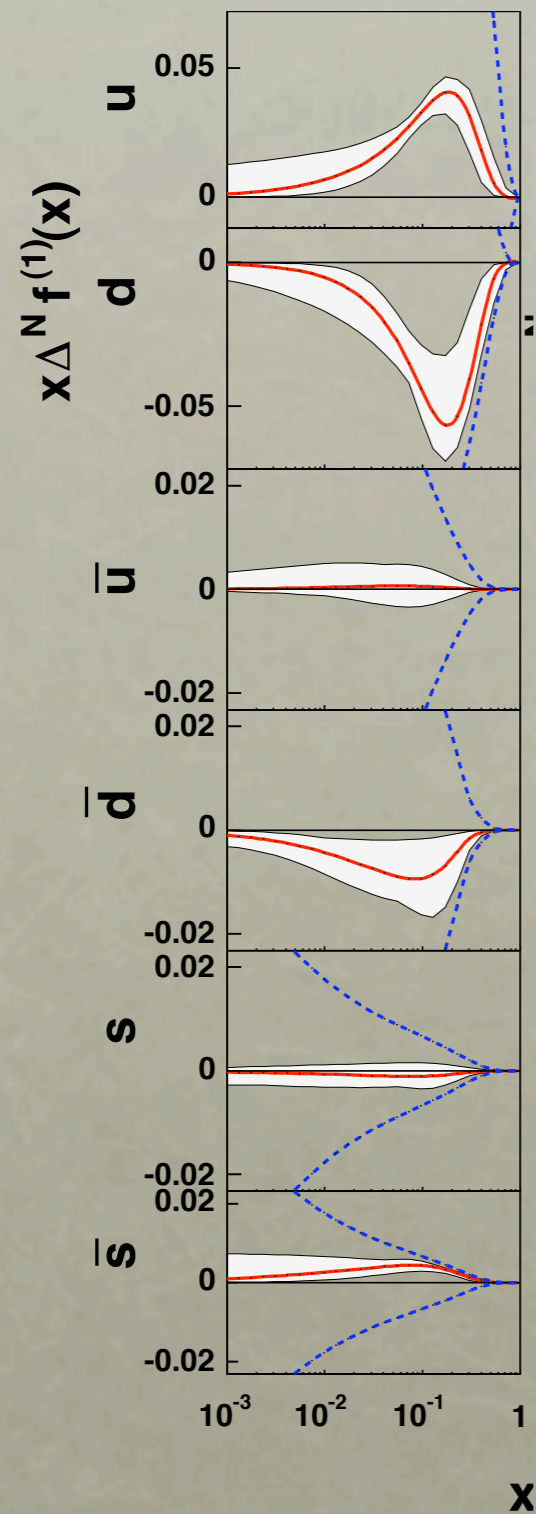
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A distortion in the distribution of quarks in transverse space can give rise to a nonzero Sivers function

Sivers function extraction

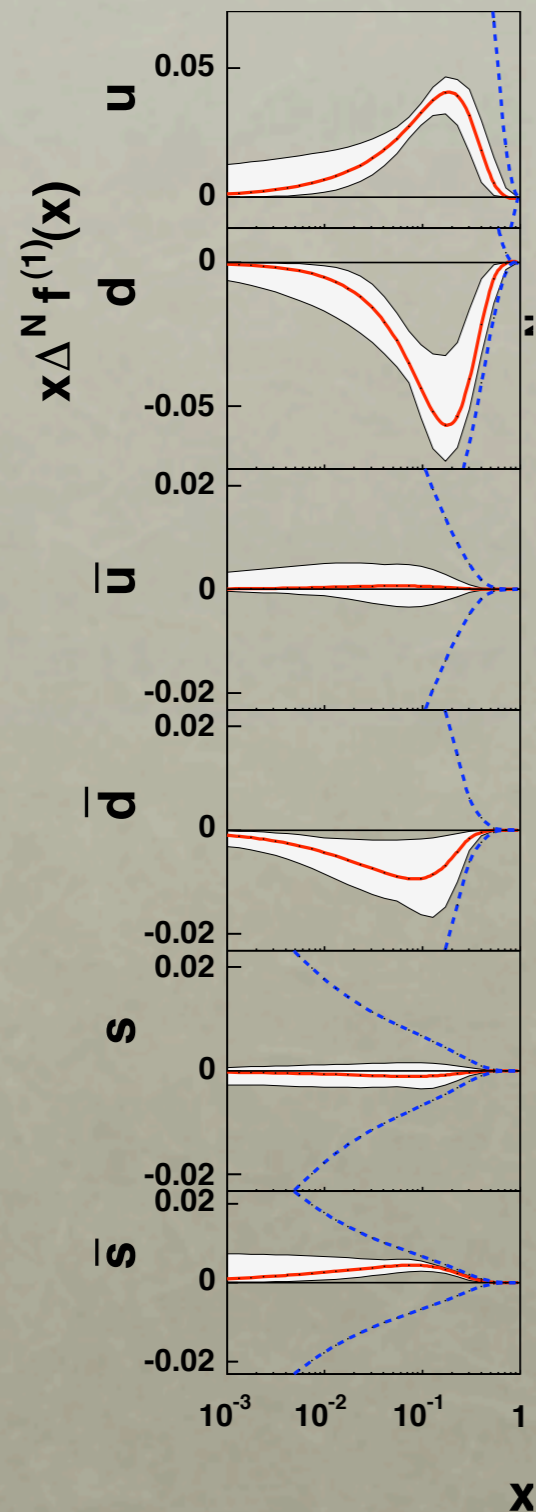


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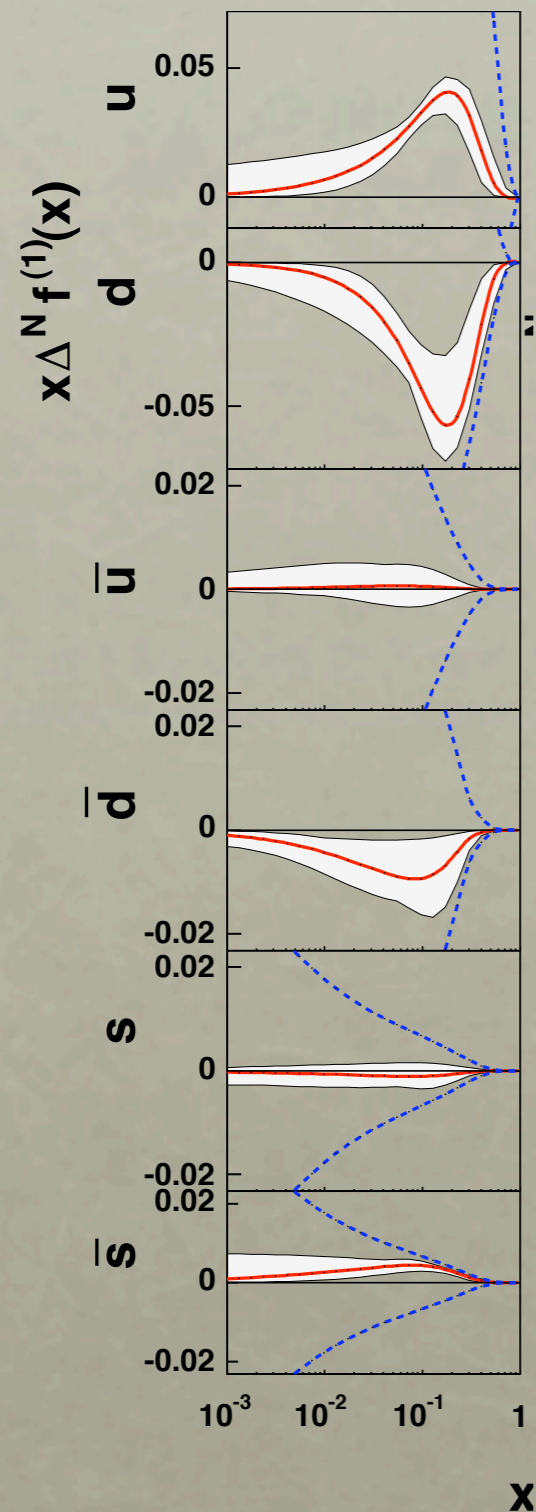
- Data from HERMES, COMPASS (deuteron)

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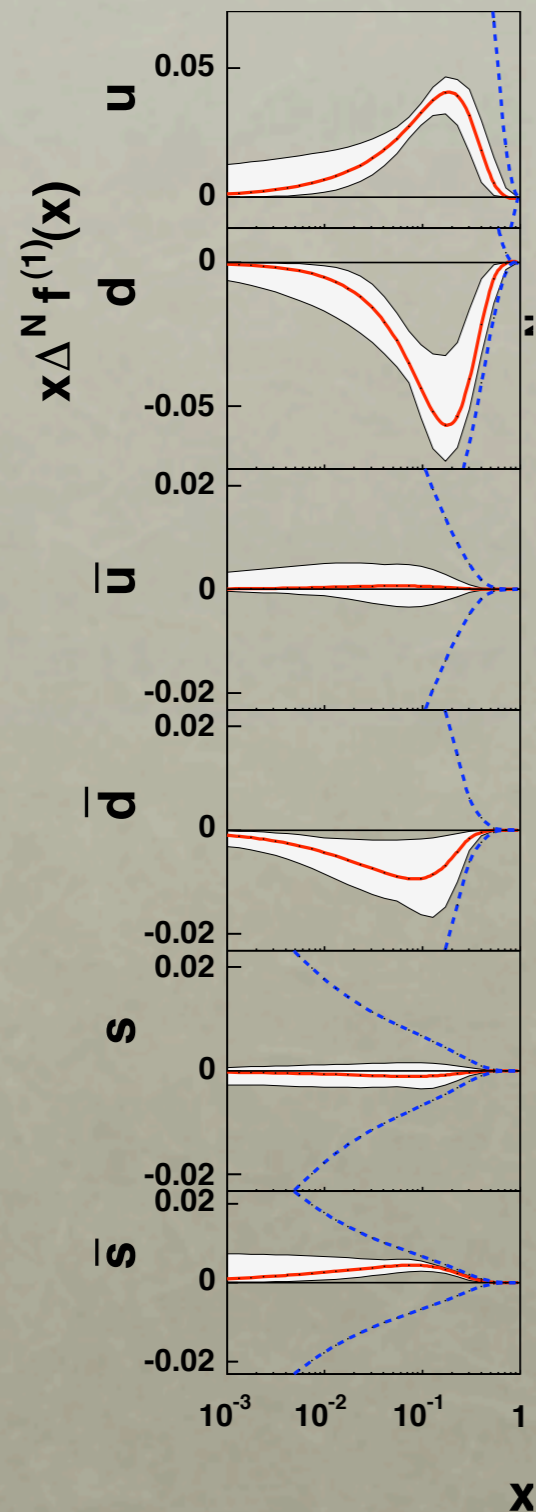
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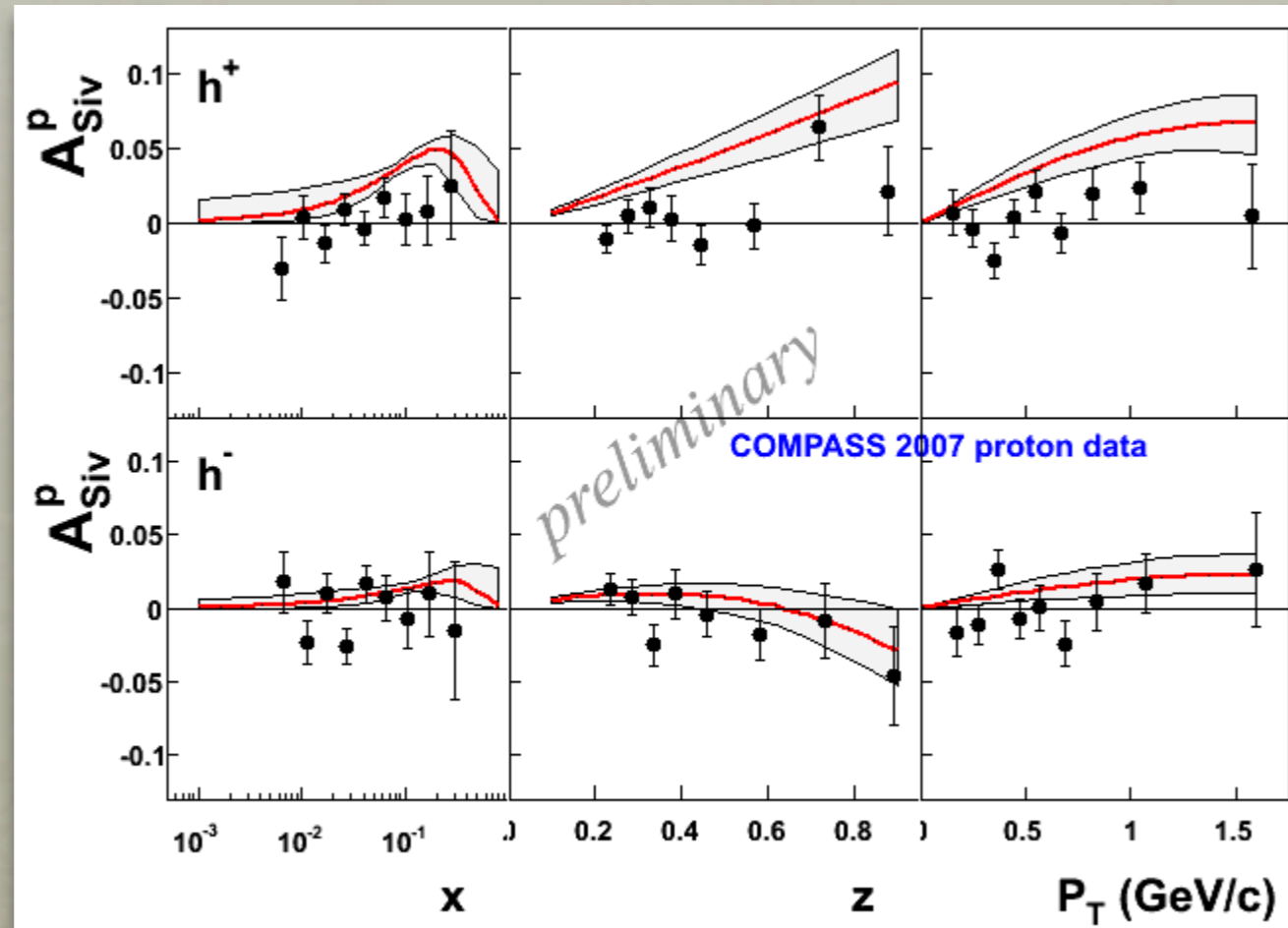
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- Statistical uncertainty only ($\Delta\chi^2 \approx 17$)

Sivers function: COMPASS



data: S. Levorato, Transversity 08
prediction: Anselmino et al., 0805.2677

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